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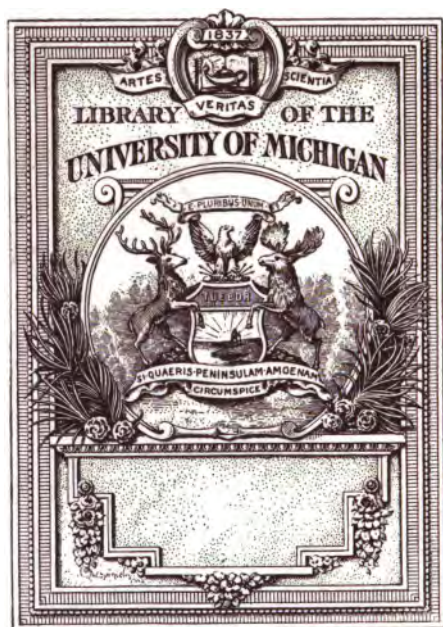
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# COMMERCIAL ARITHMETIC.



# COMMERCIAL ARITHMETIC

WITH ANSWERS.

*Samuel*  
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BY

FORMERLY POSTMASTER OF MERTON COLLEGE, OXFORD.

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## AUTHOR'S PREFACE.

IN this book it is assumed that the reader has a competent knowledge of elementary arithmetic and therefore the theoretical portions of the work are limited to the methods which are best adapted for commercial calculations. These methods are discussed chiefly in §§ I. and VI., though isolated examples occur elsewhere. The greater part of the book is devoted to the actual operations of business.

revised 15 A 26 I. B.

An endeavour has been made to give full and accurate information on all commercial subjects of first-rate importance, including British and Metric Weights and Measures, The Pricing of Goods, The Prices and Standards of Trades, Stocks and Shares, Banking, Bills of Exchange, the London Money Market, the Foreign Exchanges, etc.

Certain methods of readily saving labour have also been suggested, involving only the gradual construction of small and convenient Tables in the Counting-house itself.

The immense superiority of decimal operations over all others, especially in the application of methods of

approximation, has led me incidentally to show the advantages of a decimal system of coins and measures, with particular reference to what may be called *Mil-Metric Arithmetic*.

The adoption of the Metric System and a Decimal Coinage would simplify Arithmetic enormously, and I am hoping that the new method of decimalising money at sight adopted in this book, and the treatment of many other questions, will show the student that we have a coinage ready made to our hands, with the radical coins as the £, florin, cent, and mil (each being  $\frac{1}{10}$  of the preceding); in other words money expressed in £ and decimals of a £ to three places.

The great practical advantage of this coinage is that the sovereign is untouched. Any proposal with the penny as basis necessitates a world-wide revolution, whereas our immense foreign trade would be unaffected as long as the sovereign remains the integer.

It is true that our internal trade would be to some extent disturbed by a change in the smaller coins—which would not occur if the penny was made the basis; but this disturbance would be nothing in comparison with that caused by altering the sovereign.

The Methods of Approximation and their Developments, the Methods of Prediction, have been systematically employed throughout the book, and though care and practice are required to prevent errors, not only is there an immense saving of time and labour in their use, but they are the only scientific methods of ensuring absolute accuracy in intricate operations. It has been

the custom in Engineering Text-books, etc. to accept certain approximations as practically absolute, thus commencing with errors, the precise effect of which cannot altogether be foreseen. The Methods of Prediction show clearly the limits of error at almost every stage of the process, and produce results accurate to the exact degree required.

The information in the book is a prominent feature, and every endeavour has been made to secure accuracy. My obligations to Tate's *Modern Cambist* will be recognised as very great by those who know that standard work, but many of the facts have been verified or corrected by personal informants. The Danish Ambassador (M. de Bille) with great courtesy wrote me an exact statement of the condition of the Danish weights and measures, and many London and Provincial Bankers, Stock-brokers, Marine Insurance Agents, Merchants and Brokers have kindly volunteered aid. I wish specially to acknowledge the kindness of Sir Samuel Montagu, M.P., the well-known financier, in criticising and improving the Section on Exchanges and Exchange operations—one of the longest and most important chapters in the book.

From other written sources I have naturally derived great help. De Morgan's Methods of Approximation and their Developments have been employed throughout the book. The method of stating a Proportion sum like a pair of scales is derived from Mr. Frusher Howard's *Art of Reckoning*, and the Method of Linkages is a modified form of that given in D. O'Gorman's

*Compendious Calculations.* Mr. Moxon's monograph on Banking practice has been of great use in the Exchange Section.

In conclusion I have sincere pleasure in gratefully acknowledging the encouragement and kindness I have received from Dr. Gow in a somewhat arduous task.

S. JACKSON.



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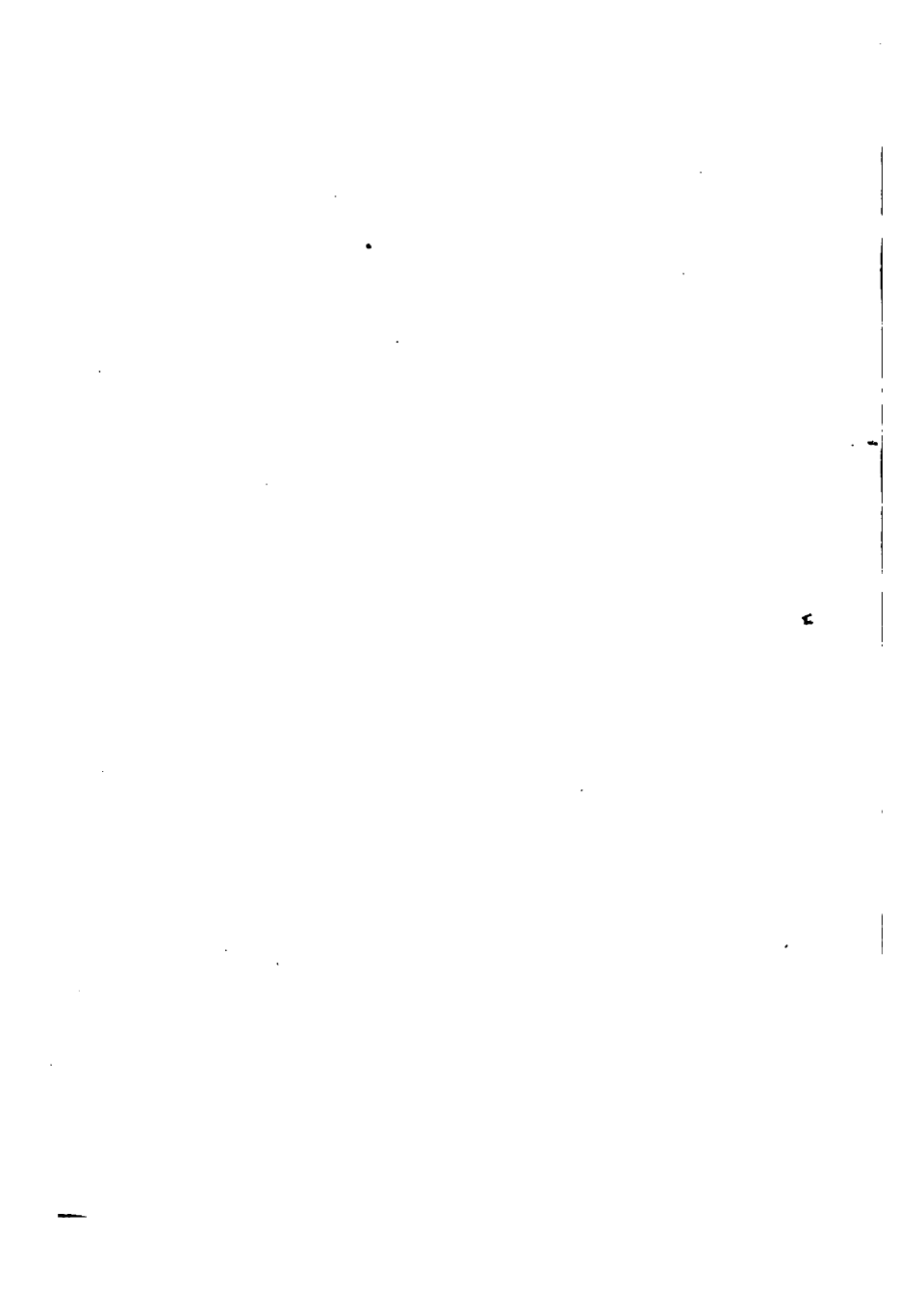
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## SECTION I.

### ARITHMETICAL OPERATIONS.

#### 1. Mental or Very Brief Rules in Multiplication and Division.

- (1) To multiply by a number entirely of nines.

Add to multiplicand as many 0's as there are 9's.  
Subtract the multiplicand.

- (2) To multiply by a number entirely of nines except in the unit-place.

Add to multiplicand as many 0's as there are digits.  
Subtract the multiplicand  $\times$  (10 - unit-figure).

e.g. 999997—add six 0's and subtract  $3 \times$  multiplicand.

- (3) To multiply by any power of 5.

Add as many 0's to multiplicand as the given power.  
Divide by same power of 2.

e.g.  $625 = 5^4$ —add four 0's and divide by 16 ( $2^4$ ).

- (4) To divide by any power of 5.

Multiply dividend by same power of 2.  
Mark off as many places as the power given.

e.g.  $3125 = 5^5$ . Multiply by 32 ( $2^5$ ), mark off 5 places.

- (5) To multiply by 11 in one line.

Add each digit of multiplicand in succession to its immediate left-hand adjacent digit, carrying when necessary and putting the unit-digit to the right and the final digit (plus remainder if any) to the left.

e.g.  $65178 \times 11 = 716958$ .

**EXAMPLES.**

1. Multiply 8693157 by 9996, 9987, 991.
2. " 739645 by 625, 3125, 250.
3. Divide 96783452 by 125, 25, 3125.
4. Multiply 876596 by 11, 121, 1331.
5. Divide 864935 by 625, 1250, 2500.

**2. To multiply in one line in the case of a two-figure multiplier.**

*Example.*  $83561 \times 37$ .

Ans. = 3091757.

Mental Process

$$\begin{array}{rcl}
 7 \times 1 & = & 7 \\
 7 \times 6 & = & 42 + 3 \times 1 = 45 \\
 4 + 7 \times 5 & = & 39 + 3 \times 6 = 57 \\
 5 + 7 \times 3 & = & 26 + 3 \times 5 = 41 \\
 4 + 7 \times 8 & = & 60 + 3 \times 3 = 69 \\
 & & 6 + 3 \times 8 = 30 \\
 & & 3 = 3
 \end{array}$$

This method may be extended to three-figure multipliers, but it is specially useful for those of two figures.

**EXAMPLES.**

Multiply in one line:

1. 65372489 by 73, 45, 84.
2. 8371656 by 85, 94, 19.
3. 5030512 by 47, 39, 26.
4. 3125671 by 24, 37, 71.
5. 4123874 by 85, 123, 304.

**3. Involution.**

In squaring and cubing numbers the following Algebraic principles are very useful:—

(1) The square of the sum of two numbers is equal to the sum of the squares of the numbers + twice the product.

(2) The square of the difference of two numbers is equal to the sum of the squares of the numbers — twice the product.

(3) The cube of the sum of two numbers is equal to the sum of the cubes of the numbers +  $3 \times$  product  $\times$  sum of the numbers.

(4) The cube of the difference of two numbers is equal to the difference of the cubes —  $3 \times$  product  $\times$  difference of the numbers.

(5) The difference of the squares of two numbers is equal to the product of the sum and difference of the numbers.

- Examples.*
1.  $305^2 = (300 + 5)^2 = 300^2 + 25 + 10 \times 300$   
 $= 90000 + 25 + 3000 = 93025.$
  2.  $493^2 = (500 - 7)^2 = 500^2 + 7^2 - 2 \cdot 500 \cdot 7$   
 $= 250000 + 49 - 7000 = 243049.$
  3.  $26^3 = (20 + 6)^3 = 20^3 + 6^3 + 3 \cdot 20 \cdot 26$   
 $= 8000 + 216 + 9360 = 17576.$
  4.  $39^3 = (40 - 1)^3 = 40^3 - 1^3 - 3 \cdot 40 \cdot 39$   
 $= 68999 - 4680 = 59319.$
  5.  $39^2 - 25^2 = 64 \times 14 = 896.$

### EXAMPLES.

1. Square 631, 8007, 9013, 15012, 848.
2. „ 739, 891, 747, 9995, 761.
3. Cube 39, 410, 937, 97, 105.
4. „ 95, 38, 729, 409, 521.
5. Find value of  $81^2 - 29^2$ ,  $375^2 - 325^2$ ,  $96^2 - 84^2$ ,  $720^2 - 711^2$ ,  $312^2 - 305^2$ .

### 4. Abbreviation in Multiplication.

This arises when certain digits or sets of digits are multiples of other digits or sets of digits following or preceding them.

Much labour is also thrown away in writing down lines twice where by a judicious arrangement of the work the repetition might be avoided.

The two principles following are of great use with easily rememberable multipliers.

1°. When 1 occurs among the digits, write down the multiplicand adding 0's according to the place of the 1. Then multiply by the remaining digits using the original multiplicand as it stands in the line written down.

$$\begin{array}{r} \text{Example 1. } 8657 \times 103. \\ 865700 \\ 25971 \\ \hline 891671 \end{array}$$

$$\begin{array}{r} \text{Example 2. } 753106 \times 3105. \\ 75310600 \\ 2259318 \\ 3765530 \\ \hline 2338394130 \end{array}$$

2°. Factors among the digits should be constantly noticed and made use of to shorten the number of lines or lessen the multipliers.

$$\begin{array}{r} \text{Example 1. } 9673 \times 315 \equiv 300 + 5 \times 3. \\ 2901900 \\ 145095 \\ \hline 3046995 \end{array}$$

$$\begin{array}{r} \text{Example 2. } 83561 \times 369 \equiv 9 + 40 \times 9. \\ 752049 \\ 3008196 \\ \hline 30834009 \end{array}$$

$$\begin{array}{r} \text{Example 3. } 5612 \times 852 \equiv 840 + 12 \equiv 12 + 70 \times 12. \\ 67344 \\ 471408 \\ \hline 4781424 \end{array}$$

The cases in which one or other or both of these principles may be used are literally without number.

A decimal point does not hinder their application.

### EXAMPLES.

- |                            |                            |
|----------------------------|----------------------------|
| 1. 8345712 $\times$ 3106.  | 2. 78934 $\times$ 1768.    |
| 3. 936412 $\times$ 78013.  | 4. 1973256 $\times$ 832.   |
| 5. 3912451 $\times$ 96412. | 6. 9178 $\times$ 2136.     |
| 7. 5614273 $\times$ 13758. | 8. 8176541 $\times$ 42731. |
| 9. 378964 $\times$ 65131.  | 10. 129875 $\times$ 8124.  |

### 5. Abbreviation in Division (the Italian method).

A great saving of labour and time is effected by subtracting in all operations of Division as you proceed. This principle admits of very wide application and will be constantly used throughout this book. Its main applications are to Long Division (Simple and Compound), Division of Decimals, Greatest Common Measure, and Division-approximation.

#### 1°. Long Division.

Draw a line under the figures required for the first step in division.

Multiply the divisor by the first figure of the quotient, *subtracting as you proceed*.

Bring down the next figure and draw a line as before.

Repeat the same process of multiplication and immediate subtraction with the second figure of the quotient and so on.

*Example.*  $7983204 \div 1875$ .

1875 | 7983204 | 4257

4832  
10820  
14454

1329 rem.

Process for First Subtraction.

	(a)	(b)	(c)
$4 \times 5 = 20 +$	3	= 2	3
$4 \times 7 = 28 + 2 = 30 +$	8	= 3	8
$4 \times 8 = 32 + 3 = 35 +$	4	= 3	9
$4 \times 1 = 4 + 3 = 7 +$		=	7

Thus practically the figure necessary to make up the figure above it is put down and the resulting tens-figure is added to the next multiplication.

(a) gives the differences to be put down,

(b) gives the figures to be carried,

(c) shows the correctness of the subtraction.

Similarly for any other line.

#### 2°. Compound Division (Money, Weights, Measures).

It is obvious that this principle may be applied and the above method used in all the actual dividing required in Compound Division of any sort.

*Example 1.* £891. 11s. 4½d. ÷ 73.

£.	s.	d.	
73		891	11 4½   £12.
		<u>161</u>	
		15	
		20	
		<u>311</u>	4s.
		19	
		12	
		<u>232</u>	3d.
		13	
		4	
		<u>54</u>	

*Example 2.* 361 days 11 hrs. 36 mins. ÷ 81.

d.	h.	m.	
81		361	11 36   4 d.
		<u>37</u>	
		24	
		<u>899</u>	11 h.
		89	
		8	
		<u>60</u>	6 m.
		516	
		<u>30</u>	
		60	
		<u>1800</u>	22 s.
		180	
		<u>18</u>	

### 3°. Division of Decimals.

This method is plainly applicable to Decimals and should always be used.

*Example.* 8765·432 ÷ 123·4.

1234		87654·32		71·03
		<u>1274</u>		
		4032		
		<u>330</u>		

The Decimalisation of Money, Weights, or Measures reduces Compound Division to Division of Decimals.

### 4°. Greatest Common Measure.

An application of this method renders the finding of G.C.M. surprisingly brief.

*Example.* G.C.M. of 33495, 106260.

5		33495		106260		3
4		4620		.. 5775		1
		....		1155		G.C.M.

Process.	Divide 33495 into 106260.	Quotient 3.	Remainder 5775
"	5775 " 33495.	" 5.	" 4620
"	4620 " 5775.	" 1.	" 1155
"	1155 " 4620.	" 4.	" 0
∴	G.C.M. is 1155.		

**EXAMPLES.**

- |  |   |
|--|---|
| 1. $865734968 \div 3714$ .   | 2. $17345834651 \div 87653$ .                 |
| 3. $98534756385 \div 47351$ .                                      | 4. $876407653126 \div 374582$ .               |
| 5. $93102356321 \div 7125$ .                                       | 6. $3567025831 \div 871$ .                    |
| 7. $9567435612 \div 931524$ .                                      | 8. $51638451784 \div 36195$ .                 |
| 9. $16573481297 \div 81756$ .                                      | 10. $731256851 \div 96145$ .                  |
| 11. £891. 5s. $6\frac{1}{4}d. \div 732$ .                          | 12. £73156. 11s. $4\frac{1}{2}d. \div 8154$ . |
| 13. £7256. 9s. $7\frac{3}{4}d. \div 4239$ .                        | 14. £631. 11s. $5\frac{1}{2}d. \div 81$ .     |
| 15. £7316. 5s. $3\frac{3}{4}d. \div 654$ .                         | 16. £351. 11s. $9\frac{1}{2}d. \div 716$ .    |
| 17. £95. 3s. $1\frac{1}{4}d. \div 403$ .                           | 18. £931. 17s. $6\frac{1}{2}d. \div 814$ .    |
| 19. £75312. 16s. $8\frac{3}{4}d. \div 8137$ .                      | 20. £516. 8s. $9\frac{1}{4}d. \div 64$ .      |
| 21. $865312805 \div 0625$ .  | 22. $7345136 \div 62314$ .                    |
| 23. $8768314 \div 609315$ .  | 24. $731058 \div 00567$ .                     |
| 25. $006785 \div 61532$ .  | 26. $8300715 \div 315814$ .                   |
| 27. $1956210 \div 83514$ .   | 28. $00004561 \div 00375$ .                   |
| 29. $00031 \div 00006$ .   | 30. $1657312 \div 81041$ .                    |
| 31. G.C.M. of 86534, 9106.   | 32. G.C.M. of 75614, 89312.                   |
| 33. „ 69751, 6349.   | 34. „ 18905, 7300.                            |
| 35. „ 743856, 9318.  | 36. „ 74165825, 93128625.                     |
| 37. „ 94378, 612436.   | 38. „ 19500618, 8954.                         |
| 39. „ 73256, 6303.   | 40. „ 856, 94732.                             |
| 41. £75. 8s. $9\frac{1}{4}d.$ (in decimals) $\div 19$ to 3 places. |   |
| 42. £861. 13s. $4\frac{1}{2}d.$ „ $\div 321$ „                     |   |
| 43. £7356. 9s. $11\frac{3}{4}d.$ „ $\div 709$ „                    |   |
| 44. £8340. 11s. $4\frac{1}{4}d.$ „ $\div 3131$ „                   |   |
| 45. £99. 10s. $4\frac{3}{4}d.$ „ $\div 834$ „                      |   |
| 46. £739. 9s. $4\frac{1}{4}d.$ „ $\div 16$ „                       |   |
| 47. £810. 8s. $7\frac{1}{2}d.$ „ $\div 1000$ „                     |   |
| 48. £537561. 10s. $11\frac{1}{2}d.$ „ $\div 851$ „                 |   |
| 49. £9007. 17s. $5\frac{3}{4}d.$ „ $\div 7014$ „                   |   |
| 50. £10000. 0s. $0d.$ „ $\div 347$ „                               |   |

**6. The Decimalisation of Money.**

The conversion of *s. d. f.* into decimals of a £ often greatly simplifies many calculations—more especially it renders of much greater service the Methods of Immediate Subtraction and Approximation.

In general it is sufficient to get any sum of money in decimals of a £ correct to 3 places, but it must be remembered that these places must be correct in the final answer. Hence it is often necessary to find the exact decimal of a £.

The methods given here will enable the student after a little practice to decimalise at sight.

**1°. To convert £. s. d. f. into £ and decimals of a £.**

(1) To three places.

Multiply the shillings by 5.

Put down the tens-figure of the product as 1st dec. place.

Reduce *d. f.* to farthings, adding 1 if pence are 6*d.* or over.

Add unit-figure of shillings-product to tens-figure of the farthings found and put down sum as 2nd dec. place.

Put unit-figure of farthings as 3rd dec. place.

(2) Completely.

Obtain the first three places as in (1).

To get the fourth

Multiply 2nd by 4, carrying from the 3rd  $\times 4$ .

Put unit-figure of this result (mentally) in the 4th place to see if its multiplication by 4 will have any effect on the carrying from the 3rd  $\times 4$  to get the true 4th place.

Put down the unit-figure (thus corrected or not) as 4th place.

Repeat this process with the 3rd and 4th to get the 5th, and then with the 4th and 5th to get the 6th and so on.

(Always carefully making the above correction when necessary.)

*Note 1.* This correction can only occur when the last figure obtained (the figure on which the carrying depends) is 2 or 7. E.g. it can only occur in getting the 4th when the 3rd is 2 or 7, in getting the 6th when the 5th is 2 or 7.



It does not always occur even then but it must be tried in these cases.

If 2 or 7 does not occur at any stage this correction need not be considered.

*Note 2.* This process brings 4999 . . . . and therefore, practically, when 48 occurs or 49 after correction, change into 5 and end the decimal.

*Note 3.* Observe—the 1st dec. place is left untouched.

*Note 4.* If it is desirable to stop at any place and make allowance for the next it is easy to calculate it as above, and if it is found to be 5 or over add 1 to the place stopped at.

*Note 5.* The operation of completing the Decimal given in (2) will be referred to as *the method of Fours* and the whole process of Decimalisation as *the method of Five and Four*.

*Note 6.* The student will very quickly know when the repeating figure is reached—the forms which always occur are 416, 83, 916, or 5 (without recurrence).

*Example 1.* £65. 14s. 8½d. = £65·735.

$$\begin{array}{r} \text{Mental process } 5 \times 14 = 70 \\ 4 \times 8\frac{1}{2} + 1 \text{ (above 6d.)} = 35 \\ \hline 735 \end{array}$$

*Example 2.* £17. 11s. 3¾d. = £17·565.

$$\begin{array}{r} \text{Mental process } 5 \times 11 = 55 \\ 4 \times 3\frac{3}{4} + 0 \text{ (below 6d.)} = 15 \\ \hline 565 \end{array}$$

*Example 3.* £86. 19s. 1¼d. = 86·9|5'5|2|0|8|3.

Mental process  $19 \times 5 = 9(5)$

Add  $1\frac{1}{4} \times 4 = 955$

$$4 \times 5 = 20 + 2 = 22 = 9552$$

(2 rejected: 2 carried from  $4 \times 5$ —3rd place)

$$4 \times 5 = 20 + 0 = 20 = 95520$$

(2 rejected: 0 to carry if no correction needed)

[Here  $4 \times 2 = 8$  ∴ put (mentally) 0 in 5th place and see its effect on  $2 \times 4$ .

This effect is of course nothing as  $4 \times 0 = 0$  ∴ no correction is needed.]

$$4 \times 2 = 8 + 0 = 8 = 955208$$

(0 carried from  $4 \times 0$ )

$$4 \times 0 = 0 + 3 = 3 = 9552083$$

(3 carried from  $4 \times 8$ )

$$4 \times 8 = 32 + 1 = 33 = 95520833$$

(3 rejected: 1 carried from  $4 \times 3$ )

and so on, the 3 obviously repeating.

*Example 4.* £183. 15s. 4½d. = £183·7697916.

Mental process $15 \times 5 = 75$	put down 7 for 1st place
$4\frac{1}{2} \times 4 = 19$	put down 6 for 2nd place
69	and 9 for 3rd place
$4 \times 9 = 36$ : carry the 3 to $4 \times 6 = 24 + 3 = 27$	put down 7 for 4th place
$4 \times 7 = 28$ : carry the 2 to $4 \times 9 = 36 + 2 = 38$	
Put 8 mentally in 5th place, then	
$4 \times 8 = 32$ : carry the 3 to $4 \times 7 = 28 + 3 = 31$	} correction
so 3 should be carried instead of 2—thus for 8	
$4 \times 9 = 36$ : carry the 3 to $4 \times 7 = 28 + 3 = 31$	put down 9 for 5th place
$4 \times 9 = 36$ : carry the 3 to $4 \times 7 = 28 + 3 = 31$	put down 1 for 6th place
$4 \times 1 = 4$ : carry 0 to $4 \times 9 = 36$	put down 6 for 7th place

and so on, the 6 repeating.

## 2°. To convert a decimal into £. s. d. f.

(1) If three places only are known.

Divide first two places by 5—quotient is shillings.

Divide remainder with 3rd place brought down by 4—quotient is pence.

Remainder (diminished by 1 if pence are 6d. or over) is farthings.

These operations *may* give results exceeding the actual value by a fraction of a farthing. To decide whether a farthing should be struck off, the fourth place is necessary.

(2) If more than three places are known.

Proceed as in (1) until the farthings-remainder is reached.

Then calculate (mentally) a fourth place from the given 2nd and 3rd by the method of Fours.

If this exceed the actual fourth place strike off a farthing from the result obtained by (1), otherwise not.

If the two fourth places are equal we must proceed to the 5th absolutely to decide—and so on. (Always using the method of Fours.)

*Example 1.* £73·896 = £73. 17s. 11½d.

Mental process  $89 \div 5 = 17s.$  rem. 4

$46 \div 4 = 11d.$  rem. 2

(above 6d.)  $2 - 1 = \frac{1}{2}$ .

*Example 2.* £73·89614 = £73. 17s. 11½d. as in Ex. 1.

Mental correction—4th place from ·896 = 8, given place is 1

∴ nearest value } below = £73. 17s. 11d.  
to farthings }

**Example 3.** £73·896916 = £73. 17s. 11½d. as in Ex. 1.

**Mental verification**—4th place from '896=8, given place is 9

$$\therefore \text{nearest value to farthings} \left\{ \begin{array}{l} \text{above the actual value} \\ \text{below the actual value} \end{array} \right\} = \pounds 73. 17s. 11\frac{1}{4}d.$$

*Note 1.* By striking off one more farthing than the rule requires in (1) all overcharge is avoided without the trouble of finding the 4th place. In doing this there can only be the loss of  $\frac{1}{4}d.$  to the seller. In most Commercial Transactions correctness to pence is all that is required.

*Note 2.* If 3 places are known with allowance for the 4th—this shows that the 4th place is at least 5—hence we may write the 3rd place as one less and consider 4th place as 5. This will in many cases decide absolutely the value correct to farthings.

**Note 3.** Certain decimals and certain decimals only have exact values in £. s. d. f.; in these cases if sufficient places are given it will be seen that the calculated 4th, 5th etc. places are identical with those given.

*Note 4.* If the decimal to 3 places is exact or is taken to be exact, the rule (1) gives the value *above* the true value to the nearest farthing and the modification in Note 1 gives the value *below* the true value to the nearest farthing. To consider three places as exact is to adopt *mils* as the division of the £.

### 3°. Reduction of Money.

An interesting application of Money-Decimalisation gives a very brief way of reducing money to farthings or pence.

(1) To reduce £. s. d. f. to farthings.

Decimalise the money as usual, to three places.  
Omit the decimal point.

Multiply by 4, commencing two places to the left and carrying from the multiplication of the previous figures what is necessary.

Subtract this from the altered Decimal.

The answer gives the farthings required.

**Example 1.** £87651. 19s. 5½d.

87651'972  
3506 078

**Ans.** 84145 894

*Example 2.* £896. 17s. 11½d.

896'896  
35 875

**Ans.** 861 021

**The subtraction can be done as you proceed.**

Example 3. £6891. 14s. 3½d.

6891'715

Ans. 6616 047

(2) To reduce £. s. d. to pence.

Decimalise to three places.

Omit the Decimal point.

Divide altered Decimal by 4, reject remainder.

Subtract the Integral part only of  $\frac{1}{100}$  of altered Decimal from the quotient.

Answer gives the pence required.

Example. £113. 5s. 8d. to pence.

4 | 113'283

28 320— 3 rejected.

1 132—83 rejected.

Ans. 27 188

Note. The explanation of the accuracy of these approximations is that the Decimal parts are the same.

e.g. £87651. 19s. 5½d.

=

87651972·916 (Altered)

Multiplying by 4 as in the rule

3506078·916

84145894·

Also £113. 5s. 8d.

=

4 | 113283·33

28320·83

1132·83

27188·

#### 4°. Extension of Method to Eighths, Sixteenths, Thirty-two'ths of a Penny.

To decimalise a sum of money including £. s. d. and lower fractions of a penny than a farthing.

Multiply shillings by 5. Put tens-figure of product as first decimal place.

Reduce decimal and fraction to farthings and a fraction of a farthing.

Add tens-figure to unit-figure of shillings-product for 2nd decimal place.

Put unit-figure and fraction (reduced to decimal form) as remaining decimal places.

(Increasing by 1 or 0 the unit-figure as the pence are over 6*d.* or not.)

The decimal arrived at is only provisional and has to be corrected to get the exact decimal—in this manner following:—

The first place is correct absolutely.

Consider the correction due to the 4th place in the manner previously shown for £. s. *d.* *f.*, using 4 to multiply and carrying as before.

Add this to 4th place and change the 3rd again if necessary.

Now see if this new form will make any difference (when multiplied by 4 in the usual way adopted in this method) to the correction applied to 4th place, make any change arrived at and proceed to correct the 5th place by means of the 3rd and 4th in the same way.

Continue the process not only until all the provisional figures are exhausted but until there is an obvious recurrence—remembering that 48 may be written 5 to avoid getting 4999....

*Example 1.* £25. 13*s.* 8½*d.* = £25·684½ = £25·68475 (provisionally).

Correction (Mental Process).

$4 \times 5 = 20$     $4 \times 7 = 28 + 2 = 30$     $4 \times 4 = 16 + 3 = 19$     $4 \times 8 = 32 + 1 = 33$   
 $\therefore$  add 3 to 7 hence decimal now is 25·68505.

Trying again with 4 we see that 4 should have been added.

So decimal should be 25·68515.

Now correct the next place

$4 \times 5 = 20$     $4 \times 1 + 2 = 6$     $4 \times 5 + 0 = 20$     $\therefore$  add 0 to 5  
 and decimal is now correct to 5 places.

Proceeding to the remaining places

$4 \times 5 = 20$     $4 \times 1 + 2 = 6$     $\therefore$  decimal is 25·685156  
 $4 \times 6 = 24$     $4 \times 5 + 2 = 22$     $\therefore$    „   is 25·6851562  
 $4 \times 2 = 8$     $4 \times 6 + 0 = 24$     $\therefore$    „   is 25·68515625.

*Example 2.* £36. 9*s.* 5½*d.* = £36·473½ = £36·4735 (provisionally).

36·4735

9

correction for 4th place

36·47447916   method of Fours.

*Example 3.* £74. 8s. 3½d. = £74.413½ = £74.413625 (provisionally).

$$\begin{array}{r}
 74\cdot413625 \\
 \underline{5} \quad \text{correction for 4th place} \\
 74\cdot414125 \\
 \underline{6} \quad \text{,, for 5th ,,} \\
 74\cdot414185 \\
 \underline{7} \quad \text{,, for 6th ,,} \\
 74\cdot414192708\bar{3} \quad \text{method of Fours.}
 \end{array}$$

### 5°. **Converse Extension.**

**To convert any decimal to £. s. d. f., 8ths, 16ths, 32ths of a penny.**

Four, Five, Six, Seven places are required respectively to approximate with absolute certainty to farthings, eighths, sixteenths, thirty-two'ths of a penny, but practically sufficiently accurate results can often be obtained with fewer places.

**Rule.** Obtain the product by 4 of the decimal with the 1st place omitted. Subtract this from the original decimal with two zeros added and neglect for the purposes of the subtraction the tens-figure of the 2nd place multiplied by 4.

Consider the decimal so obtained.

Divide first two places by 5—quotient is shillings.

Divide remainder with 3rd place brought down by 4—quotient is pence.

If pence are 6d. or over diminish the new remainder by 1.

Then this remainder (thus corrected if necessary) is farthings.

Divide this remainder with next place brought down by 5—quotient is 8ths.

Divide this remainder with next two places brought down by 25—quotient is 16ths.

Divide this remainder with next three places brought down by 125—quotient is 32ths.

*Example.* £25·6473852  
                   895408  
 £25·646489792

64 ÷ 5 = 12/-    4 rem.  
 46 ÷ 4 = 11    2 „    ∴ 1 farthing  
 14 ÷ 5 = 2    eighths  
 148 ÷ 25 = 5    sixteenths  
 1489 ÷ 125 = 11    thirty-two'ths.

*Note 1.* If a limited number of places are known, the approximations can only be carried to a certain point depending on the number of places correct in the diminished decimal and on what is known of the error in the next place.

*Note 2.* In pricing goods obtain by this method the cost to any approximation and add 1 or more to the corresponding fraction of a penny. This will always ensure against loss by retail. If the retailer wishes a certain profit on the whole sale, he will add this to the total cost and then divide—approximating by the above method and adding 1 to the corresponding fraction of a penny to ensure his full profit.

### EXAMPLES.

1. Decimalise (*a*) exactly to three places, (*b*) to three places with allowance for 4th, (*c*) exactly:

- |                                |                               |
|--------------------------------|-------------------------------|
| (1) £18. 9s. 7½ <i>d.</i>      | (2) £17. 5s. 3¾ <i>d.</i>     |
| (3) £931. 17s. 5½ <i>d.</i>    | (4) £864. 8s. 0¾ <i>d.</i>    |
| (5) £91. 13s. 5 <i>d.</i>      | (6) £72. 6s. 8¼ <i>d.</i>     |
| (7) £3306. 17s. 5¾ <i>d.</i>   | (8) £123. 14s. 6½ <i>d.</i>   |
| (9) £904. 16s. 7½ <i>d.</i>    | (10) £75. 13s. 4½ <i>d.</i>   |
| (11) £324. 0s. 10½ <i>d.</i>   | (12) £66. 15s. 9¼ <i>d.</i>   |
| (13) £8107. 11s. 0¼ <i>d.</i>  | (14) £926. 19s. 1½ <i>d.</i>  |
| (15) £841075. 3s. 3¼ <i>d.</i> | (16) £73. 1s. 0¼ <i>d.</i>    |
| (17) £96. 7s. 6½ <i>d.</i>     | (18) £875. 18s. 10½ <i>d.</i> |
| (19) £4375. 11s. 9 <i>d.</i>   | (20) £8. 12s. 2½ <i>d.</i>    |

Any sum written at random is easily decimalised—the answer being verified by actual working.

2. Convert into £. s. d. f.:

- |                       |                         |
|-----------------------|-------------------------|
| (1) £81·67154.        | (2) £793·563 (approx.). |
| (3) £184·4931.        | (4) £71·336 (exact).    |
| (5) £361·1056.        | (6) £137·004 (approx.). |
| (7) £87·3125.         | (8) £931·273 (approx.). |
| (9) £710·311 (exact). | (10) £876·950 (exact).  |

Any decimal written at random may be turned into money by the method and the answer verified by actual working.

**3. Reduce**

- (1) £3751. 19s.  $1\frac{1}{2}d.$  to farthings.
- (2) £87. 15s.  $7\frac{1}{2}d.$  to farthings.
- (3) £910. 1s.  $5\frac{1}{2}d.$  to farthings.
- (4) £193. 7s.  $5d.$  to pence.
- (5) £204. 11s.  $4d.$  to pence.
- (6) £815. 3s.  $9\frac{1}{2}d.$  to farthings.
- (7) £61. 13s.  $5\frac{1}{2}d.$  to farthings.
- (8) £76. 9s.  $11\frac{1}{2}d.$  to farthings.
- (9) £603. 9s.  $8d.$  to pence.
- (10) £71. 4s.  $10d.$  to farthings.

**4. Decimalise**

- |                                |                               |
|--------------------------------|-------------------------------|
| (1) £73. 7s. $10\frac{3}{8}d.$ | (2) £18. 6s. $4\frac{3}{8}d.$ |
| (3) 7s. $3\frac{1}{2}d.$       | (4) $5\frac{1}{8}d.$          |
| (5) 1s. $9\frac{1}{2}d.$       | (6) 3s. $4\frac{1}{8}d.$      |
| (7) £3. 10s. $4\frac{1}{2}d.$  | (8) $3\frac{1}{2}d.$          |
| (9) 1s. $1\frac{1}{8}d.$       | (10) 3s. $2\frac{5}{8}d.$     |

**5. Convert into £. s. d. f., 8ths, 16ths, 32ths, consistently with absolute accuracy:**

- |                  |                |
|------------------|----------------|
| (1) £19'3145978. | (2) £3'12784.  |
| (3) £'3567.      | (4) £'3786.    |
| (5) £7'645918.   | (6) £13'31267. |
| (7) £27'298314.  | (8) £139'765.  |
| (9) £78'8341.    | (10) £1'93756. |

**7. Methods of Approximation.**

In calculating prices or profits and in all engineering or practical work it is often only necessary to get results correct to a given number of decimal places. This is true for instance in calculating money in decimals of a £—three places ensures correctness to the nearest farthing.

Hence all working which brings results to a larger number of places is superfluous and it becomes of importance to use such methods as will give what is necessary with least work.

**1°. Addition and Subtraction.**

In adding and subtracting, the Rule for approximating is simple.



In each line stop at the place one further to the right than the number of places required correct, and make allowance for the place beyond. Then add in the ordinary manner.

*Example.* Add 83·17135 correct to 3 places.

$$\begin{array}{r} 206\cdot34127 \\ 1308\cdot12161 \\ \hline 1597\cdot6343 \end{array}$$

**Mental Process.** Consider 3 as 4 because of the 5 in line 1

$$\begin{array}{rclclcl} 2 \text{ as } 3 & & & & 7 & & 2 \\ 6 \text{ as } 6 & & & & 1 & & 3 \end{array}$$

Then add in the usual way.

The 4th place is not necessarily correct.

## 2°. On making allowance.

(1) In any Decimal Expression—to make allowance for the part beyond the place stopped at—if next place is 5 or over, add 1 to the digit stopped at, otherwise add 0.

(2) In multiplying any Decimal—to make allowance for the multiplication of the part omitted in the approximation—multiply the figure beyond the place stopped at and add the nearest multiple of 10 to the product of the figure stopped at, counting 5, 15, 25 etc. as being nearer to 10, 20, 30 etc. respectively than to 0, 10, 20 etc.

*Example 1.*  $8\cdot634578 \times 9$  correct to 3 places.

$$\begin{array}{r} 8\cdot634578 \\ 9 \\ \hline 77\cdot7111 \end{array}$$

**Mental Process.**  $9 \times 7 = 63$ , carry 6, then as usual.

*Example 2.*  $13\cdot6235 \times 7$  correct to 2 places.

$$\begin{array}{r} 13\cdot6235 \\ 7 \\ \hline 95\cdot365 \end{array}$$

**Mental Process.**  $7 \times 5 = 35$ , carry 4, then as usual.

(3) In dividing any Decimal—to make allowance at any point for further places in the quotient put down

as the final figure that exact multiple of the divisor which is nearest (above or below) to the remainder used as the dividend.

*Example.*  $84.7312 \div 19$  correct to 3 places with allowance for 4th.

$$\begin{array}{r} 19 \overline{) 84.7312} \quad (4.46 \\ \underline{87} \phantom{00} \\ 113 \phantom{00} \end{array}$$

*Mental Process.*  $6 \times 19 = 114$ , nearest to 113.

This is very important in Aliquotation.

### 3°. Multiplication. De Morgan's Method.

(1) **Absolute Rule** for finding a product correct to a given number of Decimal Places.

Reverse the multiplier omitting the decimal point.

Arrange this with the original unit-figure under that digit in the multiplicand which is one place further to the right than the number of places required correct.

Multiply by the digits of the multiplier, beginning with each at the digit immediately above but adding in the nearest multiple of 10 derived from the digit to the right.

Place the products in successive lines with their right-hand digits in a vertical column.

Add and mark off one more place to the left than the number of places required correct.

Strike off the final figure and the remainder will be the correct answer.

(2) **Sufficient Rule** in many commercial calculations.

Reverse the multiplier omitting the decimal point.

Arrange this with the original unit-figure under that digit of the multiplicand which is as far to the right as the number of places required correct.

Proceed as above with the multiplication.

Add and mark off the number of places required correct.

The last figure will never differ by more than 1 from the true result and therefore any answer of 3 places in money will always be correct to pence.

**(3) Another Form of the Rule** *valuable in Exchanges.*

Place the multiplier under the multiplicand so that the points are in a column.

Multiply by the digit furthest to the left—commencing with that digit in the multiplicand which is the required number of places further to the right than the multiplying digit is to the left of the decimal point (making allowance in the usual way for the multiplicand-digit to the right of the one taken).

Proceed in this way with each digit of the multiplier.

Throughout put the decimal point in its place for each line.

Add, and the required number of places will lie to the right of the decimal point.

N.B. When the decimal parts of the multiplier are reached the starting-point in the multiplicand will get nearer and nearer to the decimal point and ultimately pass to the left of it.

The Third Form shows in fact the principle on which the method is founded—the form is however trying to the eye and hence for general purposes the reversing of the multiplier is more convenient. In thus reversing the multiplier the starting-point of the multiplication for each figure of the multiplier is automatically settled.

In all cases of direct multiplication of money the second form is sufficient but if any continued operations are involved or there is division as well it is better to

use the Absolute Rule. The difference in the labour involved is not great.

In the Examples following each form is shown.

It should be noticed that form (3) is only to the same accuracy as form (2). To get results to the same accuracy as form (1) the multiplication should begin one place further to the right than the given form of the rule directs.

The importance of this method of Approximation cannot be over estimated.

*Example 1.*  $373\cdot8615 \times 27\cdot195$  to 3 places.

373·86150	373·8615	373·8615
59172	59172	27·195
<u>74772300</u>	<u>7477230</u>	<u>7477·230</u>
26170305	2617031	2617·031
373862	37386	37·386
336475	33647	33·647
18693	1869	1·869
<u>10167·1635</u>	<u>10167·163</u>	<u>10167·163</u>

*Example 2.* £37569. 19s. 7½d. by 87 correct to pence.

37569·9802083	37569·98020	37569·9802083
78	78	87·000
<u>30055984166</u>	<u>3005598416</u>	<u>3005598·416</u>
2629898614	262989861	262989·861
<u>£3268588·2780</u>	<u>£3268588·277</u>	<u>£3268588·277</u>
5s. 6½d.	5s. 6½d.	5s. 6½d.

*Example 3.* Value of 1751·96875 tons at £13. 7s. 8½d. per ton.

1751·96875	1751·96875	1751·96875
57348331	57348331	13·884375
<u>175196875</u>	<u>17519688</u>	<u>17519·688</u>
52559063	5255906	5255·906
5255906	525590	525·590
1401574	140157	140·157
70078	7008	7·008
5256	525	·525
1226	123	·123
88	9	·009
<u>£23449·0068</u>	<u>£23449·006</u>	<u>£23449·006</u>

**EXAMPLES.**

1. Multiply 876·314 by 38·72 correct to 3 places.
2. Multiply 9615·83416 by ·001375 correct to 5 places.
3. Multiply 3751·407 by ·6158 correct to 4 places.
4. Multiply 76510·3 by 8·91035 correct to 3 places.
5. Multiply ·008135 by 71·324 correct to 5 places.
6. Multiply 3125·006 by 1·834 correct to 3 places.
7. Multiply ·0001417 by 831 correct to 6 places.
8. Multiply 97·125 by ·0000456 correct to 7 places.
9. Multiply ·00005678 by ·000375 correct to 9 places.
10. Multiply 61·32174 by 89371·25 correct to 2 places.
11. Find price of 876 things at £3. 7s. 10½*d.* each.
12. Find price of 3725½ things at £7. 13s. 4½*d.* each.
13. Find price of 8000 gross at £51. 17s. 9¾*d.* per gross.
14. Find price of 715 dozen at £3. 8s. 9½*d.* per dozen.
15. Find price of 45225 ozs. of gold at £3. 17s. 10½*d.* per oz.
16. Find price of 9124 tons at £84. 19s. 1¼*d.* per ton.
17. Find price of 3225 pipes of wine at £46. 13s. 9*d.* the pipe.
18. Find price of 727 hides of land at £115. 18s. 11¼*d.* per hide.
19. Find price of 1959 barrels of herrings at £2. 15s. 7½*d.* per barrel.
20. Find price of 5625 yds. of silk at £1. 17s. 8½*d.* per yd.
21. Find price of 473 tons 19 cwts. 57 lbs. at £116. 8s. 4¼*d.* per ton.
22. Find price of 3125 qrs. 7 bushels 1 peck at £2. 11s. 5¾*d.* per qr.
23. Find price of 825000 things at £731. 19s. 4¾*d.* per thousand.
24. Find price of 383000 dozens at £905. 12s. 3¼*d.* per thousand dozen.

25. Find price of 751 rods of brickwork at £31. 10s. 5½d. per rod.

26. Find price of 825 sq. chains 1375 sq. links at £205. 14s. 9¼d. per sq. chain.

27. Find price of 1372 acres 3 roods 29 poles at £93. 7s. 11d. per acre.

28. Find price of 247 miles 1312 yards of railway at £8351. 17s. 6d. per mile.

29. Find price of 7032 cubic yds. of earth at £5. 16s. 7¼d. per cubic yd.

30. Find price of 8743 cwts. at £121. 9s. 8¼d. per cwt.

#### 4°. Division. De Morgan's Method.

*To find a quotient correct to a given number of decimal places.*

By inspection decide on the number of digits in the integral part of the quotient.

This added to the no. of dec. places required gives the no. of digits in the quotient.

Proceed in the ordinary way until the no. of quotient-digits to be found is *one* less than the no. of divisor-digits.

Then strike off a digit from the end of the divisor for each new figure in the quotient (making allowance for the figure struck off in the usual way) instead of bringing down the remaining figures of the dividend.

If there are more divisor-digits than there are to be quotient-digits, retain one more and strike out the rest.

*Example 1.* Divide 373·81956 by 87·243 correct to 3 places.

No. of places in  
quotient = 1 + 3 = 4.

No. of places in  
divisor = 5.

$$\begin{array}{r}
 87,243 \overline{) 373819\cdot56} \quad (4\cdot284 \\
 \underline{2484} \\
 739 \\
 \underline{41} \\
 6
 \end{array}$$

*Example 2.* Divide 2·7183615 by 4·1451297 correct to 8 places.

No. of places in 4,145,12,9,7 ) 271836150 ( ·65579640  
quotient =  $0 + 8 = 8$ .

No. of places in  
divisor = 8.

$$\begin{array}{r} 23128368 \\ \hline 2402719 \\ \hline 330154 \\ \hline 39996 \\ \hline 2690 \\ \hline 203 \\ \hline 37 \end{array}$$

*Example 3.* Divide £781. 10s. 7½d. by 85·643 correct to pence.

$$\begin{array}{r|l} 85,643 & 781.530 \\ & 4639 \\ \hline & 357 \\ & 15 \\ \hline & 6 \end{array} \quad \begin{array}{l} £8.541 \\ 10s. 10d. \end{array}$$

### 5°. Modification of De Morgan's Method for Division.

Decide by inspection the no. of digits in the integral part of the quotient or the no. of cyphers following the dec. point.

Make the no. of figures in the divisor 1 more than the no. of required places in the quotient + the no. of integral digits in the quotient or – the no. of cyphers in the quotient.

Then proceed by the method at once.

*Example 1.* Divide 373·8651 by 8514·37 to 4 places.

No. of cyphers = 1. ∴ Take  $(4 - 1) = 3$  figures in divisor.

$$\begin{array}{r} 85,1437 ) 373.8651 ( .0439 \\ \hline 332 \\ \hline 77 \\ \hline \end{array}$$

*Example 2.* Divide 7431·26 by 351 to 3 places.

No. of integers = 2. ∴ Take  $(3 + 2) = 5$  figures in divisor.

$$\begin{array}{r} 35,100 ) 743126 ( 21.171 \\ \hline 4112 \\ \hline 602 \\ \hline 251 \\ \hline 5 \\ \hline 1 \end{array}$$

This is a most valuable method in Division of Money.

*Example 3.* Divide £73965. 19s. 8½d. by 8901.

No. of integers = 1. ∴ Take 4 figures in the divisor.

$$\begin{array}{r} 8,90,1 \overline{) 73965} \quad (\text{£}8\cdot309 \\ \underline{2757} \phantom{00} \\ 87 \phantom{00} \\ \underline{7} \phantom{00} \end{array} \quad \begin{array}{l} 6s. 2d. \end{array}$$

*Example 4.* Divide £83. 9s. 7½d. by 203.

No. of ciphers = 0. ∴ Take 3 figures in the divisor.

$$\begin{array}{r} 2,0,3 \overline{) 83\cdot4} \quad (\text{£}4\cdot11 \\ \underline{22} \phantom{00} \\ 2 \phantom{00} \\ \underline{\phantom{00}} \phantom{00} \end{array} \quad \begin{array}{l} 8s. 2\frac{3}{4}d. \end{array}$$

In the case of Interest [which involves Division by 36500] this method may be used but the Method of Prediction following supersedes it.

## 6°. Method of Prediction.

*To determine the number of places which must be correct in a Multiplication so as to ensure an ensuing Division being correct to a given number of places.*

Decide by inspection the number of integers resulting from the Multiplication.

Then from this and the given divisor decide the number of integers which may arise in the final answer.

Add to this number, the number of places required correct.

Subtract from this the number of integers in the multiplication-result and you will thus get the no. of places which must be correct in the multiplication.

Arrange accordingly.

*Example.* Int. on £311. 8s. 6½d. for 174 days at 3½ per cent.

Equivalent to £311426 × 1218 ÷ 73.

No. of Integers in Multiplication = 3 } ∴ no. of Integers which may  
 No. of Integers in Divisor = 2 } occur in Quotient is 2,  
 No. of Places required correct is 3.      2 + 3 = 5,



No. of Integers in Multiplication is 3.  $5 - 3 = 2$ .

$\therefore$  No. of places required correct in the Multiplication is 2.

$$\begin{array}{r}
 3114260 \\
 8121 \\
 \hline
 311426 \\
 62285 \\
 3114 \\
 2491 \\
 \hline
 73 \overline{) 379315} \quad ( \quad \pounds 5 \cdot 196 \\
 \underline{1481} \qquad \qquad \qquad 3s. 11\frac{1}{4}d. \\
 701 \\
 \underline{44} \\
 \dots
 \end{array}$$

It is obvious that this preliminary determination removes all unnecessary work from the operations.

### EXAMPLES.

1. Divide 87391·631 by 39·275 correct to 5 places.
2. Divide 35·8435 by 3978·34 correct to 8 places.
3. Divide 8341·976 by 731·25 correct to 4 places.
4. Divide 39·125678 by 80047·5 correct to 7 places.
5. Divide ·00012356 by ·967831 correct to 5 places.
6. Divide 93215·6789 by 6345·831 correct to 4 places.
7. Divide 473·87652 by 72·961 correct to 3 places.
8. Divide ·00469857 by ·043278 correct to 6 places.
9. Divide 9·65784321 by 5136·813 correct to 5 places.
10. Divide ·000005678 by 2·1378562 correct to 8 places.
11. Divide  $\pounds 87156. 17s. 10\frac{1}{2}d.$  by 931.
12. Divide  $\pounds 371. 9s. 11\frac{1}{4}d.$  by 8047.
13. Divide  $\pounds 8296. 7s. 4\frac{3}{4}d.$  by 325.
14. Divide  $\pounds 93756. 3s. 4\frac{1}{4}d.$  by 37·75.
15. Divide  $\pounds 874259. 19s. 8\frac{3}{4}d.$  by 893·125.
16. Divide  $\pounds 7432. 13s. 11\frac{1}{4}d.$  by 71235.
17. Divide  $\pounds 310561. 9s. 8\frac{1}{2}d.$  by 8673.
18. Divide  $\pounds 81. 7s. 10\frac{1}{4}d.$  by  $91\frac{1}{2}$ .
19. Divide  $\pounds 312. 15s. 3\frac{3}{4}d.$  by  $139\frac{3}{4}$ .
20. Divide  $\pounds 8096. 12s. 11d.$  by 5760.
21. Find Int. on  $\pounds 712. 9s. 5\frac{1}{4}d.$  for 27 days at 4 p.c.

22. Find Int. on £814. 4s.  $4\frac{1}{2}d.$  for 93 days at  $3\frac{1}{2}$  p.c.
23. Find Int. on £857. 5s.  $6\frac{1}{2}d.$  for 271 days at 5 p.c.
24. Find Int. on £817. 6s.  $3\frac{3}{4}d.$  for 89 days at  $2\frac{1}{4}$  p.c.
25. Find Int. on £1218. 7s.  $5\frac{1}{4}d.$  for 91 days at  $1\frac{1}{8}$  p.c.
26. Find Int. on £83. 13s.  $7\frac{1}{4}d.$  for 357 days at 3 p.c.
27. Find Int. on £3765. 9s.  $8\frac{1}{2}d.$  for 183 days at  $6\frac{1}{4}$  p.c.
28. Find Int. on £17631. 11s.  $1\frac{1}{4}d.$  for 71 days at 5 p.c.
29. Find Int. on £9061. 17s.  $4\frac{1}{2}d.$  for 109 days at  $2\frac{3}{4}$  p.c.
30. Find Int. on £91. 7s.  $2d.$  for 153 days at  $2\frac{1}{2}$  p.c.

## SECTION II.

**THE CALCULATION OF PRICES.**

**1. Mental or Very Brief Rules.**—*Applicable to Articles, Weights and Measures.*

**1°. Given the price of one.**

(1) to find price of a dozen—consider price in d. as s.

(2) to find price of a score—consider price in s. as £.

(3) to find price of 48—consider price in f. as s.

(4) to find price of 96—consider price in eighths of a penny as s.

(5) to find price of a gross—multiply price in f. as s. by 3 or price in d. as s. by 12 or consider price in d. as s. and this again in d. as s.

(6) to find price of 192—consider price in sixteenths of a penny as s.

(7) to find price of 240—consider price in d. as £.

(8) to find price of 960—consider price in f. as £.

(9) to find price of 1920—consider price in eighths of a penny as £.

(10) to find price of 3840—consider price in sixteenths of a penny as £.

(11) to find price of any power of 10—decimalise the price into £ and move the point to the right as many places as there are 0's in the number.

(12) to find price of any *multiple* of any of the preceding numbers (including powers of 10)—multiply price changed according to the corresponding rule by the multiple.

(13) to find price of any *factor* of any of the preceding numbers (including powers of 10)—take the corresponding aliquot of the price changed according to the proper rule.

(14) to find price of any number—find price of nearest multiple of dozens, score, gross, or other of above numbers and add or subtract price of remainder.

- Examples.*
1. 240 articles at 1s.  $1\frac{1}{4}d.$  = £13. 5s. 0d.
  2. 1920 articles at  $3\frac{1}{4}d.$  = £31. 0s. 0d.
  3. 1000 articles at 2s.  $5\frac{1}{4}d.$  = £121·875 = £121. 17s. 6d.
  4. 720 articles at  $3\frac{1}{4}d.$  = £9. 15s. 0d.
  5. 960 articles at  $2\frac{1}{4}d.$  =  $\frac{1}{4}$  of £43 = £10. 15s. 0d.
  6. 1120 articles at 1s.  $2\frac{3}{4}d.$  =  $1\frac{1}{8} \times £59$  = £68. 16s. 8d.
  7. 16 doz. at 4s. 3d. per doz. = £40. 16s. 0d.
  8. 123 articles at  $3\frac{1}{2}d.$  = £1. 15s.  $10\frac{1}{2}d.$
  9. 1 gross at  $3\frac{3}{4}d.$  = 12 at 3s. 9d. = 45s.

*Note 1.* Variations on the above rules are easily made when prices are quoted by the dozen, score, gross, thousand etc.

*Note 2.* Which of the above rules is to be used depends on the character of the number or the price.

*Note 3.* All the above numbers are very useful in the application of Rules (12), (13) and (14).

## 2°. **Conversely. To find the price of one.**

- (1) given price of a dozen—consider price in s. as d.
- (2) given price of a score—consider price in £ as s.
- (3) given price of 48—consider price in s. as f.
- (4) given price of 96—consider price in s. as eighths of a penny.
- (5) given price of a gross—divide price in s. as d. by 12.

(6) given price of 192—consider price in s. as sixteenths of a penny.

(7) given price of 240—consider price in £ as d.

(8) given price of 960—consider price in £ as f.

(9) given price of 1920—consider price in £ as eighths of a penny.

(10) given price of 3840—consider price in £ as sixteenths of a penny.

(11) given price of any power of 10—reduce price to pence and mark off as many places as there are 0's in the number.

(12) given price of any *multiple* of the preceding numbers—divide the price changed according to the proper rule by the multiple.

(13) given price of any *factor* of the preceding numbers—multiply the price changed according to the proper rule by the aliquot denominator corresponding to the factor.

(14) given price of any number—split number into factors and divide the price successively using decimals when advantageous.

- Examples.*
- |    |                       |  |
|----|-----------------------|--|
| 1. | 96 at £2. 3s. 7d.     | $1 = 5\frac{1}{2}d.$ nearly.   |
| 2. | 240 at £5. 6s. 8d.    | $1 = 5\frac{1}{2}d.$   |
| 3. | 1000 at £15. 10s. 7d. | $1 = 3\cdot727d. = 3\frac{3}{4}d.$ nearly.                               |
| 4. | 320 at £3. 7s. 6d.    | $1 = \frac{1}{8} \times 3\frac{3}{4}f. = 1\frac{1}{4}f. = \frac{1}{2}d.$ |

### EXAMPLES.

*Note.*  $\frac{1}{3}$  of a s. =  $1\frac{1}{2}d.$ ,  $\frac{1}{6}$  of a s. =  $\frac{3}{4}d.$ ,  $\frac{1}{8}$  of a £ = 2s. 6d.,  $\frac{1}{16}$  of a £ = 1s. 3d.

1. Find prices of

- (a) 1 dozen at  $2\frac{3}{4}d.$ , 1s.  $5\frac{1}{2}d.$ ,  $3\frac{5}{8}d.$ , 9s.  $7\frac{3}{4}d.$ ,  
 (b) 1 score at  $47\frac{5}{8}d.$ , 2s.  $9\frac{1}{4}d.$ , 7s. 6d.,  $9\frac{5}{12}d.$ ,  
 (c) 1 gross at  $10\frac{1}{2}d.$ , 1s.  $3\frac{3}{4}d.$

2. Find prices of 10, 50, 6, 9, 300, dozen at 1s.  $1\frac{3}{4}d.$ ,  $5\frac{1}{2}d.$ ,  $3\frac{1}{4}d.$ , 3s.  $1\frac{1}{2}d.$ ,  $9\frac{5}{8}d.$  each.

3. Find prices of 2, 5, 7, 11, 12 score at  $1\frac{1}{2}d.$ ,  $7\frac{1}{2}d.$ ,  $1s.$   $2\frac{1}{2}d.$ ,  $3s.$   $4\frac{1}{2}d.$ ,  $5s.$   $6d.$  each.

4. Find prices of 5, 7, 10, 24, 100, gross at  $7\frac{1}{2}d.$ ,  $6\frac{1}{2}d.$ ,  $1s.$   $5\frac{1}{2}d.$ ,  $2s.$   $1\frac{1}{2}d.$ ,  $3s.$   $5\frac{1}{2}d.$  each.

5. Find prices of 720, 320, 600, 360, 160, articles at  $1s.$   $1\frac{1}{2}d.$ ,  $4s.$   $2\frac{1}{2}d.$ ,  $1s.$   $7\frac{1}{2}d.$ ,  $2s.$   $0\frac{3}{4}d.$ ,  $5s.$   $1\frac{1}{8}d.$

6. Find prices of 10, 100, 1000, 50, 80, things at  $1s.$   $4\frac{1}{2}d.$ ,  $2s.$   $3\frac{1}{2}d.$ ,  $3s.$   $1\frac{1}{2}d.$ ,  $4\frac{1}{2}d.$ ,  $5s.$   $9\frac{1}{2}d.$

7. Find prices of 837, 965, 65, 74, 68, 312, 198, things at  $2s.$   $2\frac{1}{2}d.$ ,  $11\frac{1}{2}d.$ ,  $3s.$   $4\frac{1}{2}d.$ ,  $5s.$   $1\frac{3}{8}d.$ ,  $3s.$   $11\frac{1}{4}d.$ ,  $1s.$   $11\frac{1}{2}d.$ ,  $1s.$   $10\frac{3}{4}d.$

8. Find price of a stone, cwt., ton, pack and sack at  $1s.$   $5d.$ ,  $9\frac{1}{2}d.$ ,  $1s.$   $5\frac{1}{2}d.$ ,  $2s.$   $9\frac{1}{4}d.$ ,  $3\frac{1}{2}d.$  per lb.

9. The price of 12, 20, 80, 64, 1920, things is £3. 10s. 0d., £4. 5s.  $11\frac{1}{2}d.$ , £18. 9s. 6d., £75. 0s. 1d., £214. 7s.  $9\frac{1}{2}d.$  respectively; find the price of 1.

10. Calculate the cost of a dozen, score, and gross at these prices— $3\frac{1}{2}d.$ ,  $5\frac{3}{4}d.$ ,  $6\frac{1}{2}d.$ ,  $10\frac{1}{2}d.$ ,  $11\frac{1}{2}d.$ ,  $1s.$   $1\frac{1}{2}d.$ ,  $1s.$   $1\frac{1}{2}d.$ ,  $1s.$   $11\frac{1}{4}d.$ ,  $2s.$   $1\frac{1}{2}d.$ ,  $4s.$   $11d.$ ,  $5s.$   $7d.$ ,  $7s.$   $6d.$

## 2. The Method of "Nine Multiples" and "Moving the Points."

The construction of small Tables containing 1, 2, 3, 4, 5, 6, 7, 8, 9 times any unit, quantity, price or decimal expression enables us to perform the conversion of quantities from one unit to another, and the calculation of prices and exchanges without further labour than addition by simply moving the decimal points—to any required accuracy.

### 1°. Conversion of Units. *English.*

#### (1) Grains to Troy ounces for Bullion Operations.

1 = .002083

2 = .00416

3 = .00625

4 = .0083

5 = .010416

6 = .0125

7 = .014583

8 = .016

9 = .01875

*Example.* 325 oz. 10 dwts. 15 grs.

= 325 oz. 255 grs.

255 grs = .416666

.104166

.010416

.53125

*Ans.* 325.53125.

## (2) lbs. to cwts. for Commerce generally.

1 =	·0089285714	<i>Example.</i>	79 cwts. 2 qrs. 17 lbs.
2 =	·0178571428		= 79 cwts. 73 lbs.
3	·0267857142		
4	·0357142857		79·625
5	·0446428571		·0267...
6	·0535714285		79·6517...
7	·0625		
8	·0714285714		
9	·0803571428		

## (3) sq. inches to sq. feet for Engineers etc.

1 =	·00694	<i>Example.</i>	27 sq. ft. 97 sq. miles.
2	·01388		= 27·625
3	·02083		·04861
4	·02777		27·67861
5	·03472		
6	·04166		
7	·04861		
8	·05555		
9	·0625		

## (4) cubic ins. to cub. ft. for Engineers etc.

1 =	·00057865740	<i>Example.</i>	91 cub. ft. 347 cub. inches.
2	·00115731481		= 91·173597
3	·00173597222		·023146
4	·00231462962		·004051
5	·00289328703		91·200794
6	·00347194444		
7	·00405060184		
8	·00462925925		
9	·00520791666		

For *English-Metric* conversions see the Chapter on the Metric System.

Multiples like (2) may also be used to calculate the price per lower unit, given the price of the higher.

*Example.* Suppose price per cwt. is 18s. 9d.—to find price per lb.

18s. 9d. = 225d. Now use (2).

∴ Price per lb. = 1·79

·18

·04

2·11 = 2½d.

## 2°. Calculation of Prices by Multiples of the Quantities (Inverse Method).

The use of Multiples of well-known quantities is not very advantageous with our coinage because it is not a decimal one. By constructing the multiples of these quantities decimalised to certain bases, however, and employing denomination-changes, full advantage may be taken of the principle of moving the points.

The best bases to use are 960 (f. as £) and 3840 (16ths as £). We give both sets of multiples, the actual and the decimalised, and the examples are worked in both ways.

With the actual multiples it is easiest to use pence — afterwards changing to £, s., d.

When we come to metric quantities at foreign prices we shall see the value of the actual multiples. They would be of as great importance with us if we used *mils* as the division of the £.

### (1) cwts. to lbs.—base 3840.

1 =	112	·02916
2 =	224	·0583
3 =	336	·0875
4 =	448	·116
5 =	560	·14583
6 =	672	·175
7 =	784	·20416
8 =	896	·23
9 =	1008	·2625

*Example 1.* Price per lb. is  $3\frac{1}{8}d. = \frac{25}{8}d.$

∴ Price per cwt. =	336	1·458
	35	·088
	<hr/>	
	371d. = £1·546 (10s. 11d.).	

*Example 2.* Price per lb. is  $1s. 1\frac{1}{8}d. = \frac{21}{8}d.$

∴ Price per cwt. =	1120	5·833
	336	·292
	70	·233
	<hr/>	

1526d. = £6·358 (7s. 2d.).

### (2) sacks to lbs. (flour)—base 960.

1 =	280	·2916
2 =	560	·583
3 =	840	·875
4 =	1120	1·16
5 =	1400	1·4583
6 =	1680	1·75
7 =	1960	2·0416
8 =	2240	2·3
9 =	2520	2·625

*Example 1.* Price per lb. is  $2\frac{1}{2}d. = 10\frac{1}{2}f.$

∴ Price per sack =	560	2·917
	175	·146
	<hr/>	
	735d. = £3·063 (1s. 3d.).	

*Example 2.* Price per lb. is  $97\frac{1}{2}$  mils.

∴ Price per sack =	25200
	1960
	140
	<hr/>

£27·300



These and such similar multiples as may be required should be carried in the pocket-book ready for use.

The bases 240 (d. as £) and 1920 (8ths as £) may also be used, but 960 is the most convenient for all kinds of prices.

### 3°. Calculation of Prices by Multiples of the Price (Direct Method).

The Price is first converted into a decimal of a £ and then the nine multiples are taken.

With commonly-occurring prices the method is very advantageous as its speed is always great and in many cases the working is instantaneous.

It is obvious also that the method can equally well apply to dozens, scores, cwts., gross, or any special quantities in which goods are quoted.

#### (1) Price = 3½d.

1 = £·01510416	Examples. 1. 5 doz. = £·906 = 18s. 1½d.
2 ·0302083	2. 73 arts. = 1·057
3 ·0453125	·045
4 ·060416	1·102 = £1. 2s. 0½d.
5 ·07552083	
6 ·090625	3. 150 things = ½ (300)
7 ·10572916	= £2·265 = £2. 5s. 3½d.
8 ·12083	
9 ·1359375	4. 27 things = (3 × 9)
	= £·408 = 8s. 2d.

#### (2) Bar Silver per oz. = 40 <sup>7</sup>/<sub>16</sub>d.

1 = £·168489583	Examples. 1. 4000 oz. = £673·958
2 ·33697916	= £673. 19s. 2d.
3 ·50546875	2. 3125 oz. = 505·469
4 ·6739583	16·849
5 ·842447916	3·370
6 1·0109375	·842
7 1·179427083	£526·530
8 1·347916	= £526. 10s. 7½d.
9 1·51640625	

With practice all two-figure quantities can be written down at sight after mental addition.

The method can be applied to Sterling Exchanges with great effect.

(3) Rupee = 1s.  $5\frac{1}{2}d.$

1	£·073046875	<i>Example.</i>	8560 rupees = 584·375
2	·146093750		36·523
3	·219140625		4·383
4	·292187500		£625·281
5	·365234375		5s. $7\frac{1}{2}d.$
6	·438281250		
7	·511328125		
8	·584375000		
9	·657421875		

Such exchanges fluctuate as a rule within very small limits and therefore the Tables required are very few. Hence those whose dealings with the East are frequent can readily make tables of the kind as they are required—keeping them for future use.

#### EXAMPLES (to be done by Multiples).

- Decimalise in ounces
  - 612 oz. 19 dwts. 23 grs.
  - 481 oz. 13 dwts. 20 grs.
  - 1362 oz. 9 dwts. 16 grs.
- Decimalise in cwts.
  - 2 tons 13 cwts. 3 qrs. 17 lbs.
  - 83 cwts. 1 qr. 19 lbs.
  - 980 cwts. 2 qrs. 16 lbs.
- Decimalise
  - in sq. ft. 823 sq. ft. 119 sq. in.
  - in sq. yds. 28 sq. yds. 7 sq. ft. 56 sq. in.
  - in cub. ft. 141 c. ft. 953 c. inches.
  - in sq. ft. 71 sq. ft. 91 sq. in.
  - in cub. ft. 84 c. ft. 1152 c. in.
- Construct the Tables for Converting
  - perches to acres. *Ex.* 19 ac. 3 ro. 29 poles.
  - sq. in. to sq. yds. 27 sq. yds. 7 sq. ft. 110 sq. in.
  - lbs. to tons. 15 tons 13 cwts. 59 lbs.
  - gallons to quarters (Imp.). 18 qrs. 7 bushels 5 galls.
  - links to yards. 81·96 chains.
- Construct the Tables for Reducing
  - tons to lbs. *Ex.*  $3\frac{1}{4}d.$ ,  $2\frac{1}{8}d.$ , 1s.  $3\frac{1}{8}d.$
  - acres to sq. yds. 1s.  $6\frac{1}{4}d.$ , 2s.  $1\frac{1}{2}d.$ ,  $9\frac{1}{8}d.$
  - puncheons of prunes to lbs.  $2\frac{1}{4}d.$ ,  $3\frac{1}{2}d.$ ,  $5\frac{3}{4}d.$
  - barrels of soap to lbs.  $1\frac{1}{4}d.$ ,  $7\frac{1}{2}d.$ ,  $4\frac{1}{2}d.$
  - Lisbon pipes to gallons.  $8\frac{1}{4}d.$ , 1s.  $1\frac{1}{4}d.$ , 2s.  $9\frac{1}{2}d.$

Find the prices per higher unit at the given prices per lower unit in each case.

6. From the Tables given below find the values respectively of

(a) 320, 94, 87, 201, 416, 512, 1000, 745, 621, 83, 192, 560 articles.

(b) 841, 325, 612, 7518, 8619, 25000, 3000, 5436, 10525, 8765 rupees.

(c) 1480, 5236, 6125, 8175, 25615, 40000, 87655, 147580 oz. of silver.

(a) Article 4½d.	(b) Rupee 1s. 3½d.	(c) Silver 31½d.
£	£	£
1=·01927083	1=·0640625	1=·129427083
2=·0385416	2=·128125	2=·25885416
3=·0578125	3=·1921875	3=·38828125
4=·077083	4=·25625	4=·5177083
5=·09635416	5=·3203125	5=·647135416
6=·115625	6=·384375	6=·7765625
7=·13489583	7=·4484375	7=·90989583
8=·15416	8=·5125	8=·1·035416
9=·1734375	9=·5765625	9=1·16484375

### 3. The Method of Inversion.

A most convenient way of finding the price of common quantities consists in constructing a small table for each in the following manner.

Consider the quantity in succession shillings, pence, farthings, eighths and sixteenths of a penny and find the corresponding sums of money.

Keep this little table for use.

The following quantities are of frequent occurrence and their Tables are given as examples, but any other quantities may be dealt with similarly as the requirements of business render advisable. All such should be carried in the pocket ready for use.

## (1) The cental (100 lbs.).

	£	s.	d.	Example 1.	Price of 1 lb. = 1s. 1½d.
s.	5	0	0		£ s. d.
d.		8	4	∴ Price per cental =	5 0 0
f.		2	1		8 4
e.	1	0	½		6 3
st.		6	¼		£5 14 7

Example 2. Price of 1 lb. = 7s. 1½d.

	£	s.	d.
∴ Price per cental =	2	18	4
		2	7½
	£3	0	11½

## (2) The cwt. (112 lbs.).

	£	s.	d.	Example.	Price of 1 lb. = 1s. 8⅔d.
s.	5	12	0		£ s. d.
d.		9	4	∴ Price per cwt. =	5 12 0
f.		2	4		1 8 0
e.	1	2			5 10
st.		7			£7 5 10

## (3) The gross (144).

	£	s.	d.	Example.	Price of 1 art. = 8s. 9d.
s.	7	4	0		£ s. d.
d.		12	0	∴ Price per gross =	4 16 0
f.		3	0		6 9
e.	1	6			£5 2 9
st.		9			

## (4) The Lisbon Pipe (115 gallons).

	£	s.	d.	Example.	Price per gallon = 1s. 9½d.
s.	5	15	0		£ s. d.
d.		9	7	∴ Price per Pipe =	5 15 0
f.		2	4½		4 6 3
e.	1	2	¾		4 9½
st.		7	⅞		£10 6 0½

**The Principle of Inversion** on which the above Tables are based is one of great value in all price calculations.

It may be stated thus:—Consider the number of articles whose price is wanted as shillings, pence, farthings, eighths, or sixteenths (one only) and the

price in the denomination taken as the number of articles—reduce the former to £. s. d. f. and multiply by the latter.

Which denomination is taken depends on the price.

*Examples.* 1. 37 arts. at  $6\frac{1}{2}d.$  =  $6\frac{1}{2}$  arts. at  $3s. 1d.$   
 $= 18s. 6d. + 1s. 6\frac{1}{2}d. = 20s. 0\frac{1}{2}d.$

2. 23 cwt. at 5s. 6d. per cwt. =  $5\frac{1}{2}$  cwt. at £1. 3s. 0d.  
= £5. 15s. 0d. + 11s. 6d. = £6. 6s. 6d.

3. 112 lbs. at  $3\frac{3}{8}d.$  per lb. = 51 lbs. at  $7d.$   
= 357d. = £1. 9s. 9d.

**EXAMPLES** (to be done by making the necessary Inversion Tables).

1. Find price of a cental at  $1s. 2\frac{3}{4}d.$ ,  $3s. 1\frac{3}{4}d.$ ,  $9\frac{1}{2}d.$ ,  $2s. 4\frac{1}{4}d.$ ,  $7\frac{5}{8}d.$  per lb.

2. Find price of a cwt. at  $3s. 4\frac{3}{4}d.$ ,  $11\frac{3}{4}d.$ ,  $1s. 7\frac{9}{16}d.$ ,  $5s. 0\frac{3}{4}d.$ ,  $10\frac{3}{4}d.$  per lb.

3. Find price of 50 dozen at  $3\frac{1}{4}d.$ ,  $2\frac{3}{4}d.$ ,  $1s. 1\frac{1}{2}d.$ ,  $10\frac{1}{4}d.$ ,  $11\frac{1}{2}d.$  each.

4. Find price of 812 dozen at 2s. 1d., 1s. 6½d., 1s. 3½d., 10d., 5s. 1d. each.

5. Find price of a Lisbon Pipe at  $2s. 7d.$ ,  $3s. 1\frac{1}{2}d.$ ,  $5s. 4\frac{1}{4}d.$ ,  $9s. 7d.$ ,  $6s. 3\frac{1}{4}d.$  per gallon.

6. Find price of 1000 yds. at  $3s. 1\frac{1}{4}d.$ ,  $1s. 11\frac{1}{2}d.$ ,  $9s. 10\frac{1}{4}d.$ ,  $1s. 5\frac{3}{4}d.$ ,  $2s. 7\frac{3}{4}d.$  per yd.

7. Find price of 950 things at  $1s. 1\frac{1}{2}d.$ , 847 things at  $2s. 5\frac{3}{4}d.$ , 912 things at  $8\frac{1}{4}d.$

8. Find price of 1000 things at  $2s. 3\frac{1}{4}d.$ , 642 things at  $1s. 4\frac{1}{4}d.$ , 900 things at  $3s. 5\frac{1}{4}d.$

9. Find price of 512 gallons at  $13s. 7\frac{1}{4}d.$ ,  $10s. 9d.$ ,  $8s. 4\frac{1}{2}d.$ ,  $5s. 6\frac{1}{4}d.$ ,  $9s. 8\frac{1}{2}d.$  per gallon.

10. Find price of 752 grains at  $3s. 1\frac{1}{4}d.$ ,  $8s. 6\frac{1}{2}d.$ ,  $4s. 1\frac{1}{4}d.$ ,  $15s. 3d.$ ,  $1s. 10\frac{1}{4}d.$  per grain.

11. Find price of a firkin of butter at  $1s. 2d.$ ,  $1s. 3\frac{1}{2}d.$ ,  $1s. 5d.$ ,  $11\frac{3}{4}d.$ ,  $1s. 6\frac{1}{4}d.$  per lb.

12. Find price of a puncheon of prunes at  $2\frac{1}{4}d.$ ,  $3\frac{1}{2}d.$ ,  $7\frac{1}{4}d.$ ,  $6\frac{1}{4}d.$ ,  $4\frac{1}{4}d.$  per lb.

13. Find price of a barrel of anchovies at  $5\frac{1}{4}d.$ ,  $1s. 1\frac{1}{4}d.$ ,  $10\frac{1}{2}d.$ ,  $1s. 7\frac{1}{4}d.$ ,  $12\frac{3}{4}d.$  per lb.

#### 4. Aliquotation.

An aliquot part is a fraction whose numerator is 1.

The principle has been long used in Practice of dividing the Price into successive portions which are aliquot parts of one of the preceding portions and thus the Price is calculated by a series of short divisions.

The extension of this principle so as to get a series of fractions each an aliquot part of the preceding one as an equivalent for a given fraction produces great simplifications in various calculations.

It is of special use in its application to Prices and Percentages.

We give here the Method of Aliquotation and the Notation adopted in this book.

$$1. \text{ To aliquotise } \frac{7}{8} = \frac{4+2+1}{8} = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left\{\frac{1}{2}\left(\frac{1}{2}\right)\right\} \\ = 0-222 \text{ (Notation).}$$

$$2. \text{ To aliquotise } 1\frac{1}{8} = 1 + \frac{8+4+1}{16} = 1 + \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left\{\frac{1}{2}\left(\frac{1}{2}\right)\right\} \\ = 1-224 \text{ (Notation).}$$

$$3. \text{ To aliquotise } 3\frac{1}{8} = 3 + \frac{32+16+1}{64} = 3 + \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{8}\left\{\frac{1}{2}\left(\frac{1}{2}\right)\right\} \\ = 3-22(16) \text{ (Notation).}$$

$$4. \text{ To aliquotise } 15 \text{ p.c.} = \frac{10+5}{100} = \frac{1}{10} + \frac{1}{2}\left(\frac{1}{10}\right) = 0-(10)2.$$

**The Method** employed, then, is to split up the numerator in such a way that each part is an aliquot of the preceding.

**The Notation** adopted is the following:—the denominators of the successive aliquots are written down in order without any marks of division—numbers above 9 being enclosed in brackets and multiples being placed before the dash.

Occasionally the aliquotation is shorter if the aliquots are not all successive but two or more aliquots of a previous one. This is denoted by a plus between them with a bar above.

$$\text{Example. } \frac{2}{20} = \frac{5+4}{20} = \frac{1}{4} + \frac{1}{5} = 0 - \overline{4+5}.$$

It may also be advisable to attach successive further aliquots to one of two or more under a bar—this is expressed thus.

$$\text{Example. } \frac{378}{960} = \frac{480+320+64+8+4}{960} = 0 - \overline{2+3582}.$$

When 1 occurs among the aliquots it denotes that the previous line of division is to be repeated—a device of frequent occurrence in practice.

There are often two or three ways of aliquotising the same fraction.

### EXAMPLES.

1. Aliquotise the fractions  $1\frac{17}{102}$ ,  $3\frac{57}{240}$ ,  $13\frac{21}{384}$ ,  $\frac{113}{256}$ ,  $\frac{112}{960}$ .
2. Aliquotise 89, 77, 292, 512, 786 to the bases 144, 240, 384, 960, 192.
3. Aliquotise 13, 23, 32, 40 p.c.
4. Aliquotise 74, 81, 59, 61 per mille.

#### 1°. Direct Aliquotation (Practice).

The ordinary Method of Practice is to aliquotise parts of the price in succession till it is exhausted. This is as a rule the most convenient and shortest method. Sometimes however difficulties arise as to the proper parts to take at the end. These may be avoided by the repetition of lines and other devices.

There may be more aliquots in one way of aliquotising a price than in another but as a compensa-

tion the divisors will be smaller. Aliquots above 16 which are not multiples of 10 are inconvenient.

Decimals should always be used as their use avoids all disagreeable fractions of a penny in the addition, and makes multiples of 10 convenient aliquots.

Carry the lines to 4 places, making allowance for the 5th—the addition will then be correct to 3 places.

Prices may be thus aliquotised without regard to the quantities. Hence if any prices are of frequent occurrence their aliquot-formulae may be made and kept for use.

*Example 1.* £3. 7s. 11½d. = £3—426 (10).

*Example 2.* 4s. 7½d. = £0—584.

*Example 3.* £1. 6s. 5¾d. = £1—458422.

*Example 4.* 2½d. = £0—(120) 442.

**Examples of Prices of frequent occurrence.**

(1) 3½d. = £0—(80) 64.

(2) 5⅞d. = £0—(60) 4422.

(3) 1s. 3⅞d. = £0—(20) 4 (12) 4.

(4) £3. 17s. 10½d. = £3—222(10) 2.

*Example.* 824 things at 1s. 3⅞d.

824	
41·2	
10·3	
·8588	
·2146	
£52·5729	= £52. 11s. 5½d.

### EXAMPLES.

1. Construct Aliquot formulae for these prices.

(1) £1. 18s. 5¾d.

(2) 19s. 4½d.

(3) £3. 16s. 5½d.

(4) £17. 15s. 4d.

(5) 17s. 6½d.

(6) 27s. 9¾d.

(7) 18s. 4½d.

(8) 56s. 7d.

(9) 39s. 8¾d.

(10) 65s. 4½d.

2. Construct Aliquot formulae for these prices.

(1) 3⅞d.

(2) 4½d.

(3) 6⅞d.

(4) 1s. 3¾d.

(5) 2⅞d.

(6) 9½d.



## 3. Calculate

- (1) Prices of 8315 things at  $5s. 7\frac{1}{2}d.$ ,  $8s. 10\frac{1}{2}d.$ ,  $\pounds 1. 7s. 10d.$
- (2) Prices of 9640 things at  $3\frac{5}{8}d.$ ,  $4\frac{3}{8}d.$ ,  $9\frac{3}{8}d.$ ,  $\pounds 2. 8s. 4\frac{1}{2}d.$
- (3) Prices of 856 things at  $7\frac{3}{4}d.$ ,  $9\frac{1}{4}d.$ ,  $\pounds 2. 7s. 6\frac{1}{2}d.$
- (4) Prices of 932 things at  $5s. 6\frac{3}{4}d.$ ,  $7\frac{1}{2}d.$ ,  $9\frac{1}{2}d.$ ,  $11\frac{1}{2}d.$
- (5) Prices of 1000 things at  $8s. 11\frac{1}{2}d.$ ,  $1s. 3\frac{5}{8}d.$ ,  $10\frac{3}{8}d.$ ,  $11\frac{5}{8}d.$
- (6) Prices of 1250 things at  $10s. 1\frac{3}{8}d.$ ,  $1s. 5\frac{9}{8}d.$ ,  $2s. 1\frac{5}{8}d.$ ,  $3s. 2\frac{9}{8}d.$
- (7) Prices of 3000 things at  $4\frac{7}{8}d.$ ,  $1\frac{3}{8}d.$ ,  $3\frac{7}{8}d.$ ,  $1\frac{1}{8}d.$ ,  $1s. 2\frac{1}{2}d.$
- (8) Prices of 784 things at  $3s. 8\frac{3}{4}d.$ ,  $3s. 2\frac{5}{8}d.$ ,  $10\frac{5}{8}d.$ ,  $1s. 1\frac{3}{8}d.$
- (9) Prices of 940 things at  $5\frac{1}{2}d.$ ,  $7\frac{1}{8}d.$ ,  $5\frac{3}{8}d.$ ,  $1s. 3\frac{3}{8}d.$ ,  $5s. 7\frac{3}{8}d.$
- (10) Prices of 2000 things at  $7\frac{3}{8}d.$ ,  $1\frac{3}{8}d.$ ,  $9\frac{1}{8}d.$ ,  $8s. 4\frac{1}{2}d.$ ,  $9s. 6\frac{1}{2}d.$

This Method of Direct Aliquotation may be applied to **Weights and Measures** with Decimalisation of the Price.

*Example.* 17 cwts. 3 qrs. 16 lbs. at  $17s. 7\frac{1}{2}d.$  per cwt.

Quantity =  $17\frac{7}{8}$  cwts.  
=  $17 - 2227$

	£	17
	6	16875
	8	81250
	4	4063
	2	2032
	1	1016
	0	1574
	£15	76810
		15s. $4\frac{1}{2}d.$

It is in general advantageous to Decimalise the Price.

**EXAMPLES.**

1. 37 cwts. 2 qrs. 19 lbs. at  $19s. 5d.$  per cwt.
2. 3 tons 19 cwts. 8 lbs. at  $\pounds 3. 8s. 7d.$  per ton.
3. 841 yds. 1 ft. 10 inches at  $22s. 8d.$  per yd.
4. 95 acres 2 roods 37 poles at  $\pounds 5. 12s. 6d.$  per acre.
5. 11 oz. 13 dwts. 8 gr. at  $\pounds 3. 17s. 11d.$  per oz.

**2°. Inverse Aliquotation.**

Aliquotation of the price is advantageous when the number of articles constantly varies, the prices being nearly the same but when the prices vary largely and the number of articles is well known Inverse Aliquotation is to be preferred.

The Rule may be stated in two parts.

(1) Reckon pence, farthings, eighths, or sixteenths as shillings and divide the given number of articles by 12, 48, 96, 192 respectively—aliquotising the remainder.

(2) Reckon shillings, pence, farthings, eighths, or sixteenths as pounds and divide the given number of articles by 20, 240, 960, 1920, 3840 respectively—aliquotising the remainder.

*Example 1.* Price of 840 articles at  $1s. 5\frac{3}{4}d. = 17\frac{3}{4}d.$

$$840 \div 48 = 17\frac{3}{4}$$

£	s.	d.
17	15	0
		3
<hr/>		
53	5	0
8	17	6
<hr/>		
£62	2	6

*Example 2.* Price of 713 articles at  $5\frac{3}{8}d. = \frac{1}{8}d.$

$$713 \div 28 = 25\frac{3}{4} = 25 - 2348.$$

£4	15
8	30
2	075
·	6916
·	1729
·	0216
<hr/>	
£15	4111
8s.	2½d.

*Example 3.* Price of 1356 articles at  $2s. 5\frac{1}{8}d. = 117\frac{3}{4}$  farthings.

$$1356 \div 11\frac{3}{4} = 116\frac{2}{3} = 116 - 3582.$$

£	s.	d.
117	15	0
39	5	0
7	17	0
	19	7½
	9	9½
<hr/>		
£166	6	5½

The most useful bases to take are 192, 240, 960.

By using Decimals we avoid difficult fractions of a penny.

If the remainder is small (under 12) it is shorter to obtain the price direct. This is always done in the case of dozens and score by commercial men.

If the remainder is large we may adopt the Principle of Interchanges (that is reckon the price something else for the remainder) in preference to Aliquotation.

The Aliquot-formulae for any quantities may be calculated for the various bases without regard to the Prices. Hence if certain quantities are of frequent occurrence suitable Inverse-Aliquotations may be made and kept for use.

e.g. 112 to base 240 = 0-351; hence if price per lb. is  $2\frac{3}{4}d.$

	£	s.	d.
price per cwt. =	2	15	0
		18	4
		3	8
		3	8
	£1	5	8

### EXAMPLES.

1. Price of 854 things at  $1s. 9\frac{3}{8}d.$ ,  $2s. 7\frac{3}{8}d.$ ,  $3s. 1\frac{1}{4}d.$
2. Price of 1260 things at  $5s. 7\frac{1}{2}d.$ ,  $9s. 6\frac{3}{4}d.$ ,  $1s. 7\frac{3}{8}d.$
3. Price of 3000 things at  $4s. 1\frac{3}{4}d.$ ,  $5s. 9\frac{1}{8}d.$ ,  $3s. 2\frac{1}{2}d.$
4. Price of 972 things at  $3s. 11\frac{1}{4}d.$ ,  $2s. 9\frac{3}{4}d.$ ,  $4s. 3\frac{3}{8}d.$
5. Price of 1008 things at  $7s. 9\frac{3}{4}d.$ ,  $5\frac{1}{4}d.$ ,  $3s. 1\frac{1}{2}d.$
6. Price of 1892 things at  $3s. 5\frac{7}{8}d.$ ,  $4s. 2\frac{9}{16}d.$ ,  $2s. 11\frac{1}{4}d.$
7. Price of 736 things at  $2s. 3\frac{5}{8}d.$ ,  $7s. 3\frac{3}{8}d.$ ,  $4s. 5\frac{1}{16}d.$
8. Price of 4050 things at  $3s. 3\frac{7}{8}d.$ ,  $3s. 4\frac{9}{16}d.$ ,  $1s. 11\frac{3}{8}d.$
9. Price of 1500 things at  $2s. 9\frac{5}{8}d.$ ,  $5s. 1\frac{1}{2}d.$ ,  $1s. 5\frac{3}{4}d.$
10. Price of 1648 things at  $3s. 1\frac{3}{4}d.$ ,  $3s. 2\frac{5}{8}d.$ ,  $1s. 10\frac{1}{2}d.$
11. Price of 1250 things at  $1s. 7\frac{5}{16}d.$ ,  $2s. 3\frac{3}{8}d.$ ,  $3s. 1\frac{1}{4}d.$
12. Price of 974 things at  $2s. 3\frac{3}{8}d.$ ,  $4s. 7\frac{3}{4}d.$ ,  $1s. 9\frac{1}{2}d.$
13. Price of 2546 things at  $3s. 5\frac{3}{4}d.$ ,  $2s. 4\frac{7}{8}d.$ ,  $1s. 5\frac{3}{8}d.$
14. Price of 3120 things at  $1s. 7\frac{1}{2}d.$ ,  $1s. 9\frac{3}{4}d.$ ,  $1s. 11\frac{1}{4}d.$
15. Price of 760 things at  $2s. 11\frac{1}{2}d.$ ,  $3s. 1\frac{3}{8}d.$ ,  $4s. 2\frac{1}{2}d.$

### 5. Interchanges.

In preference to Aliquotation the Method of Interchanges may be employed, i.e. we may split the number into convenient parts and consider the price as changed from one denomination to another to suit the parts chosen.

An example will show the principles involved.

Price of 844 arts. at 2s. 3½d.

2s. 3½d. = 27½d. = 109 f.

844 = 720 + 124 = 720 + 120 + 4 = 3 × 240 + 10 × 12 + 4.

∴ price = 3 × £27. 5s. 0d. (d. as £) + 10 × £1. 7s. 3d. (d. as s.) + 9s. 1d.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 = \quad 81 \quad 15 \quad 0 \\
 \quad \quad 18 \quad 12 \quad 6 \\
 \quad \quad \quad 9 \quad 1 \\
 \hline
 \text{£}95 \quad 16 \quad 7
 \end{array}$$

The Divisions practically useful are these:—

- (1) Into 960ths—reckon f. as £.
- (2) Into 240ths—reckon d. as £.
- (3) Into 48ths—reckon f. as s.
- (4) Into 20ths—reckon s. as £.
- (5) Into 12ths—reckon d. as s.

However unless the price is an easy fraction of a shilling (4) is not to be employed.

For very large quantities we may divide into 1920ths, 3840ths, reckoning eighths as £ and sixteenths as f.

The Best Rule to observe is:—

Write price as shillings, pence, farthings.

Divide number of articles by 960, remainder by 240, remainder by 48, remainder by 12.

Multiply appropriately.

*Example 1.* 1754 arts. at 1s. 9½d. = 21½d. = 87 f.

960	1754	1	£	s.	d.
240	794	3	87	0	0
48	74	1	65	5	0
12	26	2	4	7	0
	2		2	3	6
			3	7½	

I. = 13122 (Formula). £158 19 1½

*Example 2.* 920 arts at 2s. 11½d. = 35½d. = 141 f.

240	920	3	£	s.	d.
48	200	4	105	15	0
	8		28	4	0
			1	3	6

I. = 03408 (Formula). £135 2 6

Interchanges may be calculated for common quantities and kept for use.

e.g. 500 = 02018 (Formula), i.e.  $2 \times 240 + 1 \times 12 + 8$ .

### EXAMPLES.

1. Prices of 15 score, 30 dozen, 3 gross at 3¼d., 9½d., 1s. 1¼d. each.

2. Prices of

912 articles at 1s. 2¼d., 3s. 1½d., 4s. 2¾d., 1s. 9¾d., 2s. 10½d.

1845 articles at 7¼d., 8¾d., 9¾d., 1s. 0¾d., 2s. 1¾d.

9764 articles at 2¾d., 3¾d., 4¾d., 5d., 1s. 7¼d., 1s. 8½d.

3126 articles at 11¼d., 1s. 0¾d., 6¾d., 7¾d., 5¾d., 20d.

640 articles at 1s. 4¼d., 1s. 2½d., 1s. 3¾d., 1s. 5¼d., 1s. 6½d., 1s. 9¾d.

852 articles at 8¼d., 9½d., 7d., 11¼d., 1s. 3½d., 1s. 4¾d.

900 articles at 10¼d., 7½d., 3s. 2¼d., 4s. 3½d., 5s. 0½d., 4s. 11¾d.

800 articles at 1s. 7¼d., 1s. 9¾d., 2s. 5¼d., 3s. 7¾d., 4s. 1¼d.

1264 articles at 9¾d., 1s. 2¼d., 1s. 1¾d., 19¼d., 10¾d.

3. Prices of 720, 192, 72, 100, 2880 things at 1¼d., 2¾d., 1s. 3½d., 1s. 7¼d., 9½d.

4. Cost of 1 cwt., 5 Lisbon Pipes, 1 acre, 1 year's wage, 1 pint at 1s. 1¼d. per lb., 9s. 7½d. per gallon, 5s. 4½d. per sq. yd., 30s. 10½d. per week, 3¼d. per grain.

**6. To find the Price of any whole Number of Articles at a given price per Article.**

(1) Multiply the price as it stands by the number of Articles in the compact form given below.

(2) Decimalise the price and multiply, using the Method of Approximation.

(3) Use Direct Aliquotation with Decimal working.

These methods apply to Weights and Measures when the quantities are all of one kind or are easily reduced to one kind.

They also apply to quantities quoted by the dozen, gross, score, at so much per dozen etc.

*Example.* 7316 articles at £3. 17s. 9½d. each.

(1) Compact Form.		(2) Approximation.	(3) Practice.
4	21948	3 × 7316	3·8906250
	5487		6137
	65844	9 × 7316	272343750
12	71331		11671875
	5944-3		389063
	51212	17 × 7316	233437
	7316		£28463·8125
20	130816		16s. 3d.
	6515-16		
	21948	3 × 7316	
	£28463 16 3		

**EXAMPLES.**

- 8614 things at £2. 15s. 7½d. each.
- 395 things at £3. 11s. 1½d. each.
- 7218 things at £1. 14s. 7½d. each.
- 4500 dozen at £7. 18s. 6½d. per dozen.
- 500 gross at £15. 17s. 4½d. per gross.
- 432 tons at £4. 13s. 5½d. per ton.
- 8560 yards at £1. 12s. 4½d. per yard.
- 9463 gallons at £1. 13s. 6½d. per gallon.
- 5412 things at £3. 1s. 5½d. each.
- 736 things at £2. 5s. 11½d. each.

**7. To find the Price of any mixed (Integer with Fraction) Number of Articles given price of a single Article.**

(1) With  $\frac{1}{4}$ ,  $\frac{1}{2}$ , or any aliquot.

Multiply by whole number and *add* the aliquot part.

(2) With  $\frac{2}{3}$ ,  $\frac{3}{4}$ , or any fraction whose numerator and denominator differ by 1.

Multiply by whole number increased by 1 and *subtract*  $\frac{1}{3}$ ,  $\frac{1}{4}$ ... etc.

(3) With any other fraction—aliquotise it if convenient.

Multiply by whole number and add the successive aliquot parts.

(4) With any fraction—Decimalise it if convenient.

Multiply by the price in any form desirable (Practice etc.).

*Example 1.* 3 qrs. 5 bush. 2 pecks at 13s. 9d. per bushel.  
i.e.  $29\frac{1}{2}$  bushels at 13s. 9d. each.

*Example 2.*  $84\frac{1}{2}$  articles =  $84 - 24$  (Aliquot Notation) at £3. 1s.  $5\frac{1}{2}$ d.

$$\begin{array}{r}
 £3\cdot072916 \\
 \quad 48 \\
 \hline
 245833 \\
 12292 \\
 \hline
 £258\cdot125 \\
 1\cdot536 \\
 \cdot384 \\
 \hline
 £260\cdot045 \quad (0s. 11d.)
 \end{array}$$

These methods are applicable chiefly in the case of Weights and Measures reducible to a whole number and a fraction in the unit whose price is given.

**EXAMPLES.**

1.  $87\frac{1}{2}$  things at £2. 11s.  $7\frac{1}{2}d$ .
2.  $93\frac{3}{4}$  things at £1. 7s.  $11\frac{1}{2}d$ .
3.  $121\frac{1}{8}$  things at £2. 9s.  $8\frac{1}{2}d$ .
4. 10 qrs. 7 bush. 3 pecks at 9s.  $2\frac{1}{2}d$ . per bush.
5.  $320\frac{1}{4}$  things at £2. 6s. 8d.
6. 119 cwt. 1 qr. at £1. 16s. 5d. per cwt.
7. 700 yds. 2 ft. at 22s.  $7\frac{1}{2}d$ . per yd.
8. 874 ac. 3 roods at £4. 7s.  $9\frac{1}{2}d$ . per acre.
9. 1126 sq. yds. 5 sq. ft. at £1. 1s.  $3\frac{1}{2}d$ . per sq. yd.
10. 712 lbs. 10 oz. at 8s.  $4\frac{1}{2}$  per lb.

**8. To find Price of a Series of Quantities (belonging to any Weight or Measure) given price of one of the Units.**

(1) Use Ordinary Aliquots of the quantities with Decimals of the Price.

(2) Convert (by Reduction and Decimalisation) the given quantities into that of which the price is given.

Then use Practice for the money—ensuring accuracy to three places.

(3) Convert quantities and price into decimals.

Multiply correct to 3 places by the Method of Approximation.

*Example 1.* 367 cwt. 3 qrs. 14 lbs. at £17. 8s. 9d. per cwt.

(1)	17-4375000	(2)	367-8750	(3)	367-8750
	763		71		573471
	523125000		3678750		3678750
	104625000		2575125		2575125
	12206250				147150
		5s. $\frac{1}{4}$	6253-875		11036
$2\frac{1}{2}$	6399-56250	2s. 6d. $\frac{1}{2}$	91-969		2575
$1\frac{1}{2}$	8-71875	1s. 3d. $\frac{1}{2}$	45-984		184
$14\frac{1}{2}$	4-35937		22-992		
	2-17968		£6414-820		£6414-820
	£6414-820		16s. 5d.		16s. 5d.
	16s. 5d.				



*Example 2.* 13 lbs. 9 oz. 7 dwts. 13 grains at £3. 16s. 4½d. per lb.

(1) Practice. (Very long in this case.)	(2)	(3)
	13·75	13·75
	·02916̄	·02916̄
	·0017361	·0017361
	·00052083	·00052083
	<hr/> 13·781423	<hr/> 13·781423
	077 183	27·56285
	<hr/> 41 344	6·890711
	11 025	3·445355
	138	·861338
	96	·071778
	9	<hr/> £52·613
	<hr/> £52·612	12s. 3d.
	12s. 3d.	

### EXAMPLES.

- 371 oz. 13 dwts. 19 grs. at 7s. 10½d. per oz.
- 84 tons 12 cwts. 3 qrs. at £1. 5s. 7½d. per ton.
- 713 acres 3 roods 15 poles at £4. 9s. 5d. per acre.
- 515 sq. yds. 7 sq. ft. 110 sq. inches at £1. 7s. 4½d. per sq. yd.
- 271 cubic yds. 13 c. ft. at £5. 18s. 9d. per cub. yd.
- 125 cwts. 79 lbs. at £2. 15s. 4½d. per cwt.
- 13 furlongs 8 poles 3 yds. at £41. 5s. 6d. per furlong.
- 7864 galls. 3 qts. 1 pt. at 11s. 9½d. per gallon.
- 8450 qrs. 7 bush. at 33s. 7½d. per qr.
- 971 tons 17 cwts. 1 qr. at £21. 9s. 7½d. per ton.

### 9. To find the price of one Article or Unit.

(1) Given price of a whole number of such articles.  
Decimalise the money and divide in succession by the factors of the given number (Short Division) or use Method of Approximation (Division).

(2) Given price of a mixed number of such articles.  
Decimalise the mixed number and the price.  
Use Method of Approximation (Division).



*Note 1.* The Retailer after deciding the cost per lb. etc. of any purchase has only to add the profit per lb. (etc.) or per cent. to get the corresponding retail price.

*Note 2.* If price of retail-unit is so small as to fluctuate in 8ths and 16ths of a penny it is advisable to get the answer correct to 5 places and then by the Money-Decimalisation-Extension we can at once find price in 8ths or 16ths.

*Example.* 7 cwts. at £10. 7s. 6d., to find price per lb.

$$\begin{array}{r}
 7,8,4 \mid 10.375 \mid .01323 \\
 \underline{3\ 535} \\
 183 \\
 \underline{26} \\
 3
 \end{array}
 \qquad
 \begin{array}{r}
 .01323 \\
 \underline{5292} \\
 .0127008
 \end{array}
 \qquad
 \therefore \text{price} = 3\frac{1}{4}d.$$

Hence by selling at  $3\frac{1}{4}d.$  there can be no loss, and of course still less so at  $3\frac{1}{2}d.$

If the dealer wishes to make 20 per cent. profit he would add  $\frac{1}{5}$  to £10. 7s. 6d. before dividing.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 10 \quad 7 \quad 6 \\
 \underline{2 \quad 1 \quad 6} \\
 12 \quad 9 \quad 0 \\
 7,8,4 \mid 12.45 \mid .0158 \\
 \underline{4\ 61} \\
 69 \\
 \underline{7}
 \end{array}
 \qquad
 \therefore \text{price} = 3\frac{3}{4}d. \text{ per lb.}$$

### EXAMPLES.

1. Price per lb. if 15 cwts. cost £20. 5s. 6d.
2. Price per lb. if 7 tons cost £60. 8s. 9d.
3. Price per yd. if 750 yds. cost £42. 8s. 4d.
4. Price per yd. if 1000 yds. cost £50.
5. Price per sq. ft. if 192 sq. yds. cost £20. 16s. 8d.

### 10. The Method of Farthings.

Convert price into farthings and (if necessary) fractions of a farthing.

Multiply by the no. of farthings—mark off 3 places.

Add  $\frac{1}{24}$ —Answer is in £.

*Example 1.* 824 articles at 2s. 7½d. each = 127 l.

$$\begin{array}{r}
 82400 \\
 16480 \\
 5768 \\
 \hline
 104\cdot648 \\
 4\cdot360 \\
 \hline
 £109\cdot008 = 0s. 1\frac{3}{4}d.
 \end{array}$$

*Example 2.* 1256 things at 5½d. each = 21½ l.

$$\begin{array}{r}
 1256 \\
 2512 \\
 628 \\
 314 \\
 \hline
 27\cdot318 \\
 1\cdot138 \\
 \hline
 £28\cdot456 = 9s. 1\frac{1}{4}d.
 \end{array}$$

The difficulty of dividing by 24 may be avoided by using in succession the divisors 4 and 6 and crossing out the line derived from the 4—thus the above examples would appear in this form.

<p>(1) <math>104\cdot648</math>  <math>\underline{96\cdot162}</math>  <math>4\cdot360</math>  <math>\hline</math> <math>£109\cdot008</math></p>	<p>(2) <math>27\cdot318</math>  <math>\underline{6\cdot029}</math>  <math>1\cdot138</math>  <math>\hline</math> <math>£28\cdot456</math></p>
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## 11. The Method of Mils.

If we were to adopt the decimal system of coinage which has been repeatedly proposed since 1838, viz. that in which the radical coins are the £, florin, cent, and mil, each being  $\frac{1}{10}$  of the preceding and the sovereign being the Integer—the calculation of prices is a mere matter of multiplication and moving the point.

Reduce the price to mils (an instantaneous operation).

Multiply by the no. of mils and mark off 3 places.

*Example 1.* 824 articles at 1 fl. 3 cents 2 mils = 132 mils.

$$\begin{array}{r}
 82400 \\
 24720 \\
 1648 \\
 \hline
 £108\cdot768
 \end{array}$$

*Example 2.* 1256 things at 2 cents  $1\frac{3}{4}$  mils =  $21\frac{3}{4}$  mils.

$$\begin{array}{r}
 1256 \\
 2512 \\
 628 \\
 314 \\
 \hline
 £27.318
 \end{array}$$

### EXAMPLES.

1. 784 arts. at 2s.  $3\frac{3}{4}d.$ , 1s.  $7\frac{5}{8}d.$ ,  $8\frac{1}{16}d.$ , 121 mils, 45 mils.
2. 1218 arts. at  $7\frac{3}{8}d.$ ,  $4\frac{5}{16}d.$ ,  $9\frac{7}{8}d.$ , 51 mils, 127 mils.
3. 384 arts. at 2s.  $1\frac{1}{4}d.$ , 3s.  $1\frac{5}{16}d.$ ,  $7\frac{9}{16}d.$ , 1s.  $3\frac{3}{4}d.$ ,  $7\frac{5}{8}d.$
4. 3000 arts. at 76,  $49\frac{1}{2}$ ,  $73\frac{3}{4}$ , 161, 325 mils.
5. 7125 arts. at 241, 1125, 657, 834,  $104\frac{1}{2}$  mils.

## SECTION III

### THE ENGLISH SYSTEM OF WEIGHTS AND MEASURES.

**The Weights and Measures Act of 1826**  
(Jan. 1) enacted that

1°. The Brass Standard Yard of 1760 is the Imperial Standard Yard when at a temperature of 62° F. and from it all measures (lineal, superficial, solid) shall be constructed and the 36th part of this yard shall be *an inch*.

The length of a Pendulum vibrating *seconds* in lat. of London in a vacuum at sea-level is 39·1393 such inches—this gives the means of estimating the standard if original is lost.

2°. The Brass weight of one Pound Troy of 1758 is the Imperial standard weight and from it all other weights shall be derived. It is to contain 5760 grains and the Avoirdupois pound 7000 grains.

The weight of a cubic inch of distilled water is 252·458 grains Troy (Barometer 30 inches and Thermometer 62° F.)—this gives the means of recovering the Imperial standard pound if original is lost.

3°. The Imperial Standard Gallon (containing 10 pounds Avoirdupois weight of distilled water at 30 inches and 62° F.) is the standard measure of capacity for Liquids and Dry Goods. The weight of 10 pounds

Avoirdupois at 30 inches and 62° F. fills 277·274 cubic inches, ∴ the Imperial standard gallon contains 277·274 cubic inches.

Thus the imperial Bushel (8 gallons) will contain 2218·192 cubic inches.

Also it follows that "a pint of pure water weighs a pound and a quarter."

By the Report of 1820 the French metre was also declared to be 39·37079 inches, the English standard being used at 62° F.

The Houses of Parliament were burnt in 1834 and the standards destroyed. Fortunately however the Royal Astronomical Society in 1832 made a one-yard scale for themselves and compared it with Bird's Imperial standard of 1760 in 1834 before the fire.

The Astronomical Society's centre yard being taken as 36 inches, Bird's was found to be 35·999624; thus if the Astronomical Society's yard be measured, and ·000376 be taken from it, we get the true standard yard.

### **The Report of 1838.**

The Commission appointed in 1838 to consider the best means of restoring the standards made the following recommendations.

1°. They advocated the retention of a certain arbitrary brass rod and brass weight as the best standard yard and pound.

They found however the pendulum experiments (then recently made) doubtful and erroneous in many ways and the determination of the weight of a cubic inch of distilled water by no means accurate enough.

At the same time they affirmed that the standards can always be better restored by comparison with accurate copies (e.g. Astronomical Society's Scale and Ordnance Bars) than by reference to natural constants.

2°. They proposed that the lb. Avoirdupois should become the standard weight instead of the lb. Troy—that the Avoirdupois lb. and oz. and the Troy oz. should be retained (the Avoirdupois dram and the Troy lb. being abolished). Also the smaller weights of Troy

and Apothecaries' scales were to be retained for a time, but a decimal scale of Avoirdupois weights be made with 7 grains (the *Millet*) as the unit. In addition 3 series of decimal weights in grains, Troy ounces, and Avoirdupois lbs. were strongly recommended. The grain-weights were to be compared with the old Troy lb. of 5760 grs. and from them a platinum weight of 7000 grs. was to be constructed as the standard lb.

The 100 lbs. weight was to be called a *centner*.

3°. The standard of capacity was defined to be "the capacity which contains 10 pounds' weight of distilled water weighed in air at 30 inches and 62° F.," and this was called the Imperial Gallon.

This capacity is to be expressed in cubic inches but only as a subsidiary standard when the principal standard is inapplicable—the object of this recommendation is to remove as far as possible the errors which are involved in the determination of the standard of length.

The Imperial Gallon contains 277·274 cubic inches.

4°. The Government were strongly recommended to introduce decimal subdivisions of the acre (10 sq. chains = 1 acre, 10000 sq. links = 1 sq. chain) and to sanction the decimal subdivision of the foot as used by engineers.

5°. They proposed a decimal system of coinage with the sovereign as the basis—the radical coins being the pound, the florin ( $\pounds \frac{1}{10}$ ), the cent ( $\pounds \frac{1}{100}$ ), the mil ( $\pounds \frac{1}{1000}$ ).

The value of the cent would be  $2\frac{1}{2}d.$  =  $2\frac{1}{2}d.$  nearly.

The value of the mil would be  $\frac{1}{4}d.$  =  $\frac{1}{4}d.$  nearly.

In 1853 another Commission reported strongly in favour of the same coinage.

The Acts of 1879 and 1887 re-affirmed the units of 1826 but the Act of 1887 abolished the use of anything but the oz. Troy and grains for silversmiths.



### A Decimal Coinage.

The adoption of the Recommendation of the Commission of 1838 has been constantly proposed by the Associated Chambers of Commerce and many other public bodies interested. There can be no doubt of the great simplification such a step would produce.

Trade would not in any real sense be disturbed because the sovereign remains the integer—there would be some confusion at first over the new coins, but this would quickly disappear when once the ratio of old prices to the new was understood.

The method of Money-Decimalisation (to 3 places) used throughout this book shows how easy it would be to get hold of the value of any number of mils in s. d. f. and vice versa, e.g. 35 mils =  $8\frac{1}{4}d.$ , 247 mils =  $4s. 11\frac{1}{4}d.$  and so on— $3s. 7\frac{1}{4}d. = 180$  mils etc.

The old coins would for the most part be retained, e.g. the florin = 100 mils, the shilling = 50 mils, the sixpence = 25 mils, the  $2s. 6d. = 125$  mils.

The  $3d.$ ,  $1d.$ ,  $\frac{1}{2}d.$ ,  $\frac{1}{4}d.$  would all disappear in favour of 10 mils (cent), 5 mils, 2 mils, 1 mil.

The ratio of the penny to the 5-mils would be 200 : 240 or 5 : 6, thus five 5-mils would be taken for 6 pence.

Money would be quoted in £ and mils—prices in mils probably.

The advantages of such a decimal coinage are very great—among them are these :—

1. Most operations are thereby reduced to purely arithmetical operations.
2. Decimals would be universally understood.
3. A great saving of time in schools and business would be the result.

4. Fractions are avoided in the majority of operations.

5. The Decimal System lends itself to finer subdivisions in prices etc.

This reform taken in conjunction with the adoption of the Metric System of Weights and Measures would greatly reduce the labour of commercial calculation and above all would allow the use of all the Methods of Approximation which depend so largely upon Decimals.

### **Troy Weight.**

#### **TABLE.**

24 grains = 1 pennyweight.

20 dwts. = 1 ounce.

12 ozs. = 1 pound (lb. T.).

---

1 lb. T. = 240 dwts. = 5760 grains.

1 oz. T. = 480 grains.

Named from Troyes in France or Troy-Novant (London).

Used in weighing gold, silver, and other costly things.

Formerly used in philosophical observations but now generally superseded by the Metric System.

The grain was originally a dried wheat grain.

The Imperial Standard Pound Troy of 1758 is the standard from which all other weights are obtained.

Gold is weighed for jewellery purposes in "carats" and "carat-grains".

The word "carat" expresses the pureness without regard to the actual weight, but strictly the gold carat is  $\frac{1}{24}$  lb. T. = 10 dwts. = 240 grains T.

1 g. carat = 4 g. carat grains,  $\therefore$  1 g. carat grain = 60 grains T.

Since 1852 the Bank of England weighs Bullion in ounces and decimals of an ounce.

Diamonds are also weighed in "carats" and "carat grains", but these are quite distinct from the gold carats and grains.

$151\frac{1}{2}$  diamond carats = 1 oz. T., i.e. 303 d. carats = 960 grs. T.,  
 $\therefore$  1 d. carat =  $3\frac{1}{17}$  grs. T., hence 1 gold carat = 240 grs. T. =  $2\frac{3}{4}$  d. carats =  $75\frac{3}{4}$  d. carats. 1 d. carat =  $3\frac{1}{4}$  grs. T. nearly.

Pearls are weighed in "grains"—600 pearl grains = 1 oz. T.,  $\therefore$  1 pearl grain =  $\frac{1}{600}$  Troy grain exactly.

The Act of 1887 abolished all weights but the Troy oz. and grain for silversmiths.

1°. Mental Rules for Troy weight. To find price

- (1) per oz. given price per grain— $2 \times$  price in d. as £.
- (2) per lb. given price per grain— $6 \times$  price in f. as £.
- (3) per lb. given price per dwt.—price in d. as £.
- (4) per lb. given price per oz.—price in d. as s.
- (5) per grain given price per oz.— $\frac{1}{2}$  price in £ as d.
- (6) per oz. given price per lb.—price in s. as d.

2°. Special Rule for Troy weight.

Given price per oz.—to find price of lb., oz., dwts., grs.

Reduce lbs. to oz.

Consider oz. as £, dwts. s., grains  $\frac{1}{4}$ d.

Decimalise as for money (Five and Four).

Then use Practice or Decimalisation of Price.

*Example.* 5 lbs. 11 ozs. 8 dwts. 14 grs. at £1. 17s. 5 $\frac{1}{2}$ d. per oz.

$$\begin{array}{r}
 \text{i.e. } £71. 8s. 7d. = 71.4291\bar{6} \\
 \text{and reversing } £1. 17s. 5\frac{1}{2}d. = 192.781 \\
 \hline
 71.429 \\
 57.143 \\
 4.999 \\
 143 \\
 64 \\
 \hline
 1 \\
 \hline
 £133.779 \quad (15s. 7\frac{1}{2}d.)
 \end{array}$$

### EXAMPLES.

1. Price per oz. given price per grain as 9s. 7 $\frac{1}{2}$ d., 3s. 10d., 2s. 6d., 3s. 7 $\frac{1}{2}$ d., 1s. 9d.

2. 41000 ozs. 320 grs. at £3. 17s. 10 $\frac{1}{2}$ d. per oz. and at £3. 17s. 9d. per oz.

3. 41 lbs. 7 ozs. 18 dwts. 16 grs. at £3. 16s. 5 $\frac{1}{4}$ d. per oz. and at £2. 11s. 5 $\frac{1}{2}$ d. per oz.

4. 5240 $\frac{5}{8}$  ozs. at 39 $\frac{5}{8}$ d. per oz.

**EXAMPLES.**

1. Find price of pack of wool at  $3\frac{1}{4}d.$ ,  $5\frac{5}{8}d.$ ,  $7\frac{1}{2}d.$ ,  $5\frac{3}{8}d.$ ,  $1s. 1\frac{1}{4}d.$  per lb.

2. Find price of sack of flour at  $2\frac{1}{4}d.$ ,  $3\frac{1}{2}d.$ ,  $6\frac{1}{4}d.$ ,  $5\frac{3}{8}d.$ ,  $7\frac{1}{2}d.$  per lb.

3. Find price of puncheon of prunes at  $3\frac{1}{2}d.$ ,  $4\frac{1}{4}d.$ ,  $5\frac{1}{2}d.$ ,  $6\frac{3}{4}d.$ ,  $7d.$  per lb.

4. Find price of cwt. at  $4\frac{1}{4}d.$ ,  $7\frac{1}{2}d.$ ,  $11\frac{3}{4}d.$ ,  $9\frac{1}{8}d.$ ,  $5\frac{3}{4}d.$  per lb.

2°. To convert long tons into short tons—add  $\frac{1}{4}$  of number of English tons.

To convert short tons into long tons—take  $\frac{1}{4}$  from number of American tons.

3°. Special Rule for Avoirdupois weight.

Given price of a ton.

To find cost of tons, cwts., qrs.

Call tons £, cwts. s., qrs. 3d.

Decimalise as for money (Five and Four).

Use practice or Decimalisation of the price.

*Example.* Given cost per ton as £7. 14s. 11d. to find cost of 9 tons 19 cwts. 3 qrs.

		s. d.
		£9·9875
		7
10	$\frac{1}{2}$	69·9125
5	$\frac{1}{2}$	4·99375
		2·496875
1	$\frac{1}{8}$	£77·403125
		·041614
		£77·361 (7/2 $\frac{1}{2}$ )

**EXAMPLES.**

1. Reduce 3125 cwts. to lbs.
2. Convert 7250 English tons to American tons.
3. Reduce 648 cwts. to lbs.
4. Convert 8364 American tons to English tons.
5. 18 tons 13 cwts. 3 qrs. at £5. 13s. 7 $\frac{1}{2}d.$  per ton.

6. 26 tons 10 cwts. 2 qrs. at £2. 6s. 8½d. per ton.
7. 13 tons 19 cwts. 1 qr. at £1. 4s. 9½d. per ton.
8. 5 tons 12 cwts. 2 qrs. 15 lbs. at £2. 3s. 9½d. per cwt.
9. 6 tons 9 cwts. 3 qrs. 19 lbs. at £1. 5s. 6d. per cwt.
10. 1841 tons 15 cwts. at £3. 17s. 9½d. per ton.

### Wine and Spirit Measure.

#### TABLE.

4 gills	= 1 pint.
2 pts.	= 1 quart.
4 qts.	= 1 gallon.

The gill is also called a noggin or quartern.

The English imperial gallon = 277·274 cub. inches and holds 10 lbs. av. of pure water.

The United States' wine gallon = 231 cub. inches and holds 8½ lbs. of pure water.

This measure is used for wines, spirits, and all liquids other than malt liquors and water.

The casks used in the trade are the *pipe* or *butt*, the *hogshead*, the *quarter-cask* and the *octave*—each one being half the preceding one.

Their contents differ considerably (depending on the place of export) but there is always the same relation between them.

The recognised values are these:—

Port (Lisbon). Pipe of 115 galls. 56 dozen to the pipe.

Sherry (Portugal). Butt of 108 galls. 52 dozen to the butt.

Madeira and Cape. Pipe of 92 galls. 44 dozen to the pipe.

Sicilian. Pipe of 93 galls. 44 dozen to the pipe.

Rhenish and Hock. Aum of 30 galls. 14 dozen to the aum (hhd.).

Clarets and French wines, including Hermitage.  
Hhd. of 46 galls. 22 dozen to the hhd.

Teneriffe and Vidonia. Pipe of 100 galls. 48 dozen to the pipe (about).

Malaga. Pipe of 105 galls. 50 dozen to the pipe.

Bucellas. Pipe of 117 galls. 57 dozen to the pipe.

Tent. Hhd. of 52 galls. 25 dozen to the hhd.

Spanish Red. Tun of 210 galls. 100 dozen to the tun.

Spirits. By imperial gallon. 63 galls. to the hhd.

The tun is rarely used except for oil. It is normally divided into 2 pipes or butts, 3 puncheons, 4 hhds., and 6 tierces. Oil is measured by the tun (of 210 galls.) but really weighed, as the imperial gallon of oil is estimated to weigh 9 lbs. av.

∴ 1 tun of oil weighs 20 cwt. 1 qr. (See Corn.)

#### 1°. Mental Methods.

As these values of the pipes etc. in gallons are fixed and standard, Inversion Tables are the most rapid means of calculating the price from the price per gallon. In any case the Principle of Inversion can be used.

e.g. per imp. hhd.—price =  $5s. 3d. \times$  price per gallon in pence.  
=  $£3. 3s. 0d. \times$  price per gallon in shillings.

### EXAMPLES.

1. Construct the Inversion Tables for the Lisbon pipe (115), the sherry butt, the Malaga pipe.

2. Find price per all three at  $1s. 7\frac{1}{4}d.$ ,  $7s. 6\frac{1}{2}d.$ ,  $5s. 6d.$ ,  $12s. 9d.$ ,  $13s. 10d.$  per gallon.

3. Find price of claret hhd. at  $3s. 6d.$  per bottle.

4. Find price of imp. hhd. at  $4s. 6d.$ ,  $3s. 9d.$ ,  $5s. 10d.$  per gallon.

5. Compare the value per gallon of the port pipe at £86 and the sherry butt at £84.

2°. Special Rule for wine measure.

To find price per gallon or dozen from price per pipe etc.

(1) Decimalise price and divide by factors of gallons or dozens—using Short Division.

(2) Decimalise price and divide by gallons or dozens—using Division Approximation.

*Example.* Lisbon butt at £71. 5s. 8½d.—find price per gallon.

$$\begin{array}{r} (1) \quad 9 \overline{) 71 \cdot 28541\dot{6}} \\ \quad 12 \overline{) 7 \cdot 9206} \\ \quad \quad \cdot 6600 \end{array} \quad \begin{array}{r} (2) \quad 10 \cdot 8 \overline{) 71 \cdot 28} \mid \cdot 660 \\ \quad \quad \quad 6 \ 40 \end{array}$$

$$\cdot 6600 = 13s. 2\frac{1}{2}d.$$

### EXAMPLES.

1. Lisbon pipe at £84. 19s. 7½d.—price per dozen.
2. Rhenish aum at £47. 16s. 5¼d.—price per gallon.
3. Imperial hhd. at £74. 18s. 6¾d.—price per gallon.
4. Madeira pipe at £95. 10s. 7½d.—price per dozen.
5. Malaga pipe at £124. 5s. 7¾d.—price per gallon.

*Note.* The measures superseded by the imp. gallon are

- (1) the old wine gallon of 231 cub. inches capacity.
- (2) the old beer gallon of 282 cub. inches capacity.
- (3) the Winchester bushel of 2150·42 cub. inches capacity.

∴ 1 imp. bushel = 1·031557 Winchester bushel.

1 Winch. bushel = ·9694472 imp. bushel.

The wine gallon and the Winchester bushel are still in use in the U.S.A. and some of our own colonies.

### Gaugers' Rule.

*To find contents of a cask in gallons.*

Add  $\frac{2}{3}$  of difference of head and bung diameters to head diameter.

Multiply square of sum by length—all in inches.

For imp. gallons  $\times \cdot 0028325$ .

For U.S. gallons  $\times \cdot 0034$ .

For old beer gallons  $\times \cdot 00287$ .

If staves are but little curved add  $\frac{1}{4}$  instead of  $\frac{2}{3}$ .

**Ale and Beer Measure.**

## TABLE.

2 pints	= 1 quart.
4 qts.	= 1 gallon.

---

$4\frac{1}{2}$ gallons	= 1 pin or keg.
9 gallons	= 1 firkin.
18 gallons	= 1 kilderkin.
36 gallons	= 1 barrel.
54 gallons	= 1 hogshead.
108 gallons	= 1 butt.
216 gallons	= 1 tun.

This measure is used for all malt liquors and water.  
The half-pint is a retail measure.

The usual casks of the beer trade are the *butt*, *hogshead*, *barrel*, *kilderkin*, *firkin*, and *pin*—all of which have the fixed values given.

The *puncheon* of  $1\frac{1}{2}$  hhd. (72 galls.) is rarely used.

The *tun* is hardly ever used for beer.

*Vats* are of any size (10 barrels to 150).

*Squares* are used for fermenting—they are of any capacity.

The *tun* is chiefly used for oil and is then only estimated to contain 210 imp. gallons.

*Note.* The measure for *liquids* is the imp. gallon.

To be a legal measure it must contain at 62° F. and 30 inches pressure 10 lbs. av. weight of distilled water.

It measures 277·274 cub. inches.

∴ 1 cub. inch of standard water weighs 252·458 Troy grains.  
and 1 cub. foot of standard water weighs 62·321 lbs. av.

The measure for *seeds* and *dry goods* is the imp. bushel (of 8 gallons).

∴ its capacity is 2218·192 cub. inches.

To reduce cub. ins. to bushels—multiply by 5 and divide by 11091.

To reduce cub. ins. to gallons—multiply by 40 and divide by 11091.



**Corn or Dry Measure.**

## TABLE.

4 quarts	= 1 gallon.
2 gallons	= 1 peck.
4 pecks	= 1 bushel.
8 bushels	= 1 quarter.

---

1 pottle	= 2 quarts.
1 strike	= 2 bushels.
1 coomb	= 4 bushels.
1 chaldron	= 4 quarters.
1 load or wey	= 5 quarters.
1 last	= 10 quarters.
1 quarter	= 64 gallons.

This Table is used for all dry commodities not generally heaped above the measure.

But all grain and many other articles of commerce are being sold more and more by weight in lbs. av.

For grain all the principal places have a local weight which is always quoted there.

At Mark Lane, wheat though nominally sold by imp. qr. is in reality sold by weight, the qr. being taken at 504 lbs. and the bushel at 63.

This estimate does not form the basis for the entire wheat trade though a large number of towns use it.

The *Mark Lane Express* takes these as the standards for comparison in its weekly list of prices:—

- |                                 |          |
|---------------------------------|----------|
| 1. White and red wheat.         | 480 lbs. |
| 2. Malting and grinding barley. | 400 lbs. |
| 3. Oats.                        | 312 lbs. |

The most usual local weights for wheat are 504 and 480, for barley are 448 and 400, and for oats 336—but the variation from these is considerable.

Owing to these confusing differences the necessity for the Mark Lane Table is very great—it gives the local price and weight and the reduced prices to the above 3 weights—thus the state of the market in any town can be at once compared with that in another town.

Coals are now sold by weight in tons, cwts., qrs. (1—2 William IV.).

### Hay and Straw.

- 1 truss of straw = 36 lbs. av.
- 1 truss of old hay = 56 lbs. av.
- 1 truss of new hay = 60 lbs. av.
- 1 load = 36 trusses.
- 1 load of straw = 11 cwts. 64 lbs. = 1296 lbs.
- 1 load of old hay = 18 cwts. = 2016 lbs.
- 1 load of new hay = 19 cwts. 32 lbs. = 2160 lbs.

Hay is considered new for 3 months and is called old on Sept. 1.

- 1 cubic yd. of new hay weighs 6 stone.
- 1 cubic yd. of oldish hay weighs 8 stone.
- 1 cubic yd. of old hay weighs 9 stone.

*To find the weight of hay contained in a stack.*

Get content in c. ft.—reckoning vertical height as the height to the eaves +  $\frac{1}{3}$  of slant height (from eaves to top).

Divide by 27 and multiply result by 6, 8, 9 according to the age or solidity of the stack.

The answer is the weight in *stones*.

If stack is round, multiply *corrected height* by square of circumference  $\times .07958$ .

Another method is to reckon 10 c. yds. to the ton for old hay and 12 c. yds. to the ton for new hay.

### Cattle.

*To calculate the weight of cattle.*

Measure round the animal just behind the shoulders, then along the back from the fore part of the shoulder-blade to the bone at the tail.

Multiply the square of the girth by 5 times the length in feet.

Divide by 21 and you have weight of the four quarters in stones.

If very fat cattle add  $\frac{1}{20}$ , if very lean subtract  $\frac{1}{20}$ .

The quarters are just above  $\frac{1}{2}$  the weight of the living animal.

The skin is  $\frac{1}{18}$  and the tallow  $\frac{1}{12}$  of whole weight.

### Length Measure.

#### TABLE.

12 inches	= 1 foot.
3 ft.	= 1 yard.
$5\frac{1}{2}$ yds.	= 1 rod, pole, perch.
40 poles	= 1 furlong.
8 fms.	= 1 mile.
3 miles	= 1 league.
<hr/>	
1 furlong	= 220 yds.
1 mile	= 5280 ft.
	= 1760 yds.

The general measure for length.

Among ordinary mechanics the *inch* is usually divided into  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ , or into twelfths, but in scientific calculations it is decimally divided or the *foot* is decimally divided and subdivided.

1 fathom = 6 ft.—used in sounding depths.

1 hand = 4 inches—used in measuring horses.

1 knot or geographical mile = 6080 ft.

1 g. mile =  $1\frac{1}{3}$  of a common mile.

1 degree = 60 g. miles =  $69\frac{1}{3}$  miles.

1 line =  $\frac{1}{12}$  of an inch.

**Surveyors' Long Measure.**

## TABLE.

100 links = 1 chain (Gunter's).

---

1 chain = 66 feet = 22 yds.

1 pole = 25 links.

1 furlong = 220 yds. = 10 chains.

1 mile = 80 chains.

Largely used in measuring land.

To multiply by 1760—multiply by 16, repeat one place to right, add and suffix 0 to the sum.

To multiply by 5280—multiply by 12, repeat one place to right, add and multiply the sum by 40.

**EXAMPLES.**

1. Multiply 86415, 312, 7125 by 1760.
2. Multiply 561, 8345, 7080 by 5280.

**Cloth Measure.**

## TABLE.

$2\frac{1}{4}$  ins. = 1 nail.

4 nails = 1 quarter.

4 quarters = 1 yard.

---

1 Flemish ell =  $\frac{3}{4}$  yd. = 3 qrs.

1 English ell =  $1\frac{1}{4}$  yd. = 5 qrs.

1 French ell =  $1\frac{1}{2}$  yd. = 6 qrs.

1 Scotch ell = 37.06 inches.

Used by linen and woollen-drapers.

The Scotch and English ells are used in measuring holland.

For cloth, the yard is divided into  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ .

**Square Measure.**

## TABLE.

144 sq. inches	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$ sq. yds.	= 1 sq. perch.
40 sq. perches	= 1 rood.
4 roods	= 1 acre.
640 acres	= 1 sq. mile.

---


$$1 \text{ acre} = 4840 \text{ sq. yds.}$$

$$= 160 \text{ sq. perches.}$$

Used for all kinds of areas.

**Standard Areas with Mental Rules.**

- (1) 1 acre—price =  $20\frac{1}{2} \times$  price per sq. yd. in d. as £.
- (2) 1 acre—price =  $8 \times$  price per sq. perch in s. as £.
- (3) 1 yard of land = 30 acres—price =  $1\frac{1}{2} \times$  price per acre in s. as £.
- (4) 1 hide of land = 100 acres—price =  $5 \times$  price per acre in s. as £.
- (5) 1 square = 100 sq. ft. in flooring, thatching, etc.—price =  $8s. 4d. \times$  price in d. per sq. ft.
- (6) 1 square = 1000 shingles in paving and roofing—price =  $\text{£}1. 0s. 10d. \times$  price in f. per shingle.
- (7) 1 rod of brickwork =  $272\frac{1}{4}$  sq. ft. ( $1\frac{1}{2}$  bricks thick)—price =  $\text{£}1. 2s. 8\frac{1}{4}d. \times$  price in d. per sq. ft.

**Surveyors' Square Measure.**

## TABLE.

10000 sq. links	= 1 sq. chain.
10 sq. chains	= 1 acre.

---

Used for measuring land.

A square whose side is 10 chains contains 10 acres.  
 A square whose side is 80 chains contains 1 sq. mile.  
 $\therefore$  1 sq. mile = 6400 sq. chains = 64 million sq. links.  
 1 rood = 25000 sq. links =  $2\frac{1}{2}$  sq. chains.  
 1 sq. perch = 625 sq. links.

For the areas of all Geometrical Figures see the Formulæ of Mensuration.

### Cross Multiplication or Duodecimals.

Generally employed by painters, bricklayers etc.

Largely used also by engineers of all kinds.

The dimensions are taken in ft., inches, parts etc. decreasing from left to right by twelfths. Inches are spoken of as primes, parts as seconds, and then there are thirds, etc.

#### TABLE.

1 ft. = 12 ins. or primes (').

1 inch = 12 parts or seconds (").

1 part = 12 thirds ("" ) etc.

#### *The Rule for Cross Multiplication.*

Write terms of multiplier under corresponding terms of multiplicand.

Multiply each term of multiplicand beginning at the lowest by each term of the multiplier beginning at the highest—using 12 as divisor for all products not feet and placing your remainder in its place in accordance with this rule—"The order of a product is the sum of the orders of its factors." Carry quotient in usual way and proceed for each part of the multiplier as if it were the multiplication of a weight or measure proceeding by twelves.

Add in the usual way, using 12 as divisor to know what to carry.

*Example.* To find area of plank  $20\frac{3}{4}$  ft. by  $10\frac{1}{2}$  inches.

$$\begin{array}{r}
 20 \text{ ft. } 9' \\
 \hline
 10' \quad 6'' \\
 17 \text{ ft. } 3' \quad 6'' \\
 \hline
 10' \quad 4'' \quad 6'' \\
 18 \text{ ft. } 1' \quad 10'' \quad 6'' \\
 \hline
 18 \text{ sq. ft. } 22\frac{1}{2} \text{ sq. inches.}
 \end{array}$$

The answer shows that the same divisions are used for sq. measure, and it is equally obvious that if we were to multiply by a further linear quantity we should get the same divisions in cubic measure.

The divisions in sq. measure are spoken of as superficial ft., primes, seconds, thirds.

The divisions in cub. measure are spoken of as solid ft., primes, seconds, thirds.

It is to be carefully observed that in linear dimensions primes are inches, in superficial dimensions seconds are sq. inches, in solid dimensions thirds are cub. inches.

This gives us very simple rules for reconversion of divisions into inches.

1. For linear divisions—reduce seconds to fractions of a prime—then primes and fractions are inches.

2. For square divisions—reduce primes to seconds and thirds to fractions of a second—the seconds and fractions are sq. inches.

3. For cubic divisions—reduce primes and seconds to thirds and fourths to fractions of a third, then thirds and fractions are cub. inches.

The Rule, “The order of a product is the sum of the orders of its factors,” is easily proved, e.g.

$$5' \times 7'' = \frac{5}{12} \times \frac{7}{144} = \frac{35}{1728} = 35''' = 2'' 11'''.$$

$$4 \text{ ft.} \times 3' = 4 \times \frac{3}{12} = \frac{12}{12} = 12' = 1 \text{ sq. ft.}$$

Among mechanics linear measurements are not taken beyond  $\frac{1}{16}$  of an inch, the divisions of the inch being  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ .

These are easily converted into cross-multiplication :

$$\frac{1}{2} \text{ in.} = \frac{6}{12} = 6'', \quad \frac{1}{4} \text{ in.} = \frac{3}{12} = 3'', \quad \frac{1}{8} = \frac{1\frac{1}{2}}{12} = 1'' 6''',$$

$$\frac{1}{16} = \frac{\frac{3}{4}}{12} = 9'''.$$

Similarly for fractions of a foot.

### EXAMPLES.

1. Find by duodecimals—area of room 16 ft. 9 ins. by 12 ft. 7 ins.

2. Find by duodecimals—volume of a sq. log having edges  $16 \times 12\frac{1}{2} \times 10\frac{1}{2}$  feet.

3. Find by duodecimals—area of floor  $17\frac{1}{2}$  ft. by  $13\frac{3}{4}$  ft.

4. Find by duodecimals—the areas of the S. Petersburg, Swedish, and Norwegian deals.

5. Find by duodecimals—volume of a solid rectangular slab whose edges are  $17\frac{1}{2}$ ,  $13\frac{1}{2}$ ,  $9\frac{3}{4}$  feet.

6. Find by duodecimals—price of paving 300 yds.  $\times$  28 ft. with granite at 3s. 7d. per sq. yd.

7. Find by duodecimals—price of building earth-work edges 150 yds., 84 yds., 38 ft. at 15s. 7d. c. yd.

8. Find by either process—price of glazing 120 yds.  $\times$  110 yds. at 5½d. per sq. ft.

9. Find by either process—price of building 312 sq. perches 10 sq. yds. at £13. 15s. 10d. per sq. perch.

10. Do all the Examples on carpeting etc. by duodecimals (see page 80).

### Solid or Cubic Measure.

#### TABLE.

$$1728 \text{ cub. ins.} = 1 \text{ cub. ft.}$$

$$27 \text{ cub. ft.} = 1 \text{ cub. yd.}$$

---

Used for all solids of any regularity.



**Standard Solids with Mental Rules.**

- (1) 1 load of rough timber = 40 cub. ft.—price =  $\frac{1}{4} \times$  price per cub. foot in d. as £.
- (2) 1 load or ton of square timber = 50 cub. ft.—price = 4s. 2d.  $\times$  price in d. per cub. foot.
- (3) 1 ton of shipping = 42 cub. ft.—price = 3s. 6d.  $\times$  price in d. per cub. foot.
- (4) 1 register ton of shipping = 100 cub. ft.—price = 8s. 4d.  $\times$  price in d. per cub. foot.
- (5) 1 load of earth = 27 cub. ft.—price = 2s. 3d.  $\times$  price in d. per cub. foot.
- (6) 1 cord of wood = 4 ft.  $\times$  4 ft.  $\times$  8 ft.—price = 10s. 8d.  $\times$  price in d. per cub. foot.

•

**TIMBER.****1. Board Measure.**

In board measure all boards are reckoned 1 inch thick.

1 board foot = 1 sq. foot (1 inch thick) =  $\frac{1}{12}$  c. ft.

12 board feet = 1 c. ft.

In board and lumber measure estimates are made on 1 inch-thickness.

$\frac{1}{4}$  of the price is added for every  $\frac{1}{4}$  inch-thickness beyond 1 inch.

To find the board measure of wood used in carpentry and joinery.

Multiply length in ft. by width  $\times$  thickness in inches and divide by 12.

**2. Logs.**

A squared log is one whose sides are flat.

A round log is one roughly lopped of its branches.

(1) To find cubic content of a squared log.

(a) Product of edges if sides are rectangles.

(b) Mean area  $\times$  length if sides taper regularly.

(2) To find cubic content of a round log.

(a) (Very accurately) Mean area  $\times$  length—the mean area being calculated by measuring the girths in various places.

(b) (Accurately) Divide mean girth by 3.545 and multiply square of quotient by length of log.

(c) (Commonly) Square  $\frac{1}{4}$  of mean girth and multiply by length of log.

(3) To find cubic content of any log.

A table of the average weight of a cubic ft. in lbs. avoird. for various woods is used and from this the c. content is calculated.

(4) To find how many board ft. (sq. ft. 1 inch thick) can be cut from a given log.

Multiply length in feet by breadth  $\times$  thickness (both in inches) and divide by 12.

### 3. The S. Petersburg Standard Hundred.

Baltic and American deals are sold in the English market by the S. Petersburg standard, which is more and more being used as the general basis of value.

It consists of 120 pieces, 12 ft.  $\times$  11 ins.  $\times$   $1\frac{1}{2}$  ins.

Hence solid content = 165 c. ft.

(1) To find no. of S. Petersburg standards in a given no. of c. ft.

Divide c. ft. by 165.

(2) To find no. of S. Petersburg standards in a given no. of loads.

Multiply loads by 2 and divide by 33.

(3) To find no. of S. Petersburg standards in a given no. of deals of any other standard.

Find c. ft. in given deals and divide by 165.

*Note.* The difference between planks, deals, and battens is that planks are wider, battens narrower, than deals.

Battens are commonly sold by the Batten Standard Hundred, i.e. 120 pieces, 12 ft.  $\times$  7 ins.  $\times$   $2\frac{1}{2}$  in., but they are increasingly sold by the S. Petersburg standard.

Planks are sold by the load of 50 c. ft. or by the S. Petersburg standard.

## 4. Prices of Timber.

Square or hewn timber is sold per c. ft. or load of 50 c. ft.  
 Wainscot logs per c. ft. (in London per 18 c. ft.) or per inch.  
 Fancy woods per foot super. or per ton.  
 Lathwood and firewood per c. fathom.  
 Flooring boards per square of 100 ft.  
 Staves per mille of pipe or per 120 pieces.  
 Cask timber per mille of 1200 pieces.  
 Deals per foot super or per running foot (1440 to S. Petersburg standard) or per S. Petersburg standard.  
 Duties on timber, deals, hewn and sawn woods are calculated per load of 50 c. ft.

(1) To compare values of other standards with S. Petersburg standard—given price per standard hundred.

Use  $11 \times 1\frac{1}{2}$  as basis =  $16\frac{1}{2}$ .

Rule. Multiply price by  $2 \times \text{width} \times \text{thickness}$  in inches and divide by 33.

*Ex.*  $2\frac{1}{2} \times 7$  battens—price per S. Petersburg standard £10.  $\therefore$  price per hundred =  $\text{£}10 \times \frac{16\frac{1}{2}}{33} = \text{£}10.12s. 1d.$

(2) To compare values of other standards with S. Petersburg standard—given price per foot run.

Use 1440 as basis to find no. of equivalent linear feet.

*Ex.*  $2\frac{1}{2} \times 9$  deals— $\frac{16\frac{1}{2}}{22\frac{1}{2}} = \frac{1}{1\frac{1}{2}}$  of 1440 = 1056.

$\therefore$  price per hundred =  $1056 \times \text{price per foot run}.$

(3) To compare value of a load of timber—given price per standard hundred.

Use 165 c. ft. as basis.

Rule. Multiply price by 10 and divide by 33.

Similarly for any number of c. ft.

(4) Special rule for timber squares and fractions.

Consider each square £5, each sq. ft. 1s., each twelfth 1d.

Then use Practice for the Price.

**EXAMPLES.**

Find price of

1. Hundred of deals  $3\frac{1}{2} \times 10$  at  $6\frac{1}{4}d.$  per ft. run S. Petersburg standard.
2. Hundred of deals  $3 \times 9$  at £10. 17s.  $8\frac{1}{2}d.$  per S. Petersburg standard.
3. Hundred of deals  $2\frac{3}{4} \times 10$  at £15. 12s.  $8d.$  per S. Petersburg standard.
4. Hundred of deals  $2\frac{1}{2} \times 9$  at  $3\frac{3}{4}d.$  per ft. run S. Petersburg standard.
5. Hundred of deals  $3 \times 11$  at  $1s. 1\frac{1}{2}d.$  per ft. run S. Petersburg standard.
6. 25 loads at £15. 11s.  $6d.$  per S. Petersburg standard.
7. 1 load at £20. 8s.  $10d.$  per S. Petersburg standard.
8. 420 c. ft. at £12. 11s.  $9d.$  per S. Petersburg standard.
9. 3 sq. 95 sq. ft. 7 sq. primes at £2. 5s.  $10d.$  per sq.
10. 17 sq. 80 sq. ft. 6 sq. primes at £3. 7s.  $11d.$  per sq.

**5. Standards and Prices in Trades.**

1. Carpentry is estimated by the foot run or sq. foot.
2. Joinery is estimated by the sq. foot or sq. of 100 sq. ft.  
The timber used in carpentry is reduced to c. ft., but in joinery it is estimated by the ft. super.  
Flooring per square of 100 sq. ft.  
Parquet floors per foot super.  
Door frames per foot cube—doors per foot super.  
Staircases per foot super.
3. Glazing is estimated per sq. ft.—by duodecimals (dimensions ft., ins., parts).  
Crossbars are included in general measure.  
Ovals are taken as rectangular.  
Glass is measured per foot super.
4. Painting is estimated per sq. yd.—allowance for difficult places.  
Charges at price per sq. yd. per coat.  
Writing—height of letters measured and charged per inch run.
5. Room-papering is estimated per sq. ft. or per sq. yd. or per square.  
Paper is sold in pieces of 12 yds. of given width (generally 21 ins. wide) or per yd. of given width.  
One piece in ten is allowed for waste except in small patterns.

(1) Rule for no. of pieces required for a room—find area of walls in sq. ft. and divide by  $36 \times$  width in ft.

(2) Rule for finding area of walls—multiply  $2 \times (\text{length} + \text{breadth})$  by height.

Allowance must be made for doors, windows, fire-places etc.

6. Carpeting is estimated per sq. yd. or per yd. of given width (or per carpet).

Rule for area of floor or ceiling—multiply length by breadth.

7. Plastering is estimated per sq. yd. or per square for plain work, per sq. ft. for plain cornices, per ft. run for carved cornices.

8. Whitewashing is estimated per sq. ft. or sq. yd.

9. Distempering is estimated per yd. super.

Mouldings are measured per ft. or yd. run.

10. Brickwork is measured by the sq. yd. or sq. perch.

Standard rod of brickwork = 272 sq. ft. ( $1\frac{1}{2}$  bricks thick).

Standard rod of reduced brickwork = 408 sq. ft. (1 brick thick).

In estimating no. of bricks—add  $\frac{1}{2}$  in. to thickness of the brick for mortar.

Rule for sq. yds. or rods of brickwork.

Multiply length  $\times$  height of wall  $\times \frac{\text{thickness in } \frac{1}{2} \text{ bricks}}{3} \div 9$  (for sq. yds.) or  $272\frac{1}{2}$  (for rods).

272 sq. ft. of bricks— $1\frac{1}{2}$  thick—require 4500 bricks with waste.

A bricklayer's standard perch is also recognised— $16\frac{1}{2}$  ft.  $\times$  1 ft.  $\times$  14 ins. =  $19\frac{1}{4}$  c. ft.

11. Masonry is estimated per ft. super, per c. ft. or yd., per standard perch.

The mason's standard perch is  $16\frac{1}{2}$  ft.  $\times$  1 ft.  $\times$  18 ins. =  $24\frac{3}{4}$  c. ft.

Concrete per c. yd. if 12 inches thick or over—per sq. yd. if under 12 inches thick.

Granite per sq. yd. Asphalte per ft. super.

12. Paving is measured per sq. foot or sq. yd.

13. Excavating is estimated per cubic yd.

1 ton of soil occupies 18 c. ft.

6 gallons 1 pint of water weigh  $62\frac{1}{2}$  lbs.

Rule for finding water in a well.

Multiply  $\frac{1}{2}$  circumference by  $\frac{1}{2}$  diameter and product by depth.

Then reckon 6 gallons 1 pint per cube foot.

14. Plumbing materials—pipes are reckoned per foot run.

Lead is charged per cwt., solder per lb.

15. Slating is measured by foot super or per sq. of 100 sq. ft.

**EXAMPLES.**

Find cost of

1. Carpeting a room 20 ft. 7 ins. by 15 ft. 6 ins. at 2s. 6d. per yd. 28 inches wide, with allowance for a hearth 11 ins. by 39 ins.

2. Carpeting a room 10 ft. by 18 ft. at 3s. 5d. per sq. yd.—hearth  $1\frac{1}{2}$  ft. by  $3\frac{1}{2}$  ft.

3. Papering a room 14 ft.  $\times$  10  $\times$  8 at 11s. 9d. per piece 26 ins. wide—with allowance of window  $4\frac{1}{4} \times 2\frac{1}{2}$  ft., fire-place  $3 \times 4\frac{1}{2}$ , door  $6 \times 2\frac{3}{4}$ .

4. Painting a room  $25\frac{1}{2}$  ft.  $\times$  16  $\times$   $12\frac{1}{2}$  at 1s. 1d. per sq. yd. per coat, with two coats—with allowance for two windows  $5 \times 3$  ft., and a mantelpiece  $4 \times 3\frac{1}{2}$  ft.

5. Papering a room 20 ft.  $\times$  16  $\times$  10 at  $6\frac{1}{2}$ d. per yd. of 27 ins. width—window  $3\frac{1}{2} \times 2\frac{1}{4}$ , door  $5\frac{1}{2} \times 2\frac{3}{4}$ , mantelpiece  $3\frac{1}{2} \times 3$ .

**Mensuration.**

Mensuration is the method of calculating the magnitude of geometrical figures.

One important branch of it is land-surveying, which is practically performed with the aid of Gunter's chain of 100 links or 22 yds.

1 sq. chain thus equals 484 sq. yds.,  $\therefore$  10 sq. chains = 1 acre, hence the connection between the ordinary measures of length and area and Gunter's chain and sq. chain.

Mensuration is not however limited to areas but deals also with solids.

Areas always involve the product of two linear magnitudes.

Volumes always involve the product of three linear magnitudes.

The ratio of the circumference of a circle to its

diameter is a fundamental constant in mensuration. It is called  $\pi$  and is of the value of 3.1416 approximately.

$$\frac{\pi}{2} = 1.5708. \quad \frac{\pi}{3} = 1.0472. \quad \frac{\pi}{4} = .7854. \quad \frac{\pi}{6} = .5236.$$

F. Macfarlane Gray's Rule for multiplying by .7854.

Multiply by 7, repeat, double, and repeat, writing each successive product one place to the right.

$$\begin{array}{r} \text{Proof.} \qquad \qquad \qquad .7 \\ \qquad \qquad \qquad \qquad \qquad 7 \\ \qquad \qquad \qquad \qquad \qquad 14 \\ \qquad \qquad \qquad \qquad \qquad 14 \\ \hline \qquad \qquad \qquad \qquad \qquad .7854 \end{array}$$

Area of a circle = sq. of diameter  $\times$  .7854.

A cylindrical foot (inch) = volume of a cylinder 1 ft. (inch) in depth and diameter.

A conical foot (inch) = volume of a cone 1 ft. (inch) in depth and base-diameter.

A spherical foot (inch) = volume of a sphere 1 ft. (inch) in diameter.

The foot in each case = 1728 inches corresponding.

1 cubic foot = 1728 cub. inches.

„ = 2200 cyl. inches = 1.273 cyl. ft.

„ = 3300 sph. inches = 1.910 sph. ft.

„ = 6600 con. inches = 3.819 con. ft.

1 cyl. ft. (inch) = .7854  $\times$  cub. ft. (inch).

1 sph. ft. (inch) = .5236  $\times$  cub. ft. (inch).

1 con. ft. (inch) = .2619  $\times$  cub. ft. (inch).

Decimal Approximations for Mensuration.

1. Lineal ft.  $\times$  .00019 = miles.
2. Lineal yds.  $\times$  .000568 = miles.
3. sq. inches  $\times$  .007 = sq. ft.
4. sq. yds.  $\times$  .0002067 = acres.
5. cub. inches  $\times$  .00058 = cub. ft.
6. cub. ft.  $\times$  .03704 = cub. yds.
7. cub. ft.  $\times$  6.232 = imp. galls.

## PLANE FIGURES.

### RIGHT-ANGLED TRIANGLE.

1. Perimeter = sum of the sides.
2. Area =  $\frac{1}{2}$  product of sides enclosing the right angle.
3. I. 47. Square of hypotenuse = sum of squares of other sides.

## ANY TRIANGLE.

1. Perimeter = sum of the sides.
2. Area =  $\frac{1}{2}$  product of any side into distance of opposite vertex.
3. Area = square root of product of semi-perimeter into the semi-perimeter diminished by each of the sides in succession.

## SQUARE.

1. Perimeter =  $4 \times$  one side.
2. Area = square of one side.

## RHOMBUS.

1. Perimeter =  $4 \times$  one side.
2. Area = product of diagonals.
3. Area = product of any side  $\times$  perpendicular distance of the parallel side.

## RECTANGLE.

1. Perimeter =  $2 \times$  sum of adjacent sides.
2. Area = product of adjacent sides.

## PARALLELOGRAM.

1. Perimeter =  $2 \times$  sum of adjacent sides.
2. Area = product of any side  $\times$  perpendicular distance of its parallel.

## TRAPEZOID OR TRAPEZIUM.

1. Perimeter = sum of sides.
2. Area =  $\frac{1}{2}$  sum of parallel sides  $\times$  perpendicular distance between.

## ANY REGULAR POLYGON.

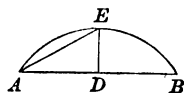
1. Perimeter = no. of sides  $\times$  length of one side.
2. Area =  $\frac{1}{2}$  sum of sides  $\times$  perpendicular distance of centre from any side.

## SIMILAR FIGURES.

1. Areas are as the squares of corresponding sides.

## CIRCLE.

1. Perimeter =  $2\pi r = 3.1416 \times$  diameter.
2. Length of arc =  $\frac{1}{360} \{8 \times \text{chord of } \frac{1}{2} \text{ arc} - \text{chord of arc}\}$  nearly.
3. Chord of  $\frac{1}{2}$  arc =  $\sqrt{\text{versed sine} \times \text{diameter}}$ .



$ED =$  versed sine.



4.  $\frac{1}{2}$  chord of arc =  $\sqrt{\text{versed sine} \times \text{diameter less versed sine.}}$
5. Arc : circumference :: angle at centre :  $360^\circ$ .
6. Area =  $\pi r^2 = .7854 \times \text{square of diameter.}$
7. Area =  $.07958 \times \text{square of circumference.}$
8. Area of sector =  $\frac{1}{2} \times \text{length of arc} \times \text{radius.}$
9. Area of sector : area of circle :: angle at centre :  $360^\circ$ .
10. Area of segment = area of sector - triangle of radii and chord.
11. Area of segment  

$$= \sqrt{\text{the square of (versed sine} \times .626) + \text{square of } \frac{1}{2} \text{ chord} \times \frac{2}{3} \text{ versed sine.}}$$
12. Area of a circular ring = difference of squares of diameters  $\times .7854$ .

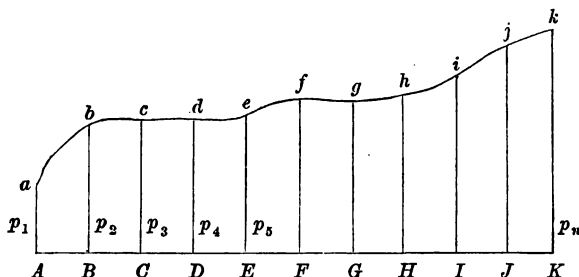
## ELLIPSE.

1. Area =  $\pi ab$  = product of major and minor axes  $\times .7854$ .

## PARABOLA.

1. Area of segment cut off by any double ordinate =  $\frac{2}{3} \times \text{rectangle of same base and height} = \frac{2}{3} \times \text{abscissa} \times \text{ordinate.}$

## SIMPSON'S RULE FOR IRREGULAR FIGURES.



$aA . bB$  etc. ordinates.

$P_1 P_2 \dots$  „

$AB = BC = CD = \dots = h.$

$$\text{Area} = \frac{h}{3} \{P_1 + P_n + 2(P_2 + P_4 + \dots) + 4(P_3 + P_5 + \dots)\}.$$

## SOLIDS.

## CYLINDER OR SEGMENT OF CYLINDER.

1. Surface =  $2\pi rh$  = circumference of base  $\times$  height =  $3.1416 \times \text{diameter} \times \text{height.}$
2. Solidity =  $\pi r^2 h$  = area of base  $\times$  height =  $.7854 \times \text{square of diameter} \times \text{height.}$

## CYLINDRIC RING.

1. Surface = circumference of perpendicular section  $\times$  mean perimeter.
2. Solidity = area of perpendicular section  $\times$  mean perimeter.
3. Surface =  $9.8698 \times (\text{thickness} + \text{inner diameter}) \times \text{thickness}$ .
4. Solidity =  $2.4674 \times (\text{thickness} + \text{inner diameter}) \times \text{square of thickness}$ .

## CONE.

1. Surface = circumference of base  $\times \frac{1}{2}$  slant side + area of base.
2. Solidity =  $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height} = \frac{1}{3} \pi r^2 h$ .

## FRUSTUM OF A CONE.

1. Surface =  $\frac{1}{2} \times \text{sum of perimeters of ends} \times \text{slant height} + \text{areas of ends}$ .
2. Solidity =  $\frac{1}{3} \times (\text{sum of areas} + \sqrt{\text{product of areas of two ends}}) \times \text{perpendicular height}$ .
3. Solidity =  $.2618 \times (\text{sum of squares of diameters} + \text{product of diameters}) \times \text{perpendicular height}$ .

## PYRAMID.

1. Surface =  $\frac{1}{2} \times \text{perimeter of base} \times \text{slant height} + \text{area of base}$ .
2. Solidity =  $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$ .

## FRUSTUM OF A PYRAMID.

1. Surface =  $\frac{1}{2} \times \text{sum of perimeters of ends} \times \text{slant height} + \text{areas of ends}$ .
2. Solidity =  $\frac{1}{3} \times (\text{sums of areas of ends} + \sqrt{\text{product of areas}}) \times \text{perpendicular height}$ .

## PARALLELEPIPED OR FRUSTUM PARALLEL TO ANY FACE.

1. Surface = sum of areas of faces.
2. Solidity = area of any face  $\times$  perpr. height.

## PRISM OR FRUSTUM PARALLEL TO BASE OR FACE.

1. Surface = sum of areas of faces.
2. Solidity = area of base  $\times$  perpendicular height.

## WEDGE.

1. Surface = sum of areas of faces.
2. Solidity =  $\frac{1}{6} \times (\text{length of edge} + \text{twice length of base}) \times \text{height} \times \text{breadth of base}$ .

N.B. In all frusta the surface and solidity may be found by considering the completed figure and the figure of completion—the difference being finally taken.

**SPHERE.**

1. Surface  $= 4\pi r^2 = \frac{1}{4} \times \text{circumference} \times \text{diameter} = 3.1416 \times \text{square of diameter}.$
2. Solidity  $= \frac{4}{3}\pi r^3 = \frac{1}{6} \times \text{circumference} \times (\text{diameter})^2 = \frac{1}{6} \times .7854 \times (\text{diameter})^3 = .5236 \times (\text{diameter})^3.$

**ZONE OF SPHERE.**

1. Surface  $= 2\pi r (h_1 - h_2) = \text{circumference of sphere} \times \text{height of zone}.$
2. Solidity  $= \frac{1}{6}\pi h (3r_1^2 + 3r_2^2 + h^2)$ , where  $r_1, r_2$  are radii of ends and  $h$  is the height of the zone.

**SEGMENT OF SPHERE.**

1. Surface  $= 2\pi r h = \text{circumference of sphere} \times \text{height of segment}.$
2. Solidity  $= \frac{1}{6}\pi h (3r_1^2 + h^2)$ , where  $r_1$  is radius of end and  $h$  height of segment.

**SPHEROID (PROLATE OR OBLATE).**

1. Solidity  $= .5236 \times \text{square of revolving axis} \times \text{fixed axis}.$
2. Solidity  $= \frac{1}{6}$  volume of circumscribing cylinder.

**PARABOLOID.**

1. Solidity  $= \frac{1}{6}$  volume of circumscribing cylinder.

**SIMILAR SOLIDS.**

1. Volumes are as the cubes of corresponding edges.

**Time.**

60 seconds (")	= 1 minute.
60 minutes (')	= 1 hour.
24 hours	= 1 day.
7 days	= 1 week.
365 days	} = 1 year.
52 weeks	
12 months	

Thirty days hath September,  
April, June, and November,  
February has twenty-eight alone,  
And all the rest have thirty-one.  
But leap-year coming once in four,  
February then has one day more.

The civil or mean solar day (the standard unit for the measurement of time) is the mean time which elapses between two successive transits of the Sun across the meridian of any place.

The civil year consists of 365 mean solar days, but the actual time taken by the Sun to return to any point in the ecliptic is 365·242218 mean solar days.

Thus the civil year is about  $\frac{1}{4}$  of a day less than the solar year.

To remedy this, Julius Caesar, as Pontifex Maximus, in B.C. 46 made every 4th year a leap-year or bissextile year of 366 days, February having a 29th (intercalary) day.

The Julian year is thus 365·25 days, while the solar year is 365·242218 days only—there is therefore an excess of ·007782 days each year.

This difference becomes in 400 years 3·1128 days ( $400 \times \cdot 007782$ ), and therefore Pope Gregory in 1582 found the Vernal Equinox to be on March 11th instead of March 21st, which was the case in 325 at the Council of Nice ( $1257 \times \cdot 007782 = 9\cdot7819$  or 10 days nearly). To correct the error, 10 days were left out, Oct. 15 following Oct. 4, 1582, and to prevent its recurrence, in every 400 years 3 of the leap-years are omitted, viz. those which complete a century not divisible by 4: thus 1200, 1600, 2000 are leap-years, but 1300, 1700, 1900 are not.

This Gregorian or New Style was adopted in England on Sept. 2, 1752, when the error was 11 days.

Russia is the only country of Europe not yet fully adopting the New Style.

The quarter days are March 25, Lady Day.  
 June 24, Midsummer Day.  
 Sept. 29, Michaelmas Day.  
 Dec. 25, Christmas Day.

A *lunar* month is the time taken by the Moon to return to the same place in its orbit.

A *calendar* month is the time between the same dates in two consecutive months.

Special rule for a *daily* wage.

Given daily wage—to find yearly payment.

Wage in d. as £ + half wage in d. as £ + 5 (or 6) days' wages.

Special rule for a *weekly* wage.

Given weekly wage—to find yearly payment.

Twice wage in s. as £ +  $\frac{1}{2}$  wage in s. as £ + 2 weeks' wages.

### EXAMPLES.

1. Construct Inversion and Interchange Tables for 365, 366, 52, 312, 313.

2. Find annual wage—given daily wages as 1s. 7 $\frac{3}{4}$ d., 1s. 10 $\frac{1}{2}$ d., 2s. 1d., 3s. 4d., 4s. 9d.

3. Find annual wage—given weekly wages as 15s. 8d., 17s. 9d., 25s. 6d., 32s., 35s., 52s., 63s.

### Angular Measure.

The English or Sexagesimal Method.

60 seconds (") = 1 minute.

60 minutes (') = 1 degree.

90 degrees (°) = 1 right angle.

30° = 1 sign (S.), 12 signs = 360° = 1 circle (C.).

The circumference of every circle is considered to be divided into 360 equal parts, each of which is called a degree as it subtends an angle of  $1^\circ$  at the centre of the circle.

The French or Centesimal Method.

100 seconds ( $''$ ) = 1 minute.

100 minutes ( $'$ ) = 1 grade.

100 grades ( $^g$ ) = 1 right angle.

The advantage of this method is that  $57^\circ 33' 5'' = 57.3305^g = 573305$  right angles at once without any working other than moving the decimal point.

1 degree =  $\frac{1}{90}$  of a right angle, 1 grade =  $\frac{1}{100}$  of a right angle,  $\therefore 90^\circ = 100^g$ ,  $\therefore 1^\circ = \frac{10^g}{9}$ , or  $1^g = \frac{9^\circ}{10}$ . Hence

(1) to convert degrees into grades—add  $\frac{1}{9}$ ,

(2) to convert grades into degrees—subtract  $\frac{1}{10}$ .

*Longitude.* A difference of  $15^\circ$  Longitude gives a difference of one hour in time.

$\therefore$  a difference of  $1^\circ = 4$  mins. in time =  $\frac{1}{15}$  hour.

„ „ of  $1' = 4$  secs. in time =  $\frac{1}{15}$  min.

„ „ of  $1'' = \frac{1}{15}$  sec. in time.

Hence

(1) *To convert angular measure into time.*

Divide degrees by 15—quotient is hours.

Multiply remainder by 4—product is minutes.

Divide minutes by 15—quotient is minutes.

Multiply remainder by 4—product is seconds.

Divide seconds by 15—quotient is seconds.

$$\begin{array}{rcl}
 \text{Ex. } 230^\circ 32' 10'' = & 15^h & 20' \\
 & & 2' \ 8'' \\
 & & 0'' \cdot 6 \\
 \hline
 & 15^h & 22' \ 8'' \cdot 7
 \end{array}$$

(2) *To convert time into angular measure.*

Multiply hours by 15—product is degrees.

Divide minutes by 4—quotient is degrees.

Multiply remainder by 15—product is minutes.

Divide seconds by 4—quotient is minutes.

Multiply remainder by 15—product is seconds.

$$\begin{array}{r} \text{Ex. } 3^{\text{h}} 14' 23'' = \\ \quad \quad \quad 45^{\circ} \\ \quad \quad \quad 3^{\circ} 30' \\ \quad \quad \quad \quad 5' 45'' \\ \hline \quad \quad 48^{\circ} 35' 45'' \end{array}$$

### Temperature (Thermometers).

The chief scales are

(1) Centigrade. Freezing point  $0^{\circ}$ , boiling point  $100^{\circ}$ , no. of degrees between = 100.

(2) Fahrenheit. Freezing point  $32^{\circ}$ , boiling point  $212^{\circ}$ , no. of degrees between = 180.

(3) Reaumur. Freezing point  $0^{\circ}$ , boiling point  $80^{\circ}$ , no. of degrees between = 80.

The Centigrade or Celsius's scale is used in scientific operations and on the Continent.

Fahrenheit is commonly used in England, Reaumur in Russia.

$$\text{Formula of conversion. } \frac{F. - 32}{180} = \frac{C.}{100} = \frac{R.}{80}.$$

**Paper Measure.****TABLE.**

24 sheets = 1 quire.

20 quires = 1 ream.

10 reams = 1 bale.

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1 ream = 480 sheets.

**Special rule for Paper.**

Given price of ream—to find price of bales, reams, quires, sheets.

Reduce bales to reams.

Consider reams £, quires s., sheets  $\frac{1}{4}$ d.

And use Practice with the price.



## SECTION IV.

### **THE METRIC SYSTEM OF WEIGHTS AND MEASURES.**

#### **1. The Metric System.**

THE French Philosophers of the time of the Great Revolution devised the metric system—entirely based upon a decimal scale. The simplicity and convenience of this system ensure its final adoption by all civilised nations. At present the English system is its only great rival owing to the vast extent of our trade. If we were prepared to give up our inconvenient moneys, weights, and measures in favour of decimal scales the boon to all students and schools as well as to business men generally would be incalculable. The mere fact that a 3 is involved in the relations of our coins (12*d.* = 1 shilling) causes most decimals of a £ to recur and thus greatly increases the labour even of decimal work. The adoption of decimal coinage and the metric system though it would cause temporary disturbance in our trade would reduce the time required for arithmetic in schools by one-half and ultimately simplify all commercial operations enormously.

The basis of the metric system is the **Metre** or unit of length.

This was originally intended to be one ten-millionth part of the arc of a meridian from the pole to the equator. Errors have since been discovered in the measurement of the half-meridian, and therefore the metre is now an arbitrary standard without exact decimal relation to any natural constant. Its value is that of a rod kept at Paris.

The unit of area is the square of 10 metres or **Are**. Areas are stated in sq. metres or hectares.

The unit of volume is the cube of 1 metre or **Stere**. Volumes are usually stated in cub. metres.

The unit of capacity is the cube of  $\frac{1}{10}$  of a metre or **Litre**.

Capacity is very often stated in cub. centimetres. One litre contains 1000 cub. centimetres.

The unit of weight is the weight of pure water at  $4^{\circ}$  C. and 760 mm. pressure contained in  $\frac{1}{1000}$  of a litre (= 1 millilitre or 1 cub. centimetre) or **Gramme**.

It follows that the weight of a litre of water is a kilogramme.

All these five units—the metre, are, stere, litre, gramme—are divided into tenths, hundredths, thousandths, or multiples of 10, 100, 1000 are taken.

The fractions are denoted by Latin prefixes, the multiples by Greek.

$\frac{1}{1000}$	milli-	} metre, are, stere, litre, gramme.
$\frac{1}{100}$	centi-	
$\frac{1}{10}$	deci-	
1		
10	dec(a)-	
100	hect(o)-	
1000	kil(o)-	

## 2. Commercial Uses.

The metre is the measure for cloths, silks, cottons and all short lengths and distances. Its value is  $\frac{1}{11}$  of our yard nearly. The pole is 5 metres very nearly, and the furlong 2 hectometres.

The kilometre is used for measuring long distances and is about  $\frac{5}{8}$  of the English mile.

The centimetre and millimetre are used in scientific calculations—as also the micron =  $\frac{1}{1000}$  of a mm. The centimetre is about  $\frac{2}{5}$  of an inch.

The are is the unit of land measure, but hectares are generally used to express large areas. The are is nearly 4 poles: the hectare  $2\frac{1}{2}$  acres. The rood equals the decare very nearly.

The square metre is the unit for measuring ordinary surfaces. Its value is nearly  $\frac{9}{16}$  of a sq. yard.

The stere is the unit for wood.

The cubic metre is used for measuring ordinary solids, viz. excavations, loads, and timber. It is very nearly  $\frac{1}{8}$  of a cubic yard.

The litre is used for both liquid and dry measures. It contains  $1\frac{3}{4}$  pint nearly. The  $\frac{1}{2}$  decalitre holds  $1\frac{1}{10}$  gallon.

The hectolitre is the unit for measuring large quantities of liquids, grain, or fruit. Its capacity is 22 gallons.

The kilogramme is the common unit of weight for trade. It weighs  $2\frac{1}{6}$  lbs. The  $\frac{1}{2}$ -kilo is also used under the name "pound" (=  $1\frac{1}{10}$  lb. Av.).

The gramme or kilogramme is the bullion weight in most Continental Countries.

The myriagramme is  $1\frac{1}{2}$  stone in weight.

The quintal and tonneau are for heavy weights. They are very nearly equal to 2 cwts. and 1 ton respectively.

The milligramme and centigramme are used in scientific calculations.

## 3. Tables of Equivalents.

LENGTH.		ENGLISH MEASURES.		FRENCH MEASURES.	
Micron = $\frac{1}{1000000}$ metre.		1 inch = 2.539954113 centimetres.		1 centimetre = .3937079 inch.	
Millimetre = $\frac{1}{1000}$ metre.		1 foot = 3.047944936 centimetres.		1 decimetre = .3280899167 foot.	
Centimetre = $\frac{1}{100}$ metre.		1 yard = .9143834307 metres.		1 metre = 1.093613298 yards.	
Decimetre = $\frac{1}{10}$ metre.		1 pole = .5029109144 decametres.		1 decametre = 1.988423737 poles.	
Metre = unit.		1 furlong = 2.011643658 hectometres.		1 hectometre = .4971059343 furlong.	
Hectometre = 100 metres.		1 mile = 1.609344 kilometres.		1 kilometre = .6213824 mile.	
Kilometre = 1000 metres.					
Myriametre = 10000 metres.					
SURFACE.					
Centiare = $\frac{1}{100}$ are = 1 sq. metre.		1 sq. inch = 6.4513668971 sq. cm.		1 sq. cm. = .1550059105 sq. in.	
Deciare = $\frac{1}{10}$ are.		1 sq. foot = 9.29033803 sq. dm.		1 sq. dm. = .1076422934 sq. ft.	
Are = a sq. decametre.		1 sq. yard = .8360971499 sq. m.		1 sq. metre = 1.19603326 sq. yds.	
Decare = 10 ares.		1 sq. pole = .2529193879 ares.		1 are = 119.603326 sq. yds.	
Hectare = 100 ares.		1 rood = 1.011677517 decares.		1 hectare = 2.4711431 acres.	
		1 acre = .404671021 hectares.			
		1 sq. mile = 258.989453 hectares.			
SOLIDITY.					
Decistere = $\frac{1}{10}$ stere.		1 cub. inch = 16.386176 c. cm.		1 cub. cm. = .061027052 cub. in.	
Stere = a cubic metre.		1 cub. ft. = 28.3153119 c. dm.		1 decistere = .3531658074 cub. ft.	
Decastere = 10 steres.		1 cub. yd. = .764554292 steres.		1 stere = 1.308021509 cub. yds.	
		1 fl. drachm = 3.5495765 cub. cm.		1 cub. cm. = .2817237 fl. drachm.	

## CAPACITY.

Millilitre =  $\frac{1}{1000}$  litre = 1 cub. cm.  
 Centilitre =  $\frac{1}{100}$  litre.  
 Decilitre =  $\frac{1}{10}$  litre.

Litre = a cubo decimetre.

Decalitre = 10 litres.

Hectolitre = 100 litres.

Kilolitre = 1000 litres = 1 cub. m.

Myrialitre = 10000 litres.

## WEIGHT.

Milligramme =  $\frac{1}{1000}$  gramme.

Centigramme =  $\frac{1}{100}$  gramme.

Decigramme =  $\frac{1}{10}$  gramme.

Gramme = weight of a c. cm. of  
 pure water at 4° C.

Decagramme = 10 grammes.

Hectogramme = 100 grammes.

Kilogramme = 1000 grammes.

Myriagramme = 10000 grammes.

Quintal = 100 kilogrammes.

Tonneau = 1000 kilogrammes.

1 gill = 1·4199306 decilitres.

1 pint = ·567982246 litres.

1 quart = 1·13596449 litres.

1 gallon = 4·543457970 litres.

1 peck = ·908691594 decalitres.

1 bushel = 3·634766376 decalitres.

1 quarter = 2·9078131 hectolitres.

1 cub. ft. = 28·3153119 litres.

1 grain = ·6479895 decigrammes.

1 dwt. = 1·55517481 grammes.

1 oz. Tr. = 3·1103496 decagrammes.

1 lb. Tr. = ·3732411954 kilogrammes.

1 scruple = 1·29597901 grammes.

1 dr. Ap. = ·3887937 decagrammes.

1 dr. Av. = 1·771847 grammes.

1 oz. Av. = 2·834954 decagrammes.

1 lb. Av. = ·4535926525 kilogrammes.

1 stone = 6·350297136 kilogrammes.

1 qr. = 1·27005943 myriagrammes.

1 cwt. = ·50802377 quintal.

1 ton = 1·0160475 tonneau.

1 centilitre = ·6102705 cub. in.

1 decilitre = ·7043093556 gill.

1 litre = 1·760773414 pints.

1 decalitre = 8·80386707 quarts.

1 litre = ·2200966768 gallon.

1 hectolitre = 2·751208459 bushels.

1 kilolitre = 3·43901057 qrs.

1 litre = ·03531658 cub. ft.

1 decigramme = 1·543234874 grains.

1 gramme = 15·43234874 grains.

1 decagramme = ·321507265 oz. Tr.

1 hectogramme = 3·5273936 ozs. Av.

1 kilogramme = 2·204621249 lbs. Av.

1 myriagramme = 1·57472946 stones.

1 quintal = 1·9684118 cwts.

1 tonneau de mer = ·98420591 ton.

#### 4. Conversion of Metric and English Quantities by the Methods of Approximation.

The equivalents given above enable us to convert metric to imperial or imperial to metric units by multiplication or division.

The methods of approximation greatly abridge the labour.

*Example 1.* To convert 8561·237 kilogrammes to lbs.

1 kilog. = 2·2046213 lbs. Multiply correct to *two* places.

$$\begin{array}{r}
 8561\cdot237 \\
 31264\ 022 \\
 \hline
 17122\ 474 \\
 1712\ 247 \\
 34\ 245 \\
 5\ 137 \\
 171 \\
 9 \\
 2 \\
 \hline
 18874\cdot28\overline{5}\ \text{lbs.}
 \end{array}$$

*Example 2.* To convert 5804 qrs. 7 bush. 3 pks. to hectolitres.

1 qr. = 2·9078 hectolitres.

5804 qrs. 7 bush. 3 pks. = 5804·96875 qrs. Multiply correct to *two* places.

$$\begin{array}{r}
 87\ 092 \\
 \hline
 11609\ 937 \\
 5224\ 471 \\
 44\ 643 \\
 \hline
 16879\cdot685\ \text{hectols.}
 \end{array}$$

*Example 3.* From the relation 1 metre = 1·0936331 yds. obtain the value of 1 yd. in decimals of a metre.

$$\begin{array}{r}
 1\cdot0,9,3,6,3,3,1\ 100000000\ (.9143834 \\
 \dots 1573021 \\
 \hline
 479388 \\
 41935 \\
 9126 \\
 377 \\
 49 \\
 \hline
 5
 \end{array}$$

*Example 4.* Given that 1 sq. yd. = .836097 sq. metre, find value in sq. yds. of 3.286581 sq. metres.

$$\begin{array}{r} .83,6,0,9,7 \quad 3.286581 \quad (3.9308 \\ \underline{778290} \\ 25803 \\ \underline{720} \\ 51 \end{array}$$

### EXAMPLES.

1. Convert correct to two places.
  - (1) 864.51 metres into feet.
  - (2) 834 sq. feet into sq. centimetres.
  - (3) 3512.45 kilogrammes into stones.
  - (4) 712 tons 13 cwt. 1 qr. into quintals.
  - (5) 9125 hectolitres into imp. qrs.
  - (6) 1151.731 kilometres into miles.
  - (7) 856.312 tonneaux into cwt.
  - (8) 315 lbs. 10 oz. to pfunde ( $\frac{1}{2}$  kilos.).
  - (9) 1876 bushels to hectols.
  - (10) 515 c. feet into c. metres.
2. Given the value of 1 yard in metres and 1 lb. in kilogrammes correct to 5 places. Find value of 726 metres in yards and 3250 kilogrammes in cwt. correct to 3 places.
3. Verify these results given among the equivalents.
  - (1) 1 hectolitre in bushels from the value of 1 gallon in litres.
  - (2) 1 acre in hectares from the value of 1 sq. yard in sq. metres.

### 5. Conversion of Metric and English Quantities by the Method of Multiples.

The following Tables give the values of 1, 2, 3, 4, 5, 6, 7, 8, 9 French or English units in terms of the main English and French units.

With the aid of these it is extremely easy to convert any metric quantity of commerce into the English equivalent desired or vice-versâ by moving the points and adding.

Very slight modifications in using the tables actually given will enable us to get the metric or imperial equivalent for any quantity expressed in any form.

Examples will render these points clear.

1. Convert 236 yds. 2 ft. 7 inches to metres.

$$\begin{array}{r}
 200 \text{ yds.} = 182 \cdot 87669 \\
 30 \text{ yds.} = 27 \cdot 43150 \\
 6 \text{ yds.} = 5 \cdot 48630 \\
 2 \text{ ft.} = \cdot 60959 \\
 7 \text{ inches} = \cdot 17780 \\
 \hline
 216 \cdot 58188 \text{ metres.}
 \end{array}$$

2. Convert 7136 hectares to acres.

$$\begin{array}{r}
 7000 = 17293 \cdot 0016 \\
 100 = 247 \cdot 1143 \\
 30 = 74 \cdot 1343 \\
 6 = 14 \cdot 8269 \\
 \hline
 17634 \cdot 0771 \text{ acres.}
 \end{array}$$

3. Convert 95 gallons to litres.

$$\begin{array}{r}
 90 = 408 \cdot 911 \\
 5 = 22 \cdot 717 \\
 \hline
 431 \cdot 628 \text{ litres.}
 \end{array}$$

4. Convert 325·6 hectolitres to quarters.

$$\begin{array}{r}
 103 \cdot 17032 \\
 6 \cdot 87802 \\
 1 \cdot 71951 \\
 \cdot 20634 \\
 \hline
 111 \cdot 97419 \text{ qrs.}
 \end{array}$$

In the case of a decimal the point moves to the left.

5. Convert 9321 kilogrammes to lbs.

$$\begin{array}{r}
 19841 \cdot 591 \\
 661 \cdot 386 \\
 44 \cdot 092 \\
 2 \cdot 205 \\
 \hline
 20549 \cdot 274 \text{ lbs.}
 \end{array}$$

6. Convert 25 tons 17 cwts. 3 qrs. to quintals.

$$\begin{array}{r}
 203 \cdot 20951 \\
 50 \cdot 80238 \\
 5 \cdot 08024 \\
 3 \cdot 55617 \\
 \cdot 38102 \\
 \hline
 263 \cdot 02932 \text{ quintals.}
 \end{array}$$

*Note.* The Tables here given are like the Government Tables but are taken to further places in general. Allowance is made for places beyond those given except in the case of those equivalents which are taken as absolute, viz. centimetres to inches, and grams to grains.



## LENGTH.

Centimetres to inches.	Metres to feet.	Metres to yards.	Kilometres to miles.
1 .3937079	1 3.280899167	1 1.093633056	1 .6213824
2 .7874158	2 6.561798333	2 2.187266111	2 1.2427648
3 1.1811237	3 9.842697500	3 3.280899167	3 1.8641473
4 1.5748316	4 13.123596667	4 4.374532222	4 2.4855297
5 1.9685395	5 16.404495833	5 5.468165278	5 3.1069121
6 2.3622474	6 19.685395000	6 6.561798333	6 3.7282945
7 2.7559553	7 22.966294167	7 7.655431389	7 4.3496769
8 3.1496632	8 26.247193333	8 8.749064444	8 4.9710593
9 3.5433711	9 29.528092500	9 9.842697500	9 5.5924418

Inches to Centimetres.	Feet to metres.	Yards to metres.	Miles to metres.
1 2.539954113	1 .3047944936	1 .9143834807	1 1.6093149
2 5.079908226	2 .6095889872	2 1.8287669615	2 3.2186299
3 7.619862339	3 .9143834807	3 2.7431504422	3 4.8279448
4 10.159816452	4 1.2191779743	4 3.6575339240	4 6.4372597
5 12.699770566	5 1.5239724679	5 4.5719174037	5 8.0465746
6 15.239724679	6 1.8287669615	6 5.4863008845	6 9.6558896
7 17.779678792	7 2.4335614551	7 6.4006843752	7 11.2652045
8 20.319632905	8 2.4383559487	8 7.3150678570	8 12.8745194
9 22.859587018	9 2.7431504422	9 8.2294513367	9 14.4838343

Note. 1 furlong = 10 chains = 1000 links.

## AREA.

Sq. cm. to sq. inches.	Sq. metres to sq. feet.	Sq. metres to sq. yards.	Hectares to acres.
1 .1550059105	1 10.76429934	1 1.196033260	1 2.4711431
2 .3100118210	2 21.52859868	2 2.392066521	2 4.9422862
3 .4650177316	3 32.29289803	3 3.588099781	3 7.4134293
4 .6200236421	4 43.05719737	4 4.784133041	4 9.8845724
5 .7750295526	5 53.82149671	5 5.980166301	5 12.3557155
6 .9300354631	6 64.58579605	6 7.176199562	6 14.8268585
7 1.0850413737	7 75.35009540	7 8.372232822	7 17.2980016
8 1.2400472842	8 86.11439474	8 9.568266082	8 19.7691447
9 1.3950531947	9 96.87869408	9 10.764299342	9 22.2402878

Sq. inches to sq. cm.	Sq. feet to sq. metres.	Sq. yards to sq. metres.	Acres to hectares.
1 6.4513668971	1 .0928996833	1 .8360971499	1 .404671021
2 12.9027337942	2 .1857993666	2 1.6721942997	2 .809342041
3 19.3541006913	3 .2786990500	3 2.5082914496	3 1.214013062
4 25.8054675884	4 .3715987333	4 3.3443885994	4 1.618684083
5 32.2568344855	5 .4644984166	5 4.1804857493	5 2.023355103
6 38.7082013826	6 .5573980999	6 5.0165828992	6 2.428026124
7 45.1595682797	7 .6502977832	7 5.8526800490	7 2.832697145
8 51.6109351768	8 .7431974665	8 6.6887771989	8 3.237368165
9 58.0623020740	9 .8360971499	9 7.5248743487	9 3.642039186

Note. 1 hectare = 10000 sq. metres. 1 acre = 10 sq. chains.

## VOLUME.

Cub. ins. to cub. cm.		Cub. ft. to cub. m.		Cub. yds. to cub. m.		Fl. drachms to cub. cm.	
1	16-886176	1	0-283153119	1	7645134224	1	3-5495765
2	32-772352	2	0-566306239	2	1-5290268447	2	7-0991531
3	49-158527	3	0-849459358	3	2-2935402671	3	10-6487296
4	65-544708	4	1-132612478	4	3-0580536895	4	14-1983062
5	81-930879	5	1-415765597	5	3-8225671119	5	17-7478827
6	98-317055	6	1-698918716	6	4-5870805342	6	21-2974592
7	114-703231	7	1-982071836	7	5-3515939566	7	24-8470358
8	131-089406	8	2-265224955	8	6-1161073790	8	28-8966123
9	147-475582	9	2-548378075	9	6-8806208014	9	31-9461889
Cub. cm. to cub. ins.		Cub. metres to cub. ft.		Cub. metres to cub. yds.		Cub. cm. to fl. drachms.	
1	0-01027052	1	35-81658074	1	1-308021509	1	2-817237
2	0-122054103	2	70-63316148	2	2-616043018	2	5-634475
3	0-183081155	3	105-94974222	3	3-924064527	3	8-451712
4	0-244108206	4	141-26682296	4	5-232086036	4	11-268950
5	0-305135258	5	176-58290370	5	6-540107545	5	14-086187
6	0-366182309	6	211-89948444	6	7-848129053	6	16-903425
7	0-427189361	7	247-21606518	7	9-156150562	7	19-720662
8	0-488216412	8	282-53264592	8	10-464172071	8	22-2537900
9	0-549243464	9	317-84922666	9	11-772193580	9	25-535187

## CAPACITY.

Pints to litres.		Gallons to litres.		Bushels to hectolitres.		Quarters to hectolitres.	
1	0-567932246	1	4-543457970	1	3634766376	1	2-907813100
2	1-135864492	2	9-086915939	2	7-269532751	2	5-815626201
3	1-708796738	3	13-630873909	3	1-0904299127	3	8-723439301
4	2-271728984	4	18-173831878	4	1-4539065502	4	11-631252402
5	2-839661231	5	22-717289848	5	1-8173331878	5	14-539065502
6	3-407598477	6	27-260747817	6	2-1808598254	6	17-446878603
7	3-975525723	7	31-804205787	7	2-5443364629	7	20-354691703
8	4-543457969	8	36-347663756	8	2-9078131005	8	23-262504804
9	5-111390215	9	40-891121726	9	3-2712897880	9	26-170317903
Litres to pints.		Litres to gallons.		Hectolitres to bushels.		Hectolitres to quarters.	
1	1-760773414	1	0-2200966768	1	2-751208459	1	3-43901057
2	3-521546828	2	0-4401933535	2	5-502416919	2	6-87802115
3	5-282320242	3	0-6602900303	3	8-253625378	3	1-031703172
4	7-043093656	4	0-8803967070	4	11-004833338	4	1-375604230
5	8-803867070	5	1-1004833338	5	13-756042297	5	1-719505287
6	10-564640787	6	1-3205800605	6	16-507250756	6	2-063406344
7	12-325413898	7	1-5406767873	7	19-258459216	7	2-407307402
8	14-086189312	8	1-7607734140	8	22-009667675	8	2-751208459
9	15-846960726	9	1-9808700908	9	24-760876135	9	3-095109517

## WEIGHT.

Kilograms to pounds.		Kilograms to stones.		Quintals to cwts.		Tonneaux to tons.	
1	2·204621249	1	·157472946	1	1·9684118	1	·9842059
2	4·409242497	2	·314945893	2	3·9368237	2	1·9684118
3	6·613863746	3	·472418839	3	5·9052355	3	2·9526177
4	8·818484994	4	·629891785	4	7·8736473	4	3·9368236
5	11·023106243	5	·787364732	5	9·8420591	5	4·9210296
6	13·227727491	6	·944837678	6	11·8104710	6	5·9052355
7	15·432348740	7	1·102310624	7	13·7788828	7	6·8894414
8	17·636969989	8	1·259783571	8	15·7472946	8	7·8736473
9	19·841591237	9	1·417256517	9	17·7157065	9	8·8578532

Lbs. to kilograms.		Stones to kilograms.		Cwts. to quintals.		Tons to tonneaux.	
1	·4535926525	1	6·350297136	1	·50802377	1	1·0160475
2	·9071853051	2	12·700594271	2	1·01604754	2	2·0320951
3	1·3607779576	3	19·050891407	3	1·52407131	3	3·0481426
4	1·8143706102	4	25·401188542	4	2·03209508	4	4·0641902
5	2·2679632627	5	31·751485678	5	2·54011885	5	5·0802377
6	2·7215559153	6	38·101782814	6	3·04814263	6	6·0962852
7	3·1751485678	7	44·452079949	7	3·55616640	7	7·1123328
8	3·6287412204	8	50·802377085	8	4·06419017	8	8·1283803
9	4·0823338729	9	57·152674220	9	4·57221394	9	9·1444279

Grams to grains.		Kilograms to ozs. Tr.		Grains to grams.		Ozs. Tr. to kilograms.	
1	15·43234874	1	32·1507265	1	·0647989504	1	·0311034962
2	30·86469748	2	64·3014531	2	·1295979007	2	·0622069923
3	46·29704622	3	96·4521796	3	·1943968511	3	·0933104885
4	61·72939496	4	128·6029062	4	·2591958014	4	·1244139847
5	77·16174370	5	160·7536327	5	·3239947518	5	·1555174809
6	92·59409244	6	192·9043592	6	·3887937022	6	·1866209770
7	108·02644118	7	225·0550858	7	·4535926525	7	·2177244732
8	123·45878992	8	257·2058123	8	·5183916029	8	·2488279694
9	138·89113866	9	289·3565389	9	·5831905532	9	·2819314655

## EXAMPLES.

## 1. Convert

- (1) 76589·45 kilogrammes to lbs.
- (2) 850·32 quintals to hundredweights.
- (3) 7656 metres to yards.
- (4) 1958 litres to gallons.
- (5) 65432 hectolitres to quarters.
- (6) 9765·874 milligrammes to grains.

## 2. Change

- (1) 8657 yds. 2 ft. 11 inches to metres.
- (2) 751 miles to kilometres.
- (3) 7586 qrs. 7 bushels to hectolitres.
- (4) 81 tons 5 cwts. to quintals.
- (5) 98768 lbs. to pfunde.
- (6) 5678·143 inches to millimetres.

## 6. The Calculation of Equivalent Prices with Foreign Rates.

1°. To find foreign price per metric unit, given sterling price per English unit.

Decimalise the sterling price to £.

Multiply this by the equivalent of the metric unit in decimals of the English units.

Finally, multiply the product by the rate of exchange.

The answer is the foreign price per metric unit.

This must be correct to 2 places.

The integral part of the Rate of Exchange consists usually of two digits.

∴ the product "price × equivalent" must be correct to 5 places (§ vi).

*Example.* Price per yd. is 1s. 9½d.—to find price per metre in Austrian florins at 117·60 (florins per £10).

1s. 9½d. = £·0885416.

1 metre = 1·0936331 yd.

1·0936331	·096832
61·45880	6711
<hr/>	<hr/>
87490	968
8749	97
547	67
44	5
2	<hr/>
·096832	1·137

∴ Price per metre = 1·14 florins.

2°. To find sterling price per English unit, given foreign price per metric unit.

Multiply foreign price by the equivalent of the English unit in decimals of the metric unit.

Divide the product by the rate of exchange.

The answer is the price in decimals of a £ per English unit.

For farthings, eighths or 16ths of a penny 3, 4 or 5 places are required in the answer.

Hence decide by the methods of Prediction (§ vi) the no. of places required correct in the product "price  $\times$  equivalent."

*Example.* Price per litre is 1.47 francs—to find price per gallon at 25.20 (francs per £).

1 gallon = 4.543458 litres.

2 | 1 | 0 + 3 = 3 - 1 = 2 places.

$$\begin{array}{r}
 4.543457 \\
 \underline{741} \\
 4543 \\
 1817 \\
 \underline{318} \\
 6.678
 \end{array}
 \qquad
 \begin{array}{r}
 2,5,20 \mid 6.678 \mid .2650 \\
 \underline{1\ 638} \mid 5s. 3\frac{3}{4}d. \\
 126 \\
 \underline{\phantom{0}1}
 \end{array}$$

### EXAMPLES.

Find price

1. per metre, given price per yd.  $8\frac{1}{4}d.$ , rate 25.18 francs.
2. per yd., given price per metre 1.07 marks, rate  $20.40\frac{1}{2}$  marks.
3. per kilog., given price per lb.  $3\frac{5}{8}d.$ , rate 18.35 kroners.
4. per lb., given price per kilog. 2.13 francs, rate  $25.19\frac{1}{2}$  francs.
5. per litre, given price per gallon 1s.  $2\frac{1}{2}d.$ , rate 96.75 roubles.
6. per Imp. qr., given price per hectol., 23.15 marks rate 20.38 marks.

### 7. The Calculation of Equivalent Prices with Sterling Rates.

1°. To find foreign price per metric unit, given sterling price per English unit.

Reduce sterling price to pence and decimals of a penny.

Multiply this by the equivalent of the metric unit in decimals of the English unit.

Divide the product by the rate of exchange.

The answer is the foreign price per metric unit.

This must usually be correct to 2 or 3 places.

The rate in pence has 2 digits in its integral part as a rule.

Hence decide by the methods of prediction (§ vi) the no. of places necessary to be correct in the product "price  $\times$  equivalent."

*Example.* Price per lb. is 1s. 1½d.—find price per kilog. in Spanish pesetas at 49½ (pence per piastre).

$$1s. 1\frac{1}{2}d. = 13.75.$$

$$1 \text{ kilog.} = 2.20462 \text{ lbs.}$$

$$2 \mid 2 \mid 1 + 3 = 4 - 2 = 2 \text{ places.}$$

$$49\frac{1}{2} \text{ per piastre} = 9.8125 \text{ per peseta.}$$

$$\begin{array}{r}
 2.20462 \\
 5731 \\
 \hline
 2\ 2046 \\
 6614 \\
 1543 \\
 110 \\
 9.8125 \overline{) 30.813} \quad 3.089 \\
 \underline{875} \\
 90 \\
 2
 \end{array}$$

$$\therefore \text{ Price per kilog.} = 3.09 \text{ pesetas.}$$

2. To find sterling price per English unit, given foreign price per metric unit.

Multiply foreign price by the equivalent of the English unit in decimals of the metric unit.

Further multiply this product by the rate in pence and decimals of a penny.

The answer is the sterling price in pence per English unit.

The rate in pence usually has 2 integral digits.

The foreign price should be correct to 2 places.

$\therefore$  the product "price  $\times$  equivalent" must be correct to 5 places (§ vi).

*Example.* Price per metre is 243 reis—find price per yd. at 50 $\frac{3}{4}$  (pence per milreis).

1 yd. = .9143884 metre.

.9143884	50.375
342	91222
182877	10075
86575	1007
2743	101
222195	5
	5
	11.193

$\therefore$  Price per yd. = 11 $\frac{1}{4}$ d.

### EXAMPLES.

Find price

1. per lb., given price per kilog. as 2.31 pes., rate 40 $\frac{1}{8}$  pence per piastre.
2. per quintal, given price per cwt. as 15s. 5 $\frac{1}{2}$ d., rate 53 $\frac{1}{8}$  pence per milreis.
3. per yd., given price per metre 1.17 rouble, rate 24 $\frac{1}{8}$  pence per rouble.
4. per gallon, given price per litre 12 reis, rate 52 $\frac{7}{16}$  pence per milreis.

### 8. Equivalent Prices by means of the Tables.

1°. To find cost in English money per English unit, given cost in foreign money per metric unit.

*Example 1.* Price per litre is 1.47 fr.—to find price per gallon at rate of exchange 25.20 (francs per £).

Refer to "gallons to litres."

Then price per gallon in francs =

	4.543458	
	1.817383	
	318042	
25.20	6.678883	£.265034
	16388	5s. 3 $\frac{3}{4}$ d.
	12688	
	88	
	12	
	2	

*Example 2.* Price per kilogramme is  $2\cdot36\frac{1}{2}$  marks—to find price per cwt. at rate of exchange  $20\cdot38$  (marks per £).

Refer to "cwts. to kilogrammes."

Then price per cwt. in marks =

$$\begin{array}{r}
 101\cdot604754 \\
 15\cdot240713 \\
 3\cdot048143 \\
 \hline
 254012 \\
 20\cdot38 \mid \frac{120\cdot147622}{18247} \mid \pounds 5\cdot8953 \\
 \hline
 1943 \\
 109 \\
 \hline
 7 \\
 1
 \end{array}$$

2°. To find cost in foreign money per metric unit, given cost in sterling per English unit.

*Example 1.* Price per yard is  $1s. 9\frac{1}{4}d.$ —to find price per metre in Austrian florins at  $117\cdot60$  (florins per £10).

Refer to "metres to yards,"

Then price per metre in pence =

$$\begin{array}{r}
 21\cdot8727 \\
 1\cdot0936 \\
 \cdot2734 \\
 24\cdot0 \mid \frac{23\cdot2397}{1639} \mid \pounds \cdot09683 \\
 \hline
 1997 \\
 77 \\
 \hline
 5 \\
 \cdot09683 \\
 6711 \\
 \hline
 968 \\
 97 \\
 67 \\
 5 \\
 \hline
 1\cdot137 \text{ fl.}
 \end{array}$$

∴ Price per metre =  $1\cdot14$  florins.

*Example 2.* Price per lb. is  $13\frac{3}{4}d.$ —find price per kilogramme in Spanish pesetas at  $49\frac{1}{4}d.$  (pence per piastre).

Refer to "kilogrammes to pounds."



Then price per kilog. in pence =

$$\begin{array}{r}
 22\cdot0462 \\
 6\cdot6139 \\
 5 \overline{) 49\cdot0625} \quad \left| \begin{array}{r} 1\cdot6535 \\ 30\cdot3136 \\ \hline 9\cdot8, 1, 2, 5 \\ \hline 911 \\ \hline 28 \end{array} \right. \quad 3\cdot089
 \end{array}$$

Ans. 3·09 pesetas.

### EXAMPLES.

1. Find price per yard, given price per metre as

- (1) 1·24 francs at 25·19½ (French).
- (2) 1·17 marks at 20·41 (German).
- (3) 1·34 florins at 12·05 (Dutch).
- (4) 1·17 florins at 117·65 (Austrian).
- (5) 294 reis at 50½*d.* (Lisbon).
- (6) 1·24 pesetas at 42½*d.* (Madrid).

2. Find price per metre, given price per yard as

1*s.* 2½*d.*, 1*s.* 3½*d.*, 1*s.* 1¾*d.*, 11½*d.*, 9⅞*d.*,

when rates of exchange are respectively

25·21½, 20·38½, 18·74 (Sweden), 49½ (Lisbon), 117·82 (Austrian).

3. Find price

- (1) per hectolitre, given price per quarter as 33*s.* 8*d.*, rate 11·95 (Holland).
- (2) per gallon, given price per litre as 2·27 marks, rate 20·39.
- (3) per bushel, given price per hectolitre as 27·25 francs, rate 25·18½.
- (4) per ½ kilogramme, given price per lb. as 11½*d.*, rate 25·26 (Belgium).
- (5) per pfund, given price per cwt. as 17*s.* 8*d.*, rate 20·38½.
- (6) per lb., given price per kilogramme as 2·23 florins, rate 118·42 (Austria).
- (7) per oz. Troy, given price per gramme as 3·82 marks, rate 20·42.

4. Find price per lb., given price per  $\frac{1}{2}$  kilogramme as below in the various countries at these exchanges:

- (1) 25·21 $\frac{1}{2}$  (France), price 1·47 francs.
- (2) 20·40 $\frac{1}{2}$  (Germany), price 2·15 marks.
- (3) 12·15 $\frac{1}{2}$  (Holland), price 1·63 florins.
- (4) 118·45 (Austria), price 2·31 $\frac{1}{2}$  florins.
- (5) 50 $\frac{3}{4}$  (Lisbon), price 0·513 mark.
- (6) 42 $\frac{1}{4}$  (Spain), price 1·27 pesetas.
- (7) 26·28 (Italy), price 1·91 $\frac{1}{2}$  liras.
- (8) 25·31 (Belgium), 2·13 francs.
- (9) 25·36 (Switzerland), 3·10 $\frac{1}{2}$  francs.
- (10) 25·76 (Bulgaria), 1·52 $\frac{1}{2}$  lei.

*Note.* For the rates of exchange in general the quotation is per £1, but if necessary the country indicated must be consulted in the Chapters on Exchanges.

### 9. Equivalent Prices with Foreign Rates by means of Fixed Numbers.

The rules given here may be used in the case of all countries giving a rate to London and using the Metric System.

#### A. 1. *Price per lb. given price per kilo.*

Fixed number for kilos. to lbs. = 108·8622366.

Multiply number by foreign price and divide by rate of exchange. Answer is in pence.

#### 2. *Price per cwt. given price per kilo.*

Fixed number for kilos. to cwts. = 1016·0475.

Multiply number by foreign price and divide by rate of exchange. Answer is in shillings.

#### 3. *Price per gallon given price per litre.*

Fixed number for litres to galls. = 1090·429913.

Multiply number by foreign price and divide by rate of exchange. Answer is in pence.

4. *Price per quarter given price per hectol.*

Fixed number for hectols. to qrs. = 58·1562620.

Multiply number by foreign price and divide by rate of exchange. Answer is in shillings.

5. *Price per yd. given price per metre.*

Fixed number for metres to yds. = 219·4520353.

Multiply number by foreign price and divide by rate of exchange. Answer is in pence.

The methods of approximation apply—especially the methods of prediction (§ vi).

Multiples (1—9) may be constructed for each of the five fixed numbers.

B. 1. *Price per kilo. given price per lb.*

Fixed number for lbs. to kilos. = ·00918592187.

Multiply price in pence by rate of exchange and the product by the fixed number. Answer is in foreign units of rate.

2. *Price per kilo. given price per cwt.*

Fixed number for cwts. to kilos. = ·0009842059.

Multiply price in shillings by rate of exchange and the product by the fixed number. Answer is in foreign units of rate.

3. *Price per litre given price per gallon.*

Fixed number for galls. to litres = ·000917069486.

Multiply price in pence by rate of exchange and the product by the fixed number. Answer is in foreign units of rate.

4. *Price per hectol. given price per Imp. qr.*

Fixed number for qrs. to hectols. = ·0171950529.

Multiply price in shillings by rate of exchange and the product by the fixed number. Answer is in foreign units of rate.

5. *Price per metre given price per yd.*

Fixed number for yds. to metres = '0045568044.

Multiply price in pence by rate of exchange and the product by the fixed number. Answer is in foreign units of rate.

The methods of prediction (§ vi) apply and multiples of the fixed numbers may be constructed.

*Example 1.* Price per kilo. = 1.72 francs. Rate = 25.21.

108.8622366	25.21   187.24   7.42
2 71	10 7
108 86	6
76 20	1
2 18	
187.24	

Price per lb. =  $7\frac{1}{2}d.$

*Example 2.* Price per Imp. qr. = 37s. 9d. Rate = 20.40.

20.40	.0171950529
37 $\frac{3}{4}$	01877
754.80	12037
15.30	1203
770.10	2
	13.242

Price per hectol. = 13.25 marks.

The modifications for a pfund ( $\frac{1}{2}$  kilo.) are easily made.

## 10. Equivalent Prices with Sterling Rates by means of Fixed Numbers.

The rules here given may be used in the case of all countries receiving a rate from London and using the Metric System.

### A. 1. *Price per lb. given price per kilo.*

Fixed number for kilos. to lbs. = .4535926525.

Multiply price by rate in pence and the product by the fixed number. Answer is in pence.

2. *Price per cwt. given price per kilo.*

Fixed number for kilos. to cwts. = 4·2335314.

Multiply price by rate in pence and the product by the fixed number. Answer is in shillings.

3. *Price per gallon given price per litre.*

Fixed number for litres to gallons = 5·4345797.

Multiply price by rate in pence and the product by the fixed number. Answer is in pence.

4. *Price per qr. given price per hectol.*

Fixed number for hectols. to qrs. = 2·423177583.

Multiply price by rate in pence and the product by the fixed number. Answer is in shillings.

5. *Price per yd. given price per metre.*

Fixed number for metres to yds. = 9143834807.

Multiply price by rate in pence and the product by the fixed number. Answer is in pence.

The methods of prediction (§ vi) apply and multiples of the fixed numbers may be constructed.

B. 1. *Price per kilo. given price per lb.*

Fixed number for lbs. to kilos. = 2·204621249.

Multiply number by price in pence and divide by rate of exchange. Answer is in foreign units of rate.

2. *Price per kilo. given price per cwt.*

Fixed number for cwts. to kilos. = 2·362094.

Multiply number by price in shillings and divide by rate of exchange. Answer is in foreign units of rate.

3. *Price per litre given price per gallon.*

Fixed number for galls. to litres = 2200966768.

Multiply number by price in pence and divide by rate of exchange. Answer is in foreign units of rate.

4. *Price per hectol. given price per Imp. qr.*

Fixed number for qrs. to hectols. = 4·1268127.

Multiply number by price in shillings and divide by rate of exchange. Answer is in foreign units of rate.

5. *Price per metre given price per yd.*

Fixed number for yds. to metres = 1·093633056.

Multiply number by price in pence and divide by rate of exchange. Answer is in foreign units of rate.

The methods of approximation apply—especially the methods of prediction (§ vi).

The multiples of the fixed numbers may be constructed.

*Example 1.* Price per metre = 1·61 pesetas at 43½d. (Spain).

1·61 pes. =	·322 piastre.	13·88625
·322 × 43½ =	966	834 19
	1288	124 97
	40½	1 59
	18·886½	65
		4

Price per yd. = 12·706 = 12¾d.

*Example 2.* Price per cwt. = 17s. 10d. Rate = 45½d. (Lisbon).

·2862092 × 17½ =	2·362092	45·25	4·2123974		·0930
	1·6534644		1398		
	196841		40		
	4·2123974				

∴ Price per kilo. = 98 reis.

In the case of Spain the rate is quoted in pence per piastre. 1 piastre = 5 pesetas.

The modifications for this and the half-kilo. are easily made.

The Portuguese milreis is divided into 1000 reis—thus one more place is necessary than with other foreign moneys.

*Note.* The Fixed Numbers are simply the invariable part of the work ready done in each case. They are obtained from the equi-

valents of the various units in a manner which is similar in all the cases, one example will therefore suffice.

9 B. 1 lbs. to kilos = .00918592187.

1 kilo = 2.204621249 lbs.,  $\therefore$  if 1 lb. costs 1d., 1 kilo will cost 2.204621249 pence. The value in £ is thus .00918592187 (2.204621249  $\div$  240) and the rule follows at once. Multiply £.00918592187 by price in pence and then by rate of exchange to convert the product in £ to the foreign price.

A most valuable exercise for the Student would be to determine some of the remaining Fixed Numbers with due regard to the rules to which they are attached.

### EXAMPLES.

1. Find price per cwt. given price per kilo. and rate of exchange.

- |                                       |  |
|---------------------------------------|--|
| (1) 1.23 m. at 20.40.                 | (2) 3.17 fl. at 11.95 $\frac{1}{2}$ .                |
| (3) 1.72 $\frac{1}{2}$ fl. at 118.20. | (4) 3.17 $\frac{1}{2}$ pesetas at 49 $\frac{3}{8}$ . |
| (5) 0.895 m. at 52 $\frac{1}{8}$ .    | (6) 1 rup. 12 annas at 17 $\frac{5}{16}$ .           |

2. Find price per kilo. given price per lb.

- |   |   |
|---|---|
| (1) 1s. 2 $\frac{3}{4}$ d. at 25.17 $\frac{1}{2}$ .         | (2) 11 $\frac{5}{16}$ d. at 20.37 $\frac{1}{2}$ .       |
| (3) 2s. 1 $\frac{1}{2}$ d. at 18.43.                        | (4) 1s. 9 $\frac{1}{8}$ d. at 42 $\frac{5}{8}$ (Spain). |
| (5) 2s. 3 $\frac{1}{2}$ d. at 51 $\frac{5}{16}$ (Portugal). |   |
| (6) 2s. 1 $\frac{1}{16}$ d. at 13 $\frac{5}{8}$ (Brazil).   |   |

3. Find price per litre given price per gallon.

- |  |
|--|
| (1) 5s. 7 $\frac{1}{4}$ d. at 26.18 (Italy).             |
| (2) 10s. 9d. at 25.27 (Switzerland).                     |
| (3) 6s. 5 $\frac{1}{2}$ d. at 47 $\frac{1}{8}$ (Spain).  |
| (4) 4s. 6 $\frac{1}{4}$ d. at 52 $\frac{3}{8}$ (Lisbon). |

4. Find price per quarter given price per hectol.

- |   |  |
|---|--|
| (1) 10.27 m. at 20.38.                  | (2) 13.50 fl. (Dutch) at 12.15 $\frac{1}{2}$ . |
| (3) 13.25 pesetas at 49 $\frac{1}{4}$ . | (4) 1.378 milr. at 51 $\frac{3}{8}$ .          |

5. Find price per metre given price per yd.

- |   |   |
|---|---|
| (1) 12s. 9 $\frac{1}{2}$ d. at 25.19.           | (2) 5s. 7 $\frac{3}{8}$ d. at 47 $\frac{5}{8}$ (Lisbon).  |
| (3) 7 $\frac{3}{4}$ d. at 20.36 $\frac{1}{2}$ . | (4) 1s. 9 $\frac{3}{8}$ d. at 23 $\frac{5}{16}$ (Russia). |

6. Find price per yd. given price per metre.

- |   |                                       |
|---|---------------------------------------|
| (1) 10.35 m. at 20.41 $\frac{1}{2}$ .               | (2) 2.350 milr. at 50 $\frac{1}{2}$ . |
| (3) 0.81 $\frac{1}{2}$ fr. at 25.20 $\frac{1}{2}$ . | (4) 1.13 fl. at 12.05 (Amst.).        |

## SECTION V.

### PROPORTION AND PERCENTAGES.

#### Proportion.

1. *The Principle of Proportion*, or in other words *the Comparison of Ratios*, underlies the vast majority of commercial calculations, but very rarely indeed does the statement of the question appear in a direct form such as a rule of three sum in ordinary text-books. In fact the usual proportion is "if 1 thing costs so much what will a given quantity cost?" and not "if 27 things cost so much what will the quantity cost?"

None the less however is the principle of proportion involved, and the essential nature of such calculations cannot be too carefully remembered however much disguised the forms in which it occurs.

It should also be noticed that the principle of *Cancellation* is of the utmost importance in reducing labour.

Proportional parts, percentages, and some calculations in interest, together with questions of mixtures etc., involve the principle most plainly, but the shortest methods of solution even of these dispenses with the rule of three or the reduction to unity forms—both these forms being too cumbrous for actual work where time is valuable.

We give an example of proportion to show the value of cancellation in reducing labour and also to show the



clearest method of stating all problems of proportion, but it must be clearly understood that no such example ever occurs in real business.

The form of solution entirely does away with such artificial distinctions as rule of three direct, rule of three inverse, simple or double rule of three.

*Example.* If 10 mowers in 18 days of 10 hours can cut 15 fields 125 ft. long by 72 ft. broad, in how many days of 12 hours can 36 mowers cut 20 fields 75 ft. long by 108 ft. broad, supposing the second set of mowers work  $\frac{1}{4}$  harder than the first set?

Days.		18	Required term.
Mowers.	2	36	10 2 Less time with 36 than with 10.
Hours.		12	10 2 Fewer days of 12 hours than of 10.
Fields.		20	15 4 More time to cut 20 fields than 15.
Length.	125	75	5 Less time to cut a length of 75 than of 125.
Breadth.	72	108	5 More time to cut a length of 108 than of 72.
Ratio.		5	4

*Ans.* = 4 days.

This is the form for examination purposes.

For rapid working a horizontal line may be used.

$$\frac{18 \times 10 \times 10 \times 20 \times 75 \times 108 \times 4}{36 \times 12 \times 15 \times 125 \times 72 \times 5} = 4 \text{ days.}$$

### Proportional Parts.

2. Many questions of division of profits, proportion of losses, shares in property or in dividends arising from joint ownership are best done by the *Theory of Proportional Parts*.

There are two fundamental cases:—

1°. To divide a number or quantity into parts proportional to certain integral numbers.

Add the integers and place each upon this sum to form fractions.

Multiply the given number or quantity by each of these fractions in succession.

The answers will be the parts required.

2°. To divide a number or quantity into parts proportional to certain fractions (vulgar or decimal).

(a) Add the fractions.

Place each fraction upon this sum-fraction and simplify.

Multiply the given number or quantity by each of these simplified fractions.

(b) Reduce the fractions to L.C.M.

Add numerators—place each numerator upon this sum to form fractions.

Multiply the given quantity by each fraction in succession as in 1.

*Example 1.* Divide £10000 among *A*, *B*, *C* in ratio of 4, 6, 8.

$$4 + 6 + 8 = 18. \quad \frac{2}{3} \text{ of } £10000 = \frac{£20000}{9} = £2222 \cdot 2222 = £2222 \cdot 4s. 5\frac{1}{2}d.$$

$$\frac{1}{3} \text{ of } £10000 = \frac{£10000}{3} = £3333 \cdot 3333 = £3333 \cdot 6s. 8d.$$

$$\frac{4}{9} \text{ of } £10000 = \frac{£40000}{9} = £4444 \cdot 4444 = £4444 \cdot 8s. 10\frac{1}{2}d.$$

*Example 2.* Divide £963. 17s. 4½d. in proportion of  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{1}{3}$ .

$$\frac{1}{2}, \frac{2}{3}, \frac{1}{3}. \quad \frac{1}{2} \times £963 \cdot 8677083 \text{ etc.}$$

The chief cases of proportional parts which arise are

- (1) Mixtures of all kinds, including alloys.
- (2) Percentages not involving time, such as commissions on sales, broker's charges, reductions for cash payment, profit and loss, bankruptcy.
- (3) Percentages involving time, such as interest, discount on bills.
- (4) Partnership with and without time.

### Mixtures.

#### 3. Diluting Spirits, Wines, etc. without raising the price.

A. To find how many gallons of plain water, sweet water, or inferior spirit at given price must be added to 1 gallon of another spirit at given price to realise a *given profit* when mixture is sold at original price.

Divide profit by difference of prices—answer is no. of gallons to be added.

*To find profit per gallon of the mixture.*

Divide profit by whole no. of gallons mixed.

In the case of plain water of course the price is nothing.

*Example.* How much sweet water at 9d. per gall. must be added to a gallon of whisky at 12s. 6d. to produce a profit of 3s.?

*Answer.*  $3 \div 12\frac{1}{2} - \frac{3}{4} = 3 \div 11\frac{1}{4} = 12 \div 47 = 1\frac{1}{4}$  gallon.

(a) *Conversely.* To find the profit arising from adding a certain number of gallons of plain water, sweet water, or inferior spirit at given price to 1 gallon of another spirit at given price when mixture is sold at this original price.

Multiply the number of gallons added by difference of prices.

*Example 1.* Profit from adding 10 gallons of plain water to 1 gallon of whisky at 10s. 6d. *Answer.*  $10 \times 10s. 6d. = £5. 5s. 0d.$

*Example 2.* Profit from adding poor brandy (8 gallons) at 1s. 6d. to 1 gallon at 15s. 6d. *Answer.*  $8 \times (15s. 6d. - 1s. 6d.) = £5. 12s. 0d.$

B. To find how many gallons of plain water, sweet water, or inferior spirit at given price must be added to 1 gallon of another spirit at given price to realise a *given profit per gallon of the mixture* when mixture is sold at original price.

Divide given profit per gallon by difference of prices less profit.

*To find total profit.*

Multiply profit per gallon of mixture by whole no. of gallons mixed.

*Example.* How much sweet water at 10d. per gallon must be added to a gallon of spirit at 12s. 6d. that profit per gallon may be 2s. 6d.?

*Answer.*  $2s. 6d. \div (12\frac{1}{2} - \frac{1}{2} - 2\frac{1}{2}) = 2\frac{1}{2} \div 9\frac{1}{2} = \frac{5}{2} \times \frac{2}{19} = \frac{5}{19}$  gall.

(b) *Conversely.* To find the profit per gallon of the mixture arising from adding a certain no. of gallons of plain water, sweet water, or inferior spirit at given price to 1 gallon of another spirit at given price when mixture is sold at this original price.

Multiply difference of prices by the no. of gallons added  $\div$  total number of gallons in mixture.

*Example.* Profit per gallon of mixture from adding 8 gallons of plain water to 1 gallon of spirit at 13s. 6d.  $13s. 6d. \times \frac{8}{9} = 12s.$

The principle underlying these two methods is "that any number of measures of diluting liquid added to one measure of original liquid gives a profit of so many measures of original liquid less the cost of the diluting measures, if the mixture be sold at the price of the original liquid."

Thus if 8 gallons of water be added to 1 gallon of wine, the profit is 8 gallons of wine, when mixture is sold at the price of the wine.

And similarly, if 6 pints of sweet water be added to 1 pint of brandy, the profit will be 6 pints of brandy less the cost of the sugar, when the mixture is sold at the price of the brandy.

The methods adopted here may be employed in blending teas or mixing chicory with coffee, but usually the methods following are better adapted, except in the cases where one of the ingredients is of little cost, as in the cases of plain or sweet water above.

It is also to be remarked that the methods following apply equally to liquids and dry mixtures, whether weighed or measured.

#### 4. **Mixtures of all kinds.**

A. To find price of a mixture—given measures (weights) and prices of ingredients.

Multiply each measure (weight) by its price.

Add and divide by sum of the measures (weights).

*Example.* 16 lbs. of coffee at  $9\frac{3}{4}d.$ , 6 at  $1s. 2d.$ , 8 at  $1s. 3d.$  are mixed.

$$\text{Answer. } \frac{16 \times 9\frac{3}{4} + 6 \times 14 + 8 \times 15}{30} = \frac{156 + 84 + 120}{30} = \frac{360}{30} = 12d. \text{ per lb.}$$

B. To find proportions of ingredients of given prices so that mixture may bear given price.

(1) For two ingredients.

Ratio =  $\frac{\text{one price} \sim \text{mixed price}}{\text{mixed price} \sim \text{other price}}$ . (A particular case of linkages.)

(2) For more than two ingredients.

Use the method of linkages explained below.

This method is based upon the principles of indeterminate equations.

In general the answers are more than one.

### **The Method of Linkages.**

Write prices in a column in descending or ascending order of magnitude.

Leave a space for the insertion of the given mixture price but place this for distinctness a little to the left of the column.

Link each number above the space with each number below—these links determine the number of the various solutions possible—the number being the ways in which we can choose the links so as to use all the prices at one time.

For any set of links chosen as a solution—find difference of each price and mixed price and place this difference opposite the price to which each price is linked.

The column thus formed gives the proportions.

In the cases where more than one link runs from any price it will have two or more differences opposite it—the sum of these gives the proportion-figure—this

happens when there is an odd number of ingredients or when the number of prices above the space is not the same as the number below.

*Example 1.* Spirits at 12s. per gallon are to be mixed with sweet water at 9d. per gallon so as to reduce the price to 10s.—find the proportions.

$$\begin{array}{rcl}
 & 12s. & 9\frac{1}{2} \\
 10s. & \left. \begin{array}{l} \\ \\ \end{array} \right\} & \text{Proportion } 37 : 8. \\
 & 9s. & 2
 \end{array}$$

*Example 2.* Teas at 5s., 4s. 2d., 3s. 6d., 2s. 4d. are to be mixed so as to bear a price of 3s. 9d.

$$\begin{array}{rcll}
 60 & 17 & 3 & \therefore 17 \text{ at } 5s. = 85s. \\
 50 & 3 & 17 & 3 \text{ at } 4s. 2d. = 12s. 6d. \\
 & \text{or} & & 3 \text{ at } 5s. = 15s. \\
 45 & \left( \begin{array}{l} 42 \\ 28 \end{array} \right) & 5 & 15 \text{ at } 3s. 6d. = 52s. 6d. \\
 & & 15 & 5 \text{ at } 2s. 4d. = 11s. 8d. \\
 & & & 15 \text{ at } 3s. 6d. = 52s. 6d. \\
 & & & 5 \text{ at } 2s. 4d. = 11s. 8d. \\
 & & & \hline
 & & & 40 \text{ at } 150s. \\
 & & & 1 \text{ at } 3s. 9d.
 \end{array}$$

*Example 3.* Whiskies at 25s., 22s., 20s., 18s., 10s. per gallon are mixed to sell at 15s.

$$\begin{array}{rcll}
 25 & 5 & & \therefore 5 \text{ at } 25s. = 125s. \\
 22 & 5 & & 5 \text{ at } 22s. = 110s. \\
 20 & 5 & & 5 \text{ at } 20s. = 100s. \\
 18 & 5 & & 5 \text{ at } 18s. = 90s. \\
 15 & & & \\
 10 & 10 + 7 + 5 + 3 & 25 \text{ at } 10s. = 250s. \\
 & & 45 \text{ at } 675s. \\
 & & 1 \text{ at } 15s.
 \end{array}$$

*Example 4.* Coffees at 2s., 1s. 10d., 1s. 8d., 1s. 4d., 1s. are mixed to sell at 1s. 6d.

$$\begin{array}{rcll}
 24 & 6 & 6 & 2 & 2 & 6 & \text{Take (4) e.g.} \\
 22 & 6 & 2 & 6 & 2 & 6 & 2 \text{ at } 2s. = 4s. \\
 20 & 2 & 6 & 6 & 6 & 2 & 2 \text{ at } 1s. 10d. = 3s. 8d. \\
 18 & \left( \begin{array}{l} 16 \\ 12 \end{array} \right) & 2 & 4 & 6 & 6 + 4 & 6 + 2 & 4 + 2 & 6 \text{ at } 1s. 8d. = 10s. \\
 & & 6 + 4 & 6 + 2 & 4 + 2 & 2 & 4 & 6 & 10 \text{ at } 1s. 4d. = 13s. 4d. \\
 & & (1) & (2) & (3) & (4) & (5) & (6) & 2 \text{ at } 1s. = 2s. \\
 & & & & & & & & \hline
 & & & & & & & & 22 \text{ at } 33s. \\
 & & & & & & & & 1 \text{ at } 1s. 6d.
 \end{array}$$

C. When some of the ingredients of given prices are limited in quantity—to find quantities of the re-

maining ingredients of given prices to join with the fixed quantities so that whole mixture may have given price.

(1) If one ingredient is limited.

Use linkages and then proportion.

(2) If more than one ingredient is limited.

Convert the given limited ingredients into a mixture of given weight and price by the method of A.

Then for this and the remaining quantities use linkages and finally proportion.

*Example 1.* 8 lbs. tea at 4s. 2d., others at 3s. 6d., 2s. 4d., to mix at 3s.

$$\begin{array}{rcl}
 50 & 8 & 8 \text{ lbs. at } 4s. 2d. = 33s. 4d. \\
 42 & 8 & 8 \text{ lbs. at } 3s. 6d. = 28s. \\
 36 & & \\
 28 & 20 & 20 \text{ lbs. at } 2s. 4d. = 46s. 8d. \\
 & & \hline
 & & 36 \text{ lbs. at } 108s. \\
 & & 1 \text{ lb. at } 3s.
 \end{array}$$

*Example 2.* 4 lbs. coffee at 1s. 10d., 6 lbs. at 1s. 4d.,—others at 1s. 8d., 1s. 3d., to mix at 1s. 5d.

$$\begin{array}{rcl}
 4 \times 1s. 10d. + 6 \times 1s. 4d. & = & \frac{15s. 4d.}{10} = 1s. 6\frac{1}{2}d. = 18\frac{1}{2}d. \\
 10 & & 10 \\
 17 & \left( \begin{array}{l} 20 \quad 2 \quad 2 \quad 10 \\ 18\frac{1}{2} \quad 2 \quad \frac{1}{2} \cdot 1\frac{1}{2} \text{ or } 4 \cdot 6 \text{ i.e.} \\ 15 \quad 4\frac{1}{2} \quad 4\frac{1}{2} \quad 22 \end{array} \right. & \begin{array}{l} 10 \text{ at } 1s. 8d. = 16s. 8d. \\ 4 \text{ at } 1s. 10d. = 7s. 4d. \\ 6 \text{ at } 1s. 4d. = 8s. \\ 22 \text{ at } 1s. 3d. = 27s. 6d. \\ \hline 42 \text{ at } 59s. 6d. \\ 1 \text{ at } 1s. 5d. \end{array}
 \end{array}$$

D. Given prices of ingredients to find quantities to form a given weight of mixture at given price.

Use linkages and then proportion.

*Example.* Currants at 1s., 10d., 6d., 4d.—240 lbs. at 8d. required.

$$\begin{array}{rcl}
 12 & 42 & \frac{1}{3} \text{ of } 240 = 80 \text{ at } 1s. = 960d. \\
 10 & 24 & \frac{2}{3} \text{ of } 240 = 40 \text{ at } 10d. = 400d. \\
 8 & \left( \begin{array}{l} 6 \\ 4 \end{array} \right) & \begin{array}{l} 24 \\ 42 \end{array} \begin{array}{l} \frac{2}{3} \text{ of } 240 = 40 \text{ at } 6d. = 240d. \\ \frac{1}{3} \text{ of } 240 = 80 \text{ at } 4d. = 320d. \end{array} \\
 & & \hline
 & & 240 \text{ lbs. at } 1920d. \\
 & & 1 \text{ lb. at } 8d.
 \end{array}$$

The proportions and price of any mixture being thus determined the retailer has only to increase the price per measure or weight by the profit he thinks fair to get the selling price of the mixture—the profit being a percentage or otherwise as the retailer thinks right.

### Alcohol Strengths.

5. *Proof Spirit* is that which weighs  $\frac{1}{3}$  of an equal measure of water—both being at  $51^{\circ}$  F.

By volume or measure it contains 57.06 p.c. of pure alcohol—at  $60^{\circ}$  F. its specific gravity is .920.

The terms proof, over proof (o.p.), under proof (u.p.) are used to indicate alcoholic values.

Proof = 1. The o.p. strengths are added to, the u.p. strengths are taken from this as decimals. 25 o.p. = 1.25. 20 u.p. = .80.

In mixtures the rules are derived from simultaneous equations or indeterminate equations as in the case of ordinary ingredients.

Use the method of linkages and (if necessary) then proportion.

*Example 1.* To mix two spirits, one 20 o.p., the other 10 u.p., so as to result in a mixture 5 o.p.

$$\begin{array}{rcl}
 1.05 & \begin{array}{l} 1.20 \\ 0.90 \end{array} \cdot 15. & \therefore \text{Proportions are 1 to 1.} \\
 & & \text{Proof. 1 gallon at 20 o.p.} = 1.20 \text{ proof galls.} \\
 & & \text{1 gallon at 10 u.p.} = .90 \text{ proof galls.} \\
 & & \quad = 2.10 \text{ proof galls.} \\
 & & \quad = 2 \text{ galls. 5 o.p.}
 \end{array}$$

*Example 2.* How many gallons at 6 u.p. are required to reduce 24 gallons 60 o.p. to 5 o.p.?

$$\begin{array}{rcl}
 1.05 & \begin{array}{l} 1.60 \\ .94 \end{array} \cdot 11. & \text{Proportions 1 to 5. } \therefore \text{ necessary quantity is} \\
 & & \quad 5 \times 24 = 120 \text{ gallons.} \\
 & & \text{Proof. 24 gallons at 60 o.p.} = 38.40 \text{ galls. proof.} \\
 & & \quad 120 \text{ gallons at 6 u.p.} = 112.80 \text{ galls. proof.} \\
 & & \quad = 151.20 \text{ galls. proof.} \\
 & & \quad = 144 \text{ galls. 5 o.p.}
 \end{array}$$



*Example 3.* With spirit 25 o.p. how much water must be mixed to make a mixture 10 u.p.?

$$\begin{array}{rcl}
 & 1.25 & \cdot 90. \quad \text{Proportions 18 to 7.} \\
 \cdot 90 & \left. \begin{array}{l} 1.25 \\ \cdot 00 \end{array} \right) & \cdot 35. \quad \text{Proof. } 18 \text{ galls. } 25 \text{ o.p.} = 22.5 \text{ galls. proof.} \\
 & & 7 \text{ galls. water} = 0 \text{ galls. proof.} \\
 & & \hline
 & & = 22.5 \text{ galls. proof.} \\
 & & = 25 \text{ galls. } 10 \text{ u.p.}
 \end{array}$$

*Example 4.* What is strength of a mixture of 9 galls. 25 o.p. and 7 galls. 16 u.p. spirit?

$$\frac{9 \times 1.25 + 7 \times .84}{16} = \frac{11.25 + 5.88}{16} = \frac{17.13}{16} = 1.07. \quad \text{Ans. } 7 \text{ o.p.}$$

### Alloys.

6. In Goldsmiths' and Silversmiths' work, according to law, the oz. Troy and decimals of the oz. are only to be employed, and fineness is to be reckoned in millièmes (thousandths).

The old way of estimating the fineness of gold was by carats—18 carats out of 24 being the standard.

The old way of estimating the fineness of silver was by ozs. and dwts.

These methods are still employed, the weights being expressed in ozs. and grains.

Millièmes are increasingly used, as their simplicity and convenience are appreciated.

The rules for *Alloys* are particular cases of mixtures.

They may be formally stated thus:—

1°. To find the proportions of two metals of differing fineness to produce when mixed an intermediate fineness.

Divide difference of higher and required fineness by difference of required and lower fineness.

For more than two metals use linkages.

2°. To make a compound of given weight and fineness.

Find proportions of the metals by 1° or linkages.

Then use proportion to find the weights required.

If the total weight required is given we can find the weight of each metal necessary.

If the weight of one metal is given we can find the weights of the others and thus the total weight.

*Example.* Mix golds 900 and 925 fine so as to make 625 ozs. 916 fine.

$$\text{Ratio} = \frac{916 - 900}{925 - 916} = \frac{16}{9}. \quad \therefore \frac{16}{9} \text{ of } 625 = 400 \text{ ozs.}, \frac{9}{16} \text{ of } 625 = 225 \text{ ozs.}$$

### EXAMPLES.

1. How much plain water must be added to 1 gallon of spirit or wine

(1) at 12s. to make a profit of 3s. 6d. or a profit per gallon of 3d.?

(2) at 11s. 9d. to make a profit of 2s. 3d. or a profit per gallon of 1s. 5d.?

(3) at 10s. to make a profit of 5s. or a profit per gallon of 3s.?

(4) at 15s. 6d. to make a profit of 7s. 3d. or a profit per gallon of 9d.?

(5) at 11s. 6d. to make a profit of 4s. 2d. or a profit per gallon of 9d.?

2. How much sweet water at 9d. per gallon must be added to 1 gallon of spirit

(1) at 13s. 9d. to make a profit of 2s. or a profit per gallon of 5d.?

(2) at 10s. 3d. to make a profit of 3s. 7d. or a profit per gallon of 1s. 1d.?

(3) at 12s. 6d. to make a profit of 5s. 6d. or a profit per gallon of 2s. 3d.?

3. Find profit from mixing (1) 9 gallons of water, (2) 13 gallons of inferior spirit at 1s. 2d., (3) 12 gallons of sweet water at 8d., with 1 gallon of wines at 15s. 4d., 13s. 7d., 12s. 9d., 17s. 8d., 11s. 6d.

4. Find price of following mixtures

(1) coffees 17 lbs. at 10½d., 13 at 1s. 3½d.

(2) teas 12 lbs. at 2s. 4d., 5 at 3s. 7d., 11 at 4s. 1d., 16 at 3s.

(3) spirits 10 galls. at 11s. 9d., 12 at 10s. 3d., 18 at 9s. 6d.

5. Find proportions of these mixtures to sell at the given prices

(1) spirits at 10*s.* 6*d.*, 13*s.* 5*d.*, 12*s.* 9*d.*, 14*s.* 2*d.*, 15*s.*, to sell at 13*s.*

(2) teas at 3*s.* 6*d.*, 2*s.* 2*d.*, 1*s.* 8*d.*, 5*s.*, 4*s.* 2*d.*, to sell at 2*s.* 6*d.*

(3) coffees at 10*d.*, 11½*d.*, 1*s.* 1*d.*, 1*s.* 4*d.*, 1*s.* 8*d.*, 2*s.*, to sell at 1*s.* 6*d.*

(4) currants at 3½*d.*, 5*d.*, 7½*d.*, 1*s.* 2*d.*, 1*s.* 3½*d.*, to sell at 1*s.*

(5) teas at 2*s.* 10*d.*, 3*s.* 1*d.*, 2*s.*, to sell at 2*s.* 9*d.*

6. (1) 10 lbs. of coffee at 1*s.* 6*d.* are mixed with coffees at 1*s.*, 1*s.* 7*d.*, 1*s.* 3*d.* to sell at 1*s.* 5*d.*, find quantities.

(2) 6 lbs. of tea at 3*s.* and 5 at 2*s.* 9*d.* with other teas at 2*s.* 6*d.*, 3*s.* 5*d.*, 4*s.* to sell at 3*s.* 2*d.*, find quantities.

(3) 348 lbs. of currants at 10*d.* are to be mixed from currants at 4½*d.*, 7*d.*, 11½*d.*, 1*s.* 2*d.*, find quantities.

(4) 100 gallons of beer at 2*s.* are to be mixed from beers at 1*s.* 5*d.*, 2*s.* 3*d.*, 1*s.* 7*d.*, 2*s.* 1*d.*, find quantities.

(5) 4 lbs. of coffee at 2*s.*, and 10 at 1*s.* 11*d.* are mixed with coffees at 10*d.*, 1*s.* 1*d.*, 1*s.* 5*d.* to sell at 1*s.* 6*d.*, find quantities.

7. Find strength of these spirit mixtures

(1) 7 galls. 10 o.p. with 8 galls. 16 u.p.

(2) 13 galls. 5 o.p., 10 galls. 3 o.p., 16 galls. 5 u.p.

8. How many gallons (1) of 8 o.p., (2) of 6 u.p., (3) of 15 o.p., (4) of 16 u.p., must be mixed with (1) 10 galls. 5 u.p. to bring mixture to proof, (2) 12 galls. 5 o.p. to bring mixture to 1 o.p., (3) 8 galls. 10 u.p. to bring mixture to 5 u.p., (4) 16 galls. 8 o.p. to bring mixture to 5 o.p.?

9. Mix the following metals to produce alloys of the given fineness

(1) golds 10 and 15 carats fine to give 20 ozs. 12 carats fine.

(2) silvers 312 and 401 grains fine to produce 150 ozs. 350 grains fine.

(3) gold 886 fine and silver 520 fine to produce alloy 840 fine.

(4) bronzes 845 and 895 fine to give 100 ozs. 870 fine.

(5) copper 512, tin 603, zinc 651 fine to give coins 600 fine.

10. (1) We have 15 ozs. of gold 17 carats fine, and a large quantity of gold at 9 carats fine and gold 12 carats fine. Find total weight of gold made from these 15 carats fine.

(2) Convert 18 carats, and 11 ozs. 2 dwts. fineness into millèmes.

(3) Express the parts of our bronze coins ½ copper, ¼ p.c. tin, 1 p.c. zinc in millèmes.

**Percentages.**

7. *Percentages* are in reality cases of proportion or proportional parts.

One hundred is taken as the basis of calculation.

So much per cent. implies so much out of every 100.

This is technically expressed by the word "Percentage" or the phrase "Rate per cent." ( $\%$  p.c.)

The use of percentages covers the whole range of commercial and scientific operations.

They may be calculated with or without regard to time.

(1) *Percentages with regard to time.*

Interest.

Bill discounting.

Partnership (compound).

(2) *Percentages without regard to time.*

Commissions on sales.

Broker's charges.

Discounts for cash.

Profit and loss.

Bankruptcy.

Population questions.

Death rate.

Dividends.

Partnership (simple).

Insurance (fire, life, marine).

Taxes.

Duties.

To the second class is the term more usually applied.

The brokerage on foreign bills is usually 1 per mille—to reckon by a thousand involves exactly the same principle as a percentage.

**Methods of Calculating Percentages.**

- (1) Decimalise the amount, moving the point two places to the left.  
Decimalise the fractional part of the rate.  
Multiply by the method of approximation.
- (2) Decimalise the amount, moving the point two places to the left.  
Aliquotise the fractional part of the rate.  
Use the rate in this form to obtain the percentage.
- (3) Decimalise the amount without moving the point.  
Aliquotise the whole rate to 100 as base.  
Use the rate in this form to obtain the percentage.

*Example 1.*  $3\frac{2}{3}$  p.c. on £312. 8s.  $5\frac{1}{2}$ d. 3·12421  
 $3\frac{2}{3} = 3-28$  (Aliquots). 8  
9·8726  
1·5621  
·1953  
£11·130 (2s.  $7\frac{1}{2}$ d.).

*Example 2.*  $1\frac{1}{2}$  p.c. on £534. 15s.  $7\frac{1}{2}$ d. 5·34782  
 $1\frac{1}{2} = 1-424$  (aliquots). 1·33695  
·66847  
·16712  
£7·520 (10s. 5d.).

*Example 3.*  $4\frac{2}{3}$  p.c. on £57. 18s.  $6\frac{1}{2}$ d. ·57926  
64  
23170  
3475  
£2·6645 (13s.  $3\frac{1}{2}$ d.).

*Example 4.* 36 p.c. on £105. 14s.  $11\frac{1}{2}$ d. 105·746875  
 $\frac{36}{100} = 0-5225$  (aliquots). 21·149375  
10·574687  
5·287344  
1·057469  
£38·168 (3s.  $4\frac{1}{2}$ d.).

**EXAMPLES.**

1.  $3\frac{5}{16}$ ,  $2\frac{1}{8}$ ,  $4\frac{1}{8}$ ,  $3\frac{5}{8}$ ,  $1\frac{1}{8}$  p.c. on £571. 6s. 9½d.
2.  $4\frac{9}{16}$ ,  $1\frac{1}{4}$ ,  $1\frac{5}{8}$ ,  $3\frac{3}{8}$ ,  $7\frac{1}{8}$  p.c. on £861. 9s. 11d.
3.  $1\frac{1}{8}$ ,  $2\frac{1}{8}$ ,  $3\frac{3}{8}$ ,  $5\frac{1}{8}$ ,  $8\frac{1}{8}$  p.c. on £1000. 0s. 0d.
4.  $3\frac{1}{2}$ ,  $4\frac{1}{4}$ ,  $1\frac{5}{8}$ ,  $3\frac{3}{8}$ ,  $5\frac{5}{8}$  p.c. on £712. 13s. 5¼d.
5.  $3\frac{1}{16}$ ,  $5\frac{7}{16}$ ,  $8\frac{1}{4}$ ,  $5\frac{7}{16}$ ,  $9\frac{1}{16}$  p.c. on £963. 7s. 8¼d.
6. Cash discounts on £5000, £712. 18s. 10d., £856. 15s. 7½d.  
at  $3\frac{1}{2}$ ,  $1\frac{1}{4}$ ,  $2\frac{1}{4}$  p.c.
7. Dividends on £20000, £7000, £900 stock at  $3\frac{1}{16}$  p.c.
8. Dividends on 30 £50 shares, 25 £10 shares at  $2\frac{3}{16}$  p.c.
9. Brokerages on £765. 18s. 5¼d. at  $\frac{1}{16}$  p.c.,  $3\frac{5}{8}$  p.c.
10. Commissions of  $2\frac{1}{2}$ , 3,  $3\frac{1}{4}$ , 5 p.c. on sales to amount of £895. 16s. 8d.

**Partnership.**

8. *Partnership* is the association of two or more persons for business purposes under an agreement to share the profits and losses according to the capital subscribed by each and the time it is employed.

Such a partnership is often termed a *firm*.

The *capital* of the firm is the joint stock (money or property) belonging to the firm and used in the business.

The *assets* of the firm are the moneys or property owing to it, the capital invested in it together with the property and stock belonging to it (valued in accordance with stated rules).

The *liabilities* of the firm are its debts.

The *net capital* is the excess of assets over liabilities.

To find each partner's share of the profit or loss.

1°. When the times are equal (simple partnership or fellowship).

(1) Multiply the whole profit or loss by the ratio of each man's capital to the whole.

(2) Find rate p.c. gained or lost and multiply each man's capital by this rate.

The proportion of a bankrupt's assets received by each creditor is determined by these rules.

Dividends on stocks and shares are calculated by the second rule.

2°. When the times are unequal (compound partnership).

Multiply each man's capital by the time it is employed.

Add the products.

Multiply the whole profit or loss by the ratio of each product to the sum of the products.

*Example 1.* *A* and *B* invest £7000 and £10000 respectively in a business. The year's profit averages £950. What proportion should go to each?

$$\begin{aligned} \text{£7000} + \text{£10000} &= \text{£17000.} & \frac{7000}{17000} &= \frac{7}{17} \text{ of } 950 = \text{£} \frac{6650}{17} = \text{£} 391. 3s. 6\frac{1}{2}d. \\ & & \frac{10000}{17000} &= \frac{10}{17} \text{ of } 950 = \text{£} \frac{9500}{17} = \text{£} 558. 17s. 5\frac{1}{2}d. \end{aligned}$$

*Example 2.* *A* starts a business with £1000. *B* joins him in 3 months with £800. *C* enters the firm 2 months after *B* with £500. What share should each have of a yearly profit of £360?

$$\begin{array}{rcl} \text{£1000} \times 12 & = & 12000 & \text{A's share } \frac{12}{27} \text{ of } \text{£} 360. \\ 800 \times 9 & = & 7200 & \text{B's share } \frac{9}{27} \text{ of } \text{£} 360. \\ 500 \times 7 & = & 3500 & \text{C's share } \frac{7}{27} \text{ of } \text{£} 360. \\ \hline & & 22700 & \end{array}$$

*Example 3.* *A* invests £3500 on May 6. *B* £2000 on June 14. *C* £5000 on Nov. 8. The declared profit is £1856 for the year. What will each get?

$$\begin{aligned} \text{£3500} \times 239 + \text{£2000} \times 200 + \text{£5000} \times 53 &= \\ 700000 & \\ 105000 & \\ 31500 & \\ \hline \text{£836500} + \text{£400000} + \text{£265000} &= \text{£1501500.} \\ \therefore \text{A gets } \frac{339}{15015} \text{ of } \text{£} 1856, & \text{B } \frac{200}{15015} \text{ of } \text{£} 1856, \text{ C } \frac{265}{15015} \text{ of } \text{£} 1856. \end{aligned}$$

### EXAMPLES.

1. *A* and *B* invest in business £6000, £8000; find yearly incomes of each from net profits of £865. 13s. 6d., £912. 17s. 4d., £1015. 8s. 6½d.

2. *A* starts a business with £8000; *B* joins him in 5 months with £7000; *C* comes in with £5000 2 months after *B*; divide a profit on year of £1156.

3. *A* begins on Feb. 1 with £4500; *B* joins him on May 5 with £7000; divide the year's profit of £956. 17s. 8d.

4. A bankrupt fails with assets £312. 16s. 10d.; he owes three creditors £256. 7s. 8d., £300. 5s. 6d., £410. 8s. 9d.; what will each get and what can he pay in the £?

5. Find dividends at  $4\frac{3}{4}$ ,  $5\frac{1}{8}$ ,  $6\frac{1}{2}$ ,  $7\frac{1}{2}$ ,  $3\frac{1}{2}$  on investments of £8000, £7500, £3120, £6515. 10s. 6d., £8450.

6. *A*, *B*, *C* enter into partnership with capital of £5000, £6000, £7000, find division of profits of £1952, £1864. 7s. 6d., £2000.

7. *B* invests in *A*'s business £2000 on May 1; *C* also invests on June 4 £1500; and *D* lends *A* on Sept. 5 £2500; the declared rate of profit is  $4\frac{1}{2}$  p.c. per annum, find *B*'s, *C*'s, *D*'s returns.

8. A firm borrows £6000 at 4 p.c. on May 16; the capital of the partners is respectively £30000, £50000, £70000; the profit found is £25000; find share of each after repaying loan with interest.

9. A firm overdraws its account on May 7 by £2000 and on June 8 by £3000 more; its final profit is found to be £8000; what net dividend can be declared on a capital of £50000 after repaying the bank (6 p.c. int.), and what will be the share of the partners of the profit if their capitals are in the ratio of 12:13:25?

10. Three new partners are introduced into a firm on April 4, May 7, June 10; they each put into the business £5000; the net profit is declared at  $12\frac{1}{2}$  p.c. per annum; find the interest each receives.

### Purchase of Freehold Property.

9. The four things of importance in purchasing Freeholds are *the rent*, *the no. of years' purchase*, *the purchase money*, and *the rate p.c. of the rental on the money paid*. The relations between them are these:—

The purchase money = the rent  $\times$  no. of years' purchase.

$$\text{The rate p.c.} = \frac{100 \times \text{rent}}{\text{purchase money}} = \frac{100}{\text{no. of years' purchase}}.$$

$$\text{The no. of years' purchase} = \frac{\text{purchase money}}{\text{rent}} = \frac{100}{\text{rate p.c.}}.$$



The chief questions are:—

1°. To find the rate p.c.—given the rent and the purchase money, or the no. of years' purchase alone.

$$\text{The rate p.c.} = \frac{100 \times \text{rent}}{\text{purchase money}} = \frac{100}{\text{no. of years' purchase}}.$$

2°. To find the rent so that the purchase money may yield a given rate.

$$\text{The rent} = \frac{\text{rate} \times \text{purchase money}}{100}.$$

3°. To find the no. of years' purchase so as to realise a given rate.

$$\text{The no. of years' purchase} = \frac{100}{\text{rate}}.$$

*Example 1.* What rate is given by a rental of £64 on £1250 purchase money?

$$\text{Rate} = \frac{64 \times 100}{1250} = \frac{6400}{1250} = 5\frac{1}{4} \text{ p.c.}$$

*Example 2.* What must be rent so as to realise 10 p.c. on £856?

$$\text{Rent} = \frac{856 \times 10}{100} = 85.6 = £85. 12s. 0d.$$

*Example 3.* What must be no. of years' purchase to realise 7 p.c.?

$$\text{Time} = \frac{100}{7} = 14\frac{2}{7} \text{ years.}$$

*Example 4.* What must be rate on 8 years' purchase?

$$\text{Rate} = \frac{100}{8} = 12\frac{1}{2} \text{ p.c.}$$

### EXAMPLES.

Find

1. Rate given by rental of £32 on £650.
2. Rate given by 13 years' purchase.
3. Rate given by rental of £45 on £1860.
4. Rate given by 12 years' purchase.
5. Rent to realise 5 p.c. on £912 purchase.
6. Rent to realise  $7\frac{1}{2}$  p.c. on £840 purchase.

7. Rent to realise  $5\frac{1}{2}$  p.c. on £648 purchase.
8. Rent to realise  $8\frac{1}{8}$  p.c. on £725 purchase.
9. No. of years' purchase to realise  $8\frac{1}{16}$  p.c.
10. No. of years' purchase to realise  $7\frac{1}{2}$  p.c.

### Taxes.

10. An *assessment* is a specific sum charged on shares or property for specific purposes.

A *tax* is an assessment on income or property to defray public expenses.

*Real estate* is fixed property such as houses and lands.

*Personal estate* is moveable property such as money, cattle, ships, furniture.

1°. To decide on a rate or tax from the amount of rateable property and the money required.

Divide the money required by the rateable value—this will give the rate per £.

*Example.* A corporation needs £28654 for public purposes—the rateable property is £252750.

∴ rate required =  $\frac{28654}{252750} \times 20 = \frac{4719\frac{1}{2}}{10000} = 2s. 3.207d.$  in the £.

$$\begin{array}{r}
 25275 \mid 57308 \mid 2s. \\
 \underline{67580} \\
 19516 \\
 \underline{81096} \mid 8.207d. \\
 \underline{52710} \\
 19600
 \end{array}$$

2°. To find the amount of a tax on a given property.

Multiply the valuation by the rate—using decimals or fractions of a £.

*Example.* Tax is 3s. 4d. in £—find tax on property valued at £785. 0s. 0d.

$$\begin{array}{r}
 3s. 4d. = \frac{1}{2}\text{£}. \quad \begin{array}{r} 785 \\ \hline \end{array} \\
 \text{£}130.833 (16s. 8d.).
 \end{array}$$

**EXAMPLES.**

1. Decide the rate necessary for £2564 on a rateable value of £10000.
2. Decide the rate necessary for £10000 on a rateable value of £85650.
3. Find assessment at  $6\frac{1}{2}d.$  in the £ on an income of £350 with deduction of £120.
4. Find inhabited house duty on rentals of £25, £30, £60, £80 at  $3d.$ ,  $4d.$ ,  $6d.$ ,  $8d.$  in the £ respectively.
5. Find amount of rates at  $4s. 6d.$  in the £ on the following values:—£351. 18s.  $4d.$ , £96. 10s.  $6d.$ , £712. 18s.  $6\frac{1}{2}d.$ , £907. 11s.  $9d.$ , £1000.

**Duties.**

11. *Duties* are taxes on the sale of goods.

The *Customs* are duties on imports or exports.

The *Eccise* consists of duties on spirituous liquors and tobacco, and of licenses for their sale.

An *ad valorem* duty is a certain percentage of the value.

A *specific* duty is the duty on a fixed quantity (pound, gallon, etc.).

The custom-house and excise officers collect the duties.

A *tariff* is a schedule of duty-rates fixed by law.

The *gross weight* or value is the entire weight of goods without allowance.

The *net weight* or value is the weight as actually charged after deduction of all allowances.

Questions on duties are mere cases of proportion or percentages.

**EXAMPLES.**

1. Find duty on £855 worth of goods at  $\frac{1}{8}$  p.c.
2. Find duty on £10000 worth of goods at  $\frac{3}{16}$  p.c.
3. Find duty on 3 tons of currants at  $\frac{1}{8}d.$  per lb.
4. Find duty on 7564 tons of iron at  $\frac{1}{2}d.$  per lb.
5. Find duty on 8570 gallons of spirit at 2s.  $6d.$  per gallon.
6. Find duty on 27589 gallons of whisky at 1s.  $10d.$  per gallon.

**Insurance.**

12. *Insurance* is a contract of indemnity against loss or damages.

*Fire insurance* is a contract of indemnity against loss by fire.

*Marine insurance* is a contract of indemnity against loss of cargo or vessels.

*Life insurance* is a contract to pay a certain sum in case of the death of the insured to his next-of-kin.

The *insurer* or *underwriter* is the party who takes the risk.

The contract is termed a *policy*.

The *premium* is the sum paid for insurance.

1°. To find the yearly premium for a desired insurance.

Multiply the amount by the rate.

2°. To find the sum necessary to be insured to cover both property and premium.

Divide the property-value by 1 minus  $\frac{\text{the rate}}{100}$ .

*Example 1.* To find yearly premium to insure goods for £350 at 2s. 3d. per £100.

$$3 \cdot 50 \times 2\frac{3}{4} \text{ s.} = \left. \begin{array}{l} 7 \cdot 00 \\ \cdot 875 \end{array} \right\} = 7 \cdot 875 = 7 \text{ s. } 10\frac{1}{2} \text{ d.}$$

*Example 2.* To insure goods so as to cover premium and value £1260 at  $\frac{1}{8}$  p.c.

$$\text{Amount} = 1260 \div 1 - \frac{1}{8} \frac{1}{100} = 1260 \times 1\frac{1}{8} \frac{1}{100} = 1260 + 1 \cdot 577 = 1261 \cdot 577,$$

$$\therefore \text{yearly premium} = \frac{1}{8} \text{ p.c. on } 1261 \cdot 577 = £1 \cdot 577.$$

**EXAMPLES.**

Find

1. Premium on £400 at  $\frac{1}{8}$  p.c.
2. Premium on £650 at  $\frac{3}{8}$  p.c.
3. Premium on £860 at  $4\frac{1}{2}$  per mille.
4. Premium on £712. 17s. 8d. at 2s. 4d. per £100.

5. Premium on £840 at  $\frac{3}{10}$  p.c.
6. Amount to cover premium and £1200 at  $\frac{5}{16}$  p.c., also premium.
7. Amount to cover premium and £1850 at 2s. 9d. per £100, also premium.
8. Amount to cover premium and £7000 at 3s. 4d. per £100, also premium.
9. Amount to cover premium and £6450 at  $\frac{11}{20}$  p.c., also premium.
10. Amount to cover premium and £20000 at 5 p.c., also premium.

### Profit and Loss and the Pricing of Goods.

13. Questions in *Profit and Loss* are solved by the principles of proportion.

Their solution involves the constant use of percentages.

The chief point to be observed in all questions of *theory* is to reckon on the *cost price* unless the selling price is specially involved.

The *Pricing of Goods* is a very important branch of the subject.

1°. To find gain or loss p.c., given cost price and selling price.

Multiply gain or loss by  $\frac{100}{\text{cost price}}$ .

2°. To find selling price so as to gain a given p.c.

Multiply cost price by  $\frac{100 + \text{given p.c.}}{100}$ . (1.05 if p.c. is 5.)

3°. To mark goods so as to ensure a given p.c. profit after a given discount.

(1) Find selling price so as to gain the given p.c.

Multiply this by  $\frac{100}{100 - \text{given discount}}$ .

(2) Multiply cost price by  $\frac{100 + \text{given p.c. profit}}{100 - \text{given p.c. discount}}$ .

*Note.* By discounting the selling price of an article a loss may be sustained without suspecting it—this arises from the fact that the profit is thus discounted as well as the original price.

*Ex.* An article costs 5s. and is marked so as to realise 60 p.c. profit—but a discount on the selling price of 45 p.c. is allowed, is there a loss?

Selling price = 5s. +  $\frac{60}{100}$  of 5s. = 5s. + 3s. = 8s.

45 p.c. on 8s. = 3·6s.  $\therefore$  actual selling price is 4·4s., a loss of  $\frac{4}{5}$ s. on 5s., i.e. 12 p.c.

*Example 1.* Find gain p.c. by selling for 11½d. what cost 8¾d.

$$11\frac{1}{2}d. - 8\frac{3}{4}d. = 2\frac{1}{4}d. \quad \frac{250}{8\frac{3}{4}} = \frac{199\frac{1}{2}}{8\frac{3}{4}} = 23\frac{1}{2} = 23\frac{1}{2} \text{ p.c.}$$

*Example 2.* Find selling price of an article which cost 2s. 9½d. so as to gain 20 p.c. after deducting 10.

$$\frac{120}{100} \times 33\cdot25d. = \frac{12}{10} \times 33\cdot25 = 33\cdot25 + 11\cdot085 = 44\cdot335d.$$

*Example 3.* Find selling price of article costing 3s. 2½d. so as to gain 12 p.c.

$$\begin{array}{r} 38\cdot5d. \times 1\cdot12 = 38\cdot5 \\ \quad \quad \quad 3\cdot85 \\ \quad \quad \quad \cdot770 \\ \hline 43\cdot120d. \end{array}$$

### EXAMPLES.

1. Find gain or loss p.c. by selling article at 6½d. for 9d., 7¾d. for 6d., 10¾d. for 1s., 25s. for 30s., £15. 15s. for £20., £5. 17s. 4½d. for £7. 10s.

2. Find selling price of article at 10¾d. so as to gain 5 p.c., 1s. 5½d. gain 7 p.c., 3s. 7d. gain 10 p.c., 1s. 5¾d. gain 15 p.c., 5s. 11½d. gain 20 p.c., £3. 5s. 9½d. gain 24 p.c., £11. 10s. 6d. gain 40 p.c.

3. Find selling price of articles at these prices to gain the given p.c. after deducting the given discounts.

(1) 7¾d. 15 p.c. 5 p.c. (2) 1s. 9½d. 10 p.c. 5 p.c. (3) 3s. 11½d. 25 p.c. 10 p.c. (4) 5s. 6d. 18 p.c. 5 p.c. (5) 22s. 11½d. 12 p.c. 5 p.c. (6) 17s. 10¾d. 20 p.c. 7 p.c. (7) £3. 5s. 9d. 40 p.c. 25 p.c. (8) £8. 8s. 60 p.c. 20 p.c. (9) £25. 7s. 6d. 15 p.c. 2½ p.c. (10) £30. 4s. 6d. 25 p.c. 12½ p.c.

4°. *To mark Goods bought by the dozen, score, gross, hundred, thousand, so as to gain a given p.c.*

$$(1) \text{ Multiply cost price of dozen by } \frac{100 + \text{given p.c.}}{1200}.$$

$$(2) \text{ Multiply cost price of score by } \frac{100 + \text{given p.c.}}{2000}.$$

$$(3) \text{ Multiply cost price of gross by } \frac{100 + \text{given p.c.}}{14400}.$$

$$(4) \text{ Multiply cost price of 100 by } \frac{100 + \text{given p.c.}}{10000}.$$

$$(5) \text{ Multiply cost price of 1000 by } \frac{100 + \text{given p.c.}}{100000}.$$

(6) Use Denomination-changes (for dozens s. as d., for score £ as s., for gross s. as d. twice) and multiply by

$$\frac{100 + \text{given p.c.}}{100}.$$

This principle may be extended to any quantities convenient for such changes, e.g. for 12 score consider £ as d., for 80 dozen consider £ as f., etc.

**The chief Methods of Solution** are these:—

(1) Directly—multiplying cost price (e.g.) for dozens by 1·05 and dividing by 12, for score by 1·05 and dividing by 2, for gross by 1·05 and dividing by 144, for hundreds by 1·05, for thousands by 1·05.

The methods of approximation apply—more particularly the method adopted in compound interest for any year.

*Example 1.* Find selling price of each article in a dozen costing 17s. 8½d. so as to gain 6 p.c.

$$\begin{array}{r}
 212 \cdot 25 \\
 12 \cdot 735 \\
 \hline
 12 \mid \begin{array}{r} 224 \cdot 985 \\ 104 \end{array} \mid \begin{array}{l} 18 \cdot 749 \text{ nearly.} \\ 18 \cdot 75 = 1s. 6\frac{1}{2}d. \end{array} \\
 \hline
 89 \\
 58 \\
 \hline
 105
 \end{array}$$

*Example 2.* Find single selling price of 100 costing £25. 17s. 8d. so as to gain 7 p.c.

$$\begin{array}{r}
 £25 \cdot 888 \\
 \cdot 25883 \\
 \hline
 1812 \\
 \hline
 £27695 = 5s. 6\frac{1}{2}d.
 \end{array}$$

(2) By considering shillings pence etc. and then multiplying cost price so changed by 1·05 (e.g.).

*Example.* Find selling price of each in a score at £12. 7s. 8½d. so as to gain 8 p.c.

$$\begin{array}{r}
 12 \cdot 384s. \\
 \cdot 991 \\
 \hline
 13 \cdot 375 = 13s. 4\frac{1}{2}d.
 \end{array}$$

(3) By aliquotising the fractions given above.

Thus for dozens the fraction  $\frac{100 + \text{given p.c.}}{1200} = \frac{1}{10}$  for 20 p.c.,  $\frac{5}{48}$  for 25 p.c.,  $\frac{7}{80}$  for 5 p.c., etc., and these can easily be aliquotised when necessary.

Similarly the fraction  $\frac{100 + \text{given p.c.}}{144}$  can be aliquotised and  $\frac{1}{100}$  of the result taken for gross.

Thus also the fraction  $\frac{100 + \text{given p.c.}}{100}$  can often be advantageously aliquotised.

*Example.* Mark goods at 5s. 9d. per dozen so as to gain 10 p.c.

$$\begin{array}{r}
 11\frac{9}{10} = \frac{111}{10} = 0 - (12)(10). \quad \begin{array}{r} 69 \\ 5 \cdot 75 \\ \hline 575 \end{array} \\
 \hline
 6 \cdot 325d. = 6\frac{1}{2}d. \text{ or } 6\frac{1}{2}d.
 \end{array}$$



*Note.* In the case of the fraction  $\frac{100 + \text{given p.c.}}{100}$ , the practical method of using it is to add the fraction  $\frac{\text{given p.c.}}{100}$ , aliquotised when necessary, to the original cost.

### EXAMPLES.

Mark goods so as to gain following p.c.

1. Dozens at 22s. 9½d., 36s. 7d., 25s. 7d., 19s. 10½d., 47s. 6½d. so as to gain 5, 7½, 16, 20, 24 p.c. respectively.

2. Score at 63s. 7d., 85s. 9d., 115s. 10½d., 79s. 4d., £3. 16s. 9½d. so as to gain 13, 21, 18, 25, 35 p.c. respectively.

3. Gross at £12. 9s. 10½d., £15. 18s., £21. 6s., £94. 7s. 10d., £38. 9s. 6d. so as to gain 10, 15, 20, 25, 60 p.c. respectively.

4. Hundreds at £112. 9s. 6½d., £256. 7s. 8½d., £97. 8s. 10½d. so as to gain 5, 12½, 15 p.c. respectively.

5. Thousands at £320. 7s. 10½d., £640, £850 so as to gain 8, 14, 24, p.c. respectively.

6. Articles at 6½d. (5 p.c.), 3s. 1½d. (22 p.c.), 1s. 1½d. (7 p.c.), 2s. 7½d. (24 p.c.), 1s. 6½d. (16 p.c.), 5s. 9d. (26 p.c.), 2s. 0½d. (25 p.c.), 1s. 11½d. (24 p.c.), 2s. 4½d. (30 p.c.).

7. Dozens at 10s. 6d. (5 p.c.), 9s. 5d. (12½ p.c.), 11s. 4d. (17 p.c.), 24s. 7d. (40 p.c.), 17s. 10½d. (10 p.c.).

8. Score at 55s. 6d., 73s. 10½d., 96s. 8½d. to gain 5, 7, 13 p.c. respectively.

9. Gross at £12. 9s. 7½d., £16, £40. 7s. 10½d. to gain 16, 24, 32 p.c. respectively.

10. Hundreds at £64. 8s. 7d., £20. 9s. 5d. to gain 10 p.c. and 15 p.c. respectively.

5°. To price goods bought in any quantity so as to gain a given p.c.

Add to total cost required gain p.c., found by any of the percentage methods.

Divide by quantity expressed in unit whose price is required—using Division-approximation to any desired accuracy (farthings, eighths, or sixteenths of a penny).

*Example 1.* Price goods per lb. so as to gain 10 per cent. if bought at £1. 5s. 9d. per cwt.

$$\begin{array}{r}
 1.2375 \\
 12375 \\
 112 \mid \frac{1.36125}{241} \mid .012 = 3\frac{1}{2}d. \text{ per lb.}
 \end{array}$$

*Example 2.* 715 qrs. of wheat are bought for £1260: what must be the price per qr. so as to gain  $12\frac{1}{2}$  p.c.?

$$\begin{array}{r}
 £1260 \\
 157.5 \\
 71.5 \mid \frac{£1417.5}{702.5} \mid 1.982 \\
 \quad \quad \quad \frac{59.0}{1.8}
 \end{array}
 \quad \text{£1. 19s. 8d.}$$

### EXAMPLES.

Price these goods to gain the quoted p.c.

1. 7 tons 5 cwts. costing £20. 12s. 6d.—per lb. to gain 10 p.c.
2. 5 sacks of flour costing £16. 8s. 4d.—per lb. to gain 5 p.c.
3. 814 things costing £35. 7s. 6d.—per art. to gain 12 p.c.
4. 65 centals of cotton costing £85. 11s. 4d.—per lb. to gain 25 p.c.
5. 840 gallons of oil costing £38. 5s. 9d.—per gall. to gain 20 p.c.

For **Commercial Profit and Loss** see **Answers**, pp. 17, 18.

## SECTION VI.

### METHODS OF PREDICTION.

THESE are based on De Morgan's rules (pp. 18—25), and his methods of multiplication and division are always to be employed in connection with them.

#### 1. Form $a \times b$ .

Observe the number of integers or decimal ciphers in each multiplier.

The places which must be taken in each = the number of places required correct  $\pm$  the number of integers or decimal ciphers in the other + 1.

Reverse one, placing the significant digits one to the left of those in the other.

*Example 1.*  $25 \cdot 1875346 \times 3 \cdot 1871586$  to 2 places.

$$\begin{array}{r}
 2 + 2 + 1 = 5. \qquad 25 \cdot 1875 \\
 2 + 1 + 1 = 4. \qquad 517 \ 813 \\
 \hline
 75 \ 563 \\
 2 \ 519 \\
 2 \ 014 \\
 176 \\
 3 \\
 1 \\
 \hline
 80 \cdot 276
 \end{array}$$

*Example 2.*  $12 \cdot 88475 \times \cdot 00088765$  to 4 places.

$$\begin{array}{r}
 4 + 2 + 1 = 7. \qquad 12 \cdot 88 \\
 4 - 8 + 1 = 2. \qquad 678 \ 3 \\
 \hline
 38 \ 5 \\
 10 \ 2 \\
 8 \\
 1 \\
 \hline
 \cdot 00496
 \end{array}$$

**2. Form  $a \div b$ .**

Observe the number of integers or decimal ciphers in the divisor and dividend.

Hence decide by inspection the number of integers or decimal ciphers in the quotient.

The number of places which must be taken in the divisor = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of divisor integers or ciphers  $+ 1$ .

The number of places which may have to be taken in the dividend = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of dividend integers or ciphers  $+ 1$ .

Employ the significant figures so determined.

*Example 1.*  $873.8651 \div 8514.37$  to 4 places.

Quotient-ciphers = 1.

Divisor-places =  $4 - 1 - 4 + 1 = 0$ .

Dividend-places =  $4 - 1 - 3 + 1 = 1$ .

$$\begin{array}{r|l} 8,514 & 373.8 \quad .0439 \\ & \underline{332} \\ & 77 \end{array}$$

*Example 2.*  $\text{£}73965.19s. 8\frac{1}{2}d. \div 8901$  to 3 places.

Quotient-integers = 1.

Divisor-places =  $3 + 1 - 4 + 1 = 1$ .

Dividend-places =  $3 + 1 - 5 + 1 = 0$ .

$$\begin{array}{r|l} 8,901,0 & 73965 \quad \text{£}8.309 \\ & \underline{2757} \\ & 87 \\ & 7 \end{array}$$

**3. Form  $a \times b \times c$ .**

Observe the number of integers or decimal ciphers in any multiplier and decide by inspection the number in the product of the other two.

The number of places which must be correct in this product = number of places required correct  $\pm$  number of integers or ciphers in the multiplier  $+ 1$ .

The number of places which must be taken in the multiplier = number of places required correct  $\pm$  number of integers or ciphers in the product  $+ 1$ .

*Example.* £.03125  $\times$  2.20462125  $\times$  25.19 to 2 places.

2-1+1=2.	2.20462	55.53
2+2+1=5.	9152	521 3
	4 4092	166 6
	1 1023	5 6
	220	1 1
	198	3
	55.533	1.736

4. **Form**  $a \times \frac{b}{c}$ .

Observe the number of integers or decimal ciphers in either multiplier and decide by inspection the number in the remaining quotient.

The number of places which must be correct in this quotient = number of places required correct  $\pm$  number of integers or ciphers in the multiplier + 1.

The number of places which must be taken in the multiplier = number of places required correct  $\pm$  number of integers or ciphers in the quotient + 1.

*Example.* £31.14260416  $\times$  12.18  $\div$  73 to 3 places.

3+2+1=6.	73   12.18   .166849	31.1426
3+0+1=4.	4 88	948 661
	500	31 143
	620	18 685
	360	1 868
	68	249
	2	12
		3
		5.1960

See page 158.

5. **Form**  $\frac{a \times b}{c}$ .

Observe the number of integers or decimal ciphers in the divisor.

Decide by inspection the number in the dividend and hence also in the quotient.

The number of places which may have to be correct in the dividend = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of dividend integers or ciphers + 1.

The number of places which must be taken in the divisor = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of divisor integers or ciphers  $+ 1$ .

*Example.*  $\pounds 311.4260416 \times 12.18 \div 73$  to 3 places.

$$3 + 2 - 4 + 1 = 2.$$

$$3 + 2 - 2 + 1 = 4.$$

$$\begin{array}{r} 311.42604 \\ \underline{8121} \\ 311\ 4260 \\ 62\ 2852 \\ \underline{3\ 1143} \\ 2\ 4914 \\ 7,3,0,0,0,0 \mid \underline{3793.169} \mid 51.961 \\ \underline{143\ 16} \\ 70\ 16 \\ \underline{4\ 46} \\ .8 \end{array}$$

See page 24.

## 6. Form $\frac{a}{b \times c}$ .

Observe the number of integers or decimal ciphers in the dividend.

Decide by inspection the number in the divisor and hence also the number in the quotient.

The number of places which must be correct in the divisor = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of divisor integers or ciphers  $+ 1$ .

The number of places which may have to be taken in the dividend = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of dividend integers or ciphers  $+ 1$ .

*Example.*  $1.47 \div 2.20462125 \div 20.41$  to 4 places.

$$4 - 1 - 2 + 1 = 2.$$

$$4 - 1 - 1 + 1 = 3.$$

$$\begin{array}{r} 2.20462 \\ \underline{1402} \\ 4\ 4092 \\ \underline{882} \\ 22 \\ \underline{44.996} \end{array} \quad \begin{array}{r} 4,4,9,9 \mid \underline{1.470} \mid .0326 \\ \underline{120} \\ 30 \\ \underline{4} \end{array}$$

7. **Form**  $\frac{a \times b}{c \times d}$ .

Decide by inspection the number of integers or ciphers in the dividend and divisor and hence also the number in the quotient.

The number of places to which the dividend may have to be correct = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of dividend integers or ciphers + 1.

The number of places to which the divisor must be correct = number of places required correct  $\pm$  number of quotient integers or ciphers  $\mp$  number of divisor integers or ciphers + 1.

*Example.*  $27 \cdot 3125 \times 6134583 \div 17 \cdot 321 \div 5 \cdot 785$  to 3 places.

$$3 + 0 - 2 + 1 = 2.$$

$$3 + 0 - 3 + 1 = 1.$$

$27 \cdot 312$	$17 \cdot 321$
$543 \ 16$	$58 \ 75$
$\hline 163 \ 87$	$\hline 86 \ 61$
$2 \ 73$	$12 \ 12$
$82$	$1 \ 38$
$11$	$9$
$\hline 1$	$\hline 100 \cdot 20$
$\hline 16 \cdot 754$	

$$1,00,2 \mid 16 \cdot 75 \mid \cdot 167$$

$$\begin{array}{r} 6 \ 7 \\ \hline 7 \end{array}$$

These methods should be employed in all questions which involve one or more of the forms given. In every part of this book their use will greatly abridge the labour of calculation.

Examples will be found in the Metric Section and also under Proportional Parts, Partnership, and Interest.

The rules will be found to be specially valuable in Indirect Exchanges and Arbitrations.

The principles underlying the rules involving division are (1) that the number of figures correct in the dividend must be one more than or the same as the number of figures in the quotient, (2) that the number of figures correct in the divisor must be one more than or the same as the number of figures in the dividend.

The reason of the rule for a continued product will be obvious on reflection.

## SECTION VII.

### INTEREST, ANNUITIES, AND STOCKS.

#### Interest.

1. *Interest* is money paid for the use of money lent for a certain time at a fixed rate.

The money lent is called the *Principal*.

The interest on £100 for a year is called the *Rate per cent.*

The sum of the interest and principal is called the *Amount*.

*Simple Interest* is interest reckoned on the principal alone.

*Compound Interest* is the interest accumulated from year to year by the addition of the yearly interest to the principal—the interest when added also bearing interest.

Any term may take the place of a year, e.g. a quarter or half-year.

In ordinary business simple interest is the only kind largely used—first, because the time of bills is usually less than a year but also because accounts are made up every half-year and any interest accruing is either paid or added to the balance; thus though this involves in reality the principle of compound interest yet the operation is rarely performed.

It cannot however be too clearly understood that money is always bearing interest when in use (i.e. lent or invested or put into business) or would bear interest if there was any profit.



**Interest in Banks.**

2. The Calculation of Interest is an important part of Banking business. The customs adopted vary somewhat but the leading principles are always the same.

(1) *Interest Tables* are used to avoid labour and save time, and of these there are various forms.

Lawrie's tables are calculated at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , 4,  $4\frac{1}{2}$ , 5 p.c. on every £1 up to £100, then every £10 up to £1000, every £100 up to £10000, every £1000 up to £100000 for 1 to 365 days.

For any intermediate rates devices are used to get them from the given rates.

$$\text{Ex. } 1\frac{5}{8} = \frac{3\frac{1}{2} + 3}{4}.$$

King's tables are for very large amounts for 1 day. There are other convenient tables, notably Rourke Jones', giving interest at rates proceeding by eighths.

In customers' accounts banks disregard or allow for the s.d.f. according as the s.d.f. are above or below 10s. in reckoning interest.

Banks never enter farthings as interest.

(2) *For Current Accounts* interest is calculated by "*extending the Decimal*" or "*on the minimum monthly balances*."

London banks give no interest on current accounts but do not charge for cashing cheques as a compensation.

Provincial banks give interest and charge for cheques.

The term "extending the decimal" is utterly misleading—there is no decimal—the operation is simply that of multiplying pounds by days.

By this means the interest is reduced to that of a sum of money for 1 day.

An example will make it clear.

THOMAS SMITH,  
56, High Street,  
Weymouth.

Current account				Decimal					
	Cr.	£ s. d.	Dr.	£ s. d.	Cr.	£ s. d.	Days		
1892									
Mar. 1. By cheque	2673	19 4½			2673	19 4½	12	3	2 0 8 8
13. By cash	1851	4 5			4525	3 9½	6	2	7 1 5 0
19. To cheque			2000	0 0	2525	3 9½	6	1	5 1 5 0
25. To cheque			215	3 6	2310	0 3½			

2674 × 12 = 32088      4525 × 6 = 27150 etc.

When the books are balanced the decimals (so-called) are added up and interest on amount for 1 day at current-deposit rate is found and added to balance of the customer.

Similarly the decimal is extended in the case of overdrafts but the rate is much higher.

If there is *a change in the rate* two methods may be adopted.

(a) Add up decimal to date when rate changes—find interest and extend it in a further column—then begin again with your decimal for the new rate.

(b) Add up decimal to date and equalise it to new rate—then proceed with decimal as usual.

To equalise to new rate, add or subtract proportionally, e.g. 6 p.c. from 5 p.c. subtract  $\frac{1}{5}$  of decimal.

*Minimum Monthly Balances* are the lowest amounts remaining to the credit of any customers during the month—on these interest is allowed by those banks which adopt the system. It finds favour with Scotch banks and some English banks.

(3) *For deposit accounts* interest is best calculated by forming a table of days from a certain date for the rate of interest transformed to 5 p.c.

*Ex.* 2 p.c. for 93 days = 5 p.c. for 37.2 days.

Shaw's Scotch tables are an instance.

The rate for deposits varies during the year to some extent as it depends upon the bank rate of discount.

(4) *To average the income tax* when rate has changed on Ap. 5.

**Rule.** Multiply days to Ap. 5 by old rate and days from Ap. 5 by new rate and divide by the total period of days of the interest wanted.

*Ex.* A dividend has accrued from Nov. 11, 1887 to May 15, 1888. Tax changed from 7*d.* to 6*d.* on Ap. 5, 1888.

Averaged rate =  $\frac{7 \times 146 + 6 \times 40}{186} = 6.784946*d.*$  in the £.

### Simple Interest.

#### 3. Mental or very brief methods.

- (1) 5 p.c. p.a. for a year—divide by 20 (£ as s.).  
5 p.c. p.a. for a month—divide by 240 (£ as d.)

For any no. of years or months multiply by the no. or take aliquot parts of a year.

To convert any other rate to 5 p.c.—double the rate and divide by 10 or take aliquot parts of 5.

*Ex.* 1 p.c. =  $\cdot 2 \times 5$  p.c.       $3\frac{1}{2}$  p.c. =  $\cdot 65 \times 5$  p.c.  
 $3\frac{1}{2}$  p.c. =  $(0 - 252) \times 5$  p.c.

Any p.c. p.a. for a year— $\frac{1}{10}$  of £ as s.  $\times$  twice rate.

Any p.c. p.a. for a month— $\frac{1}{120}$  of £ as d.  $\times$  twice rate.

For any no. of years or months multiply by the no. or take aliquot parts of a year.

(2) The joint factors of 1200 are:— $1200 \times 1$ ,  $600 \times 2$ ,  $400 \times 3$ ,  $300 \times 4$ ,  $240 \times 5$ ,  $200 \times 6$ ,  $150 \times 8$ ,  $120 \times 10$ ,  $100 \times 12$ ,  $80 \times 15$ ,  $75 \times 16$ ,  $60 \times 20$ ,  $50 \times 24$ ,  $48 \times 25$ ,  $40 \times 30$ .

Hence when the product of the rate  $\times$  time in months comes to any of the above numbers (small or large) to obtain the Interest divide the Principal by the factor to which it is attached.

*Ex.*  $1\frac{1}{2}$  p.c. for 4 mos.  $1\frac{1}{2} \times 4 = 5 \therefore$  divide by 240,  
8 p.c. for 5 mos.  $8 \times 5 = 40 \therefore$  divide by 30,  
3 p.c. for 4 mos.  $3 \times 4 = 12 \therefore$  divide by 100.

The Principal should be decimalised.

(3) Similarly in the rare cases when years are involved (unclaimed dividends, etc.) the joint factors of 100 may be employed, viz.:— $100 \times 1$ ,  $50 \times 2$ ,  $25 \times 4$ ,  $20 \times 5$ ,  $10 \times 10$ .

*Ex.* 5 p.c. for 10 yrs.  $5 \times 10 = 50$   $\therefore$  divide by 2,  
 $12\frac{1}{2}$  p.c. for 8 yrs.  $12\frac{1}{2} \times 8 = 100$   $\therefore$  divide by 1.

*Example 1.* Interest at  $3\frac{1}{2}$  p.c. for 4 mos. on £312. 17s.  $4\frac{1}{4}$ d.  
 At 5 p.c. p.a. for 4 mos. int. = £312.867  $\div 60$  = £5.2144.

$$\begin{array}{r}
 5 \cdot 2144 \\
 2 \cdot 6072 \quad 2\frac{1}{2} \text{ p.c.} = \frac{1}{2} \text{ of } 5 \text{ p.c.} \\
 \cdot 5214 \quad \frac{1}{2} \text{ p.c.} = \frac{1}{2} \text{ of } 2\frac{1}{2} \text{ p.c.} \\
 \cdot 2607 \quad \frac{1}{4} \text{ p.c.} = \frac{1}{2} \text{ of } \frac{1}{2} \text{ p.c.} \\
 \hline
 \pounds 3 \cdot 3893
 \end{array}$$

7s. 9d.

*Example 2.* Int. on £815. 7s.  $6\frac{1}{4}$ d. at 5 p.c. for 5 mos.

$$\begin{array}{r}
 8 \mid 815 \cdot 376 \\
 6 \mid 101 \cdot 922 \\
 \hline
 \pounds 16 \cdot 987
 \end{array}$$

19s. 9d.

*Example 3.* £751. 8s. 4d. at 5 p.c. for 8 mos.

$$\begin{array}{r}
 \pounds \quad s. \quad d. \\
 37 \quad 11 \quad 5 \\
 \hline
 6 = \frac{1}{2} \quad 18 \quad 15 \quad 8\frac{1}{2} \\
 2 = \frac{1}{4} \quad 6 \quad 5 \quad 2\frac{1}{4} \\
 \hline
 \pounds 25 \quad 0 \quad 11\frac{1}{4}
 \end{array}$$

### EXAMPLES.

Find Interest on

1. £512 at 5 p.c. for 1 year, 3, 7, 8 mos.
2. £650 at 5 p.c. for 1 yr., at 3 p.c. for 1 month, at 4 p.c. for 3 mos.
3. £875. 10s. at  $3\frac{1}{2}$  p.c. for 4 mos., at 4 p.c. for 1 yr.
4. £724 at  $2\frac{1}{2}$  p.c. for 4 mos.,  $3\frac{1}{2}$  p.c. for 5 mos.
5. £1000 at  $1\frac{1}{2}$  p.c. for 3 mos.,  $2\frac{1}{2}$  p.c. for 4 mos.
6. £7642. 5s. 8d. at 4 p.c. for 1 month,  $3\frac{1}{2}$  p.c. for 2 mos.
7. £912 at 7 p.c. for 1 month, 6 p.c. for 4 mos.
8. £8635. 7s. 6d. at 6 p.c. for 2 mos., at 5 p.c. for 9 mos.

9. £540. 6s. 9d. at 5 p.c. for 10 mos., at 6 p.c. for 11 mos.
10. £1020. 17s. 6d. at 3 p.c. for 7 mos., at  $4\frac{1}{2}$  p.c. for 8 mos.
11. £250 at  $2\frac{1}{2}$  for 10 yrs., £5000 at 5 p.c. for 8 yrs.
12. £350 at  $1\frac{1}{2}$  for 4 yrs., £800 at 10 p.c. for 10 yrs.

#### 4. General Method for Years and Months.

Find interest at given rate for 1 year or any no. of years.

Take aliquot parts for the time remaining.

Add for the answer.

*Example.* £731. 8s.  $7\frac{1}{2}$ d. at 3 p.c. for 3 yrs. 7 mos.

	7·31431
	3
	21·94293
	3
6 mos. = $\frac{1}{2}$	65·82879
1 mo. = $\frac{1}{6}$	10·97146
	1·82857
	£78·62885
	12s. $6\frac{3}{4}$ d.

*Note.* This is a very rare type of question.

### EXAMPLES.

1. £834. 9s.  $7\frac{1}{4}$ d. for 4 yrs. 8 mos. at  $3\frac{1}{2}$  p.c.
2. £651. 17s.  $4\frac{3}{4}$ d. for 2 yrs. 7 mos. at  $2\frac{1}{4}$  p.c.
3. £325. 8s. 9d. for 5 yrs. 5 mos. at  $4\frac{1}{4}$  p.c.
4. £972. 12s.  $6\frac{1}{2}$ d. for 6 yrs. 3 mos. at  $2\frac{3}{4}$  p.c.
5. £761. 7s.  $4\frac{1}{4}$ d. for 1 yr. 10 mos. at  $1\frac{1}{2}$  p.c.

#### 5. Months. The Method of Aliquotation.

When the time is given in months, very often the aliquotation of the product "rate  $\times$  time" gives surprisingly brief results.

The rule to be observed is:—

Decimalise the principal to 3 places.

Move the point two places to the left.

Aliquotise the product "rate  $\times$  time" and use the aliquots to multiply and divide the altered principal in the ordinary way.

Add and change decimal to s.d.f.

*Example 1.* Int. on £315. 7s. 4d. for 5 mos. at  $3\frac{1}{4}\%$  p.c.

Rate  $\times$  time =  $\frac{5}{12} \times \frac{11}{4} = \frac{55}{48} = 1\frac{11}{48} = 1\frac{23}{96} = 1\frac{1}{4} = 1 - 444$  (aliquot notation).

Thus working is

3	15366
·	78841
·	19710
·	04928
—	
Interest =	£4.18845
	3s. 9d.

*Example 2.* Int. on £712. 9s.  $3\frac{1}{2}d.$  for 3 mos. at  $2\frac{1}{8}\%$  p.c.

Rate  $\times$  time =  $\frac{3}{4} \times 2\frac{1}{8} = \frac{3}{2} = 1\frac{1}{2} = 0 - 248$  (aliquot notation).

Thus working is

7	12374
3	56187
·	89047
·	11131
—	
Interest =	£4.56365
	11s. 3d.

It is obvious that this method is very short if the aliquot formulae are known, and these may be constructed for any commonly-occurring rates for 1, 2, 3 etc. up to 12 mos. The examples given below illustrate such tables.

$\frac{7}{8}\%$ p.c.	$1\frac{1}{4}\%$ p.c.	$2\frac{1}{8}\%$ p.c.
mos.	mos.	mos.
1=0 - (16)6	1=0 - (12)4	1=0 - 684
2=0 - 86	2=0 - 64	2=0 - 384
3=0 - 822	3=0 - 44	3=0 - 284
4=0 - 46	4=0 - 34	4=0 - 22(12)
5=0 - 4342	5=0 - 328	5=0 - 222234
6=0 - 422	6=0 - 24	6=1 - 84
7=0 - 32(16)	7=0 - 2342	7=1 - 4426
8=0 - 26	8=0 - 223	8=1 - 2(12)
9=0 - 244	9=0 - 2222	9=1 - 23424
10=0 - 2342	10=1 - (24)	10=1 - 22234
11=0 - 2264	11=1 - 86	11=2 - (12)422
12=0 - 222	12=1 - 4	12=2 - 44

$3\frac{1}{8}$ p.c.	$4\frac{1}{2}$ p.c.	5 p.c.
mos.	mos.	mos.
1=0-446	1=0-38	1=0-34
2=0-246	2=0-22	2=0-223
3=0-22222	3=1-8	3=1-4
4=1-46	4=1-2	4=1-31
5=1-2642	5=1-222	5=2-(12)
6=1-2222	6=2-4	6=2-2
7=2-628	7=2-24	7=2-2223
8=2-26	8=3-0	8=3-3
9=2-2224	9=3-38	9=3-22
10=3-642	10=3-22	10=4-6
11=3-4164	11=4-8	11=4-26
12=3-222	12=4-2	12=5-0

**EXAMPLES.**

(To be done by constructing the Aliquot Formulae.)

1. £865. 7s.  $11\frac{1}{4}d.$  for 3 mos. at  $1\frac{1}{8}\%$ , 2 mos. at  $1\frac{5}{16}\%$ , 7 mos. at  $4\frac{5}{8}\%$ .
2. £72. 9s.  $7d.$  for 5 mos. at  $2\frac{1}{4}\%$ , 7 mos. at  $3\frac{5}{8}\%$ , 10 mos. at  $1\frac{7}{8}\%$ .
3. £512. 13s.  $4\frac{1}{4}d.$  for 11 mos. at  $4\frac{1}{16}\%$ , 1 mo. at  $7\frac{1}{8}\%$ , 10 mos. at  $\frac{3}{4}\%$ .
4. £657. 9s.  $4\frac{1}{2}d.$  for 9 mos. at  $3\frac{1}{8}\%$ , 3 mos. at  $5\frac{1}{8}\%$ , 6 mos. at  $1\frac{1}{16}\%$ .
5. £802. 11s.  $5\frac{1}{4}d.$  for 4 mos. at  $2\frac{7}{16}\%$ , 7 mos. at  $7\frac{3}{8}\%$ , 8 mos. at  $3\frac{3}{4}\%$ .
6. £1002. 8s.  $7d.$  for 3 mos. at  $5\frac{5}{16}\%$ , 8 mos. at  $2\frac{7}{16}\%$ , 7 mos. at  $4\frac{9}{16}\%$ .
7. £738. 6s.  $1\frac{1}{2}d.$  for 8 mos. at  $4\frac{3}{16}\%$ , 3 mos. at  $5\frac{1}{8}\%$ , 4 mos. at  $3\frac{3}{8}\%$ .
8. £905. 10s.  $4d.$  for 9 mos. at  $7\frac{1}{8}\%$ , 11 mos. at  $1\frac{3}{8}\%$ , 8 mos. at  $1\frac{5}{8}\%$ .
9. £89. 13s.  $6\frac{3}{4}d.$  for 7 mos. at  $2\frac{1}{16}\%$ , 9 mos. at  $4\frac{7}{16}\%$ , 10 mos. at  $4\frac{3}{8}\%$ .
10. £1012. 19s.  $8\frac{1}{4}d.$  for 6 mos. at  $3\frac{1}{4}\%$ , 2 mos. at  $5\frac{3}{16}\%$ , 5 mos. at  $9\frac{1}{8}\%$ .
11. Construct the aliquot formulae for 3 mos. at  $3\frac{1}{8}\%$  and 7 mos. at  $4\frac{1}{8}\%$ .
12. Construct the aliquot formulae for 1—12 mos. at  $5\frac{1}{4}\%$ ,  $2\frac{3}{8}\%$ .
13. Construct the aliquot formulae for 1—12 mos. at  $6\frac{3}{8}\%$ ,  $3\frac{1}{16}\%$ .



14. Construct the aliquot formulae for 1—12 mos. at the rates  $3\frac{1}{8}\%$ ,  $2\frac{1}{8}\%$ .

15. Construct the aliquot formulae for 1—12 mos. at rates from  $2\frac{1}{2}\%$  to 3 (by 8ths).

# 6. Days. Method of Multiples.

Convert money into £ and decimals (to 3 places).

Multiply (using methods of prediction, § vi) by the multiplier derived from the table corresponding to the product "days  $\times$  rate."

Table of Interest on £1 at 1 p.c.

Days	
1	·00002739726
2	·00005479452
3	·00008219178
4	·00010958904
5	·00013698630
6	·00016438356
7	·00019178082
8	·00021917808
9	·00024657534

*Example 1.* Interest on £531. 11s. 7 $\frac{1}{2}$ d. for 37 days at  $3\frac{1}{2}\%$  p.c.

531·582	3 + 3 + 1 = 7 places required in the multiplier (§ vi).	
9745 300	37 $\times$ $3\frac{1}{2}$ = 129·5	·00273973
15947		·00054795
2658		·00024658
212		·00001370
37		·00854796
5		
£1·8859		
17s. 8 $\frac{1}{2}$ d.		

*Example 2.* Interest on £265. 13s. 8 $\frac{1}{2}$ d. for 92 days at  $2\frac{1}{8}\%$  p.c.

92 $\times$ $2\frac{1}{8}$ = 184	·00547945	265·684
23	·00027397	6818 500
5 $\frac{1}{2}$	·00005479	13284
212 $\frac{1}{2}$	·00002055	2125
	·00581866	27
		21
		1
		£1·5458
3 + 3 + 1 = 7 (§ vi).		10s. 11d.

**EXAMPLES.**

Find Interest on

1. £712. 9s.  $10\frac{1}{2}d.$  for 91, 95 days at  $3\frac{1}{8}\%$ ,  $2\frac{1}{16}\%$ .
2. £832. 5s.  $8\frac{3}{4}d.$  for 110, 71 days at  $5\frac{1}{8}\%$ ,  $3\frac{1}{16}\%$ .
3. £912. 7s.  $6\frac{1}{2}d.$  for 85, 94 days at  $4\frac{1}{16}\%$ ,  $3\frac{1}{8}\%$ .
4. £705. 11s.  $4\frac{1}{4}d.$  for 97, 61 days at  $2\frac{1}{8}\%$ ,  $4\frac{1}{16}\%$ .
5. £1246. 12s.  $5\frac{3}{4}d.$  for 125, 93 days at  $3\frac{5}{8}\%$ ,  $4\frac{1}{16}\%$ .
6. £3125. 16s.  $10\frac{1}{2}d.$  for 61, 92 days at  $2\frac{3}{8}\%$ ,  $5\frac{1}{16}\%$ .
7. £973. 14s.  $5\frac{3}{4}d.$  for 87, 95 days at  $1\frac{1}{8}\%$ ,  $2\frac{1}{16}\%$ .
8. £641. 7s.  $11\frac{1}{2}d.$  for 72, 81 days at  $2\frac{1}{16}\%$ ,  $1\frac{1}{8}\%$ .
9. £1432. 8s.  $10\frac{1}{4}d.$  for 271, 82 days at  $3\frac{7}{8}\%$ ,  $1\frac{1}{16}\%$ .
10. £495. 19s.  $10\frac{1}{2}d.$  for 91, 87 days at  $5\frac{1}{8}\%$ ,  $1\frac{1}{16}\%$ .

**7. Days. The Third, Tenth, and Tenth Rule.**

Make the denominator 73000.

Find the corresponding numerator.

Move the decimal point in the principal 5 places to the left.

Multiply this principal (thus changed) by the numerator.

Add to this product  $\frac{1}{3}$  of it, then  $\frac{1}{10}$  of this, then  $\frac{1}{10}$  of this again.The answer is too great by  $\frac{1}{100000}$  ( $\frac{1}{10}$  per mille). $\frac{1}{10}$  p.m. =  $\frac{1}{2}d.$  per £10 approximately.*Example.* Int. on £612. 9s.  $5\frac{1}{4}d.$  for 93 days at  $3\frac{1}{2}\%$ .

$$\frac{93 \times 3\frac{1}{2}}{36500} = \frac{93 \times 7}{73000} = \frac{651}{73000}$$

	00612471	
	156	
	367483	
	30624	
	612	
	3·98719	
	1·32906	
	·13291	
	·01329	
	5·46245	£ 5 9 3
	·00055	$\frac{1}{2}$ less
	£5·46190	£5 9 2 $\frac{1}{2}$
	9s. 2 $\frac{1}{2}d.$	

$\frac{1}{10}$   
 $\frac{1}{10} \left\{ \frac{1}{10} \right\}$   
 $\frac{1}{10} \left\{ \frac{1}{10} \left\{ \frac{1}{10} \right\} \right\}$   
 less  $\frac{1}{100000}$

**Proof of Rule.**

$$\begin{aligned}
\frac{651}{100000} \left\{ 1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300} \right\} &= \frac{651}{1000} \left\{ \frac{300+100+10+1}{30000} \right\} = \frac{651}{1000} \times \frac{411}{30000} \\
&= \frac{651}{1000} \times \frac{137}{10000} \\
\frac{137}{10000} - \frac{1}{73} &= \frac{10001-10000}{730000} = \frac{1}{730000} \therefore \frac{137}{10000} = \frac{1}{73} + \frac{1}{730000} \\
\therefore \frac{651}{100000} \left\{ 1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300} \right\} &= \frac{651}{1000} \left\{ \frac{1}{73} + \frac{1}{730000} \right\} \\
&= \frac{651}{73000} \left\{ 1 + \frac{1}{10000} \right\} \\
\therefore \frac{651}{73000} &= \frac{651}{100000} \left\{ 1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300} \right\} \div \left\{ 1 + \frac{1}{10000} \right\} \\
&= \text{Rule} - \frac{1}{10000} \text{ Rule (very nearly).}
\end{aligned}$$

**EXAMPLES.**

Find Interest on

1. £372. 8s. 4½d. for 56, 89 days at 3½, 4½.
2. £712. 9s. 7½d. for 167, 312 days at 2½, 5½.
3. £274. 11s. 6½d. for 210, 184 days at 4½, 1½.
4. £195. 17s. 4½d. for 95, 91 days at 5½, 2½.
5. £846. 13s. 7½d. for 34, 147 days at 3½, 4½.

**8. Days. Method of Prediction.**

Reduce  $\frac{\text{days} \times \text{rate}}{36500}$  to a simple fraction without cancellation.

Decimalise the principal to 3 places and move the decimal point to the left as many places as there are zeros in the denominator of the above fraction.

Then decide by the method of prediction the no. of places which must be correct in the multiplication of the altered principal by the numerator of the fraction so as to ensure correctness to three places in the final answer after division by the denominator (without zeros)—the methods of approximation being employed for the multiplication and division.

Multiply and divide accordingly.

*Example 1.* £312.13s. 4½d. for 83 days at 3 p.c.

$$\begin{array}{r} 83 \times 3 = 249 \\ 36500 = \frac{249}{36500} \end{array} \quad \begin{array}{l} 3 \cdot 12667 \times 249 \text{ will contain 3 integers.} \\ 365 \text{ has 3. } \therefore \text{ there can only be 1 integer in} \\ \text{the final answer.} \end{array}$$

Hence we shall have 4 figures in the answer.

And therefore we must have 4 figures correct in the multiplication, of which 3 are integers--thus we must get 1 place correct in the multiplication.

The argument is put down actually thus :

$$3 \cdot 12667 \times 249 \equiv 3 \text{ integers.}$$

$$3 \mid 3 \mid 1 + 3 = 4 - 3 = 1 \text{ dec. place required.}$$

$$\begin{array}{r} 3 \cdot 12667 \\ 942 \\ \hline 62538 \\ 12506 \\ 2813 \\ \hline 3,6,5 \mid \overline{778 \cdot 52} \mid \text{£}2 \cdot 132 \\ \quad \quad \underline{485} \quad \quad 2s. 7\frac{1}{2}d. \\ \quad \quad \underline{120} \\ \quad \quad \underline{10} \\ \quad \quad 3 \end{array}$$

*Example 2.* £851.17s. 3½d. for 173 days at 2½ p.c.

$$\begin{array}{r} 173 \times 2\frac{1}{2} = 6747 \\ 36500 = \frac{6747}{36500} \end{array} \quad \begin{array}{l} 851865 \times 6747 \equiv 4 \text{ integers.} \\ 3 \mid 4 \mid 2 + 3 = 5 - 4 = 1 \text{ dec. place required.} \end{array}$$

$$\begin{array}{r} 851865 \\ 7476 \\ \hline 511119 \\ 59630 \\ 3407 \\ 596 \\ \hline 5,8,4 \mid \overline{5747 \cdot 52} \mid \text{£}9 \cdot 841 \\ \quad \quad \underline{4915} \quad \quad 16s. 10d. \\ \quad \quad \underline{243} \\ \quad \quad \underline{9} \\ \quad \quad 3 \end{array}$$

*Note 1.* A rule of thumb for deciding the no. of places necessary is:—

Reverse multiplier and place the digit now in the unit-place under the last place but one of the altered principal.

*Note 2.* This method surpasses all others in speed and convenience.

Other forms of it are given on p. 143.

**EXAMPLES.**

Find Interest on

1. £653. 9s.  $10\frac{1}{2}d.$  for 129, 83 days at  $3\frac{3}{16}\%$ ,  $2\frac{1}{8}\%$  respectively.
2. £234. 11s.  $7\frac{1}{2}d.$  for 91, 94 days at  $4\frac{1}{8}\%$ ,  $3\frac{1}{4}\%$  respectively.
3. £412. 16s.  $8\frac{1}{2}d.$  for 87, 32 days at  $5\frac{1}{16}\%$ ,  $4\frac{1}{8}\%$  respectively.
4. £516. 14s.  $10\frac{1}{2}d.$  for 215, 300 days at  $6\frac{1}{8}\%$ ,  $2\frac{1}{16}\%$  respectively.
5. £617. 8s.  $5\frac{1}{2}d.$  for 74, 89 days at  $7\frac{1}{2}\%$ ,  $3\frac{5}{8}\%$  respectively.
6. £3125. 9s.  $7\frac{3}{4}d.$  for 321, 64 days at  $3\frac{1}{4}\%$ ,  $2\frac{1}{8}\%$  respectively.
7. £274. 7s.  $8\frac{1}{2}d.$  for 93, 78 days at  $4\frac{5}{16}\%$ ,  $3\frac{3}{8}\%$  respectively.
8. £427. 13s.  $4\frac{3}{4}d.$  for 36, 57 days at  $1\frac{5}{8}\%$ ,  $2\frac{3}{8}\%$  respectively.
9. £561. 12s.  $6\frac{1}{2}d.$  for 63, 49 days at  $1\frac{1}{8}\%$ ,  $3\frac{1}{2}\%$  respectively.
10. £187. 6s.  $11\frac{3}{4}d.$  for 128, 212 days at  $2\frac{9}{16}\%$ ,  $2\frac{1}{2}\%$  respectively.
11. £695. 11s.  $10\frac{1}{2}d.$  for 84, 100 days at  $3\frac{1}{4}\%$ ,  $4\frac{1}{4}\%$  respectively.
12. £834. 9s.  $7\frac{1}{2}d.$  for 75, 92 days at  $4\frac{1}{8}\%$ ,  $3\frac{1}{16}\%$  respectively.
13. £721. 4s.  $10\frac{3}{4}d.$  for 65, 178 days at  $5\frac{7}{16}\%$ ,  $2\frac{3}{8}\%$  respectively.
14. £392. 5s.  $8\frac{1}{2}d.$  for 81, 200 days at  $3\frac{1}{8}\%$ ,  $5\frac{9}{16}\%$  respectively.
15. £268. 7s.  $4\frac{3}{4}d.$  from May 10 to June 13, March 5 to June 8, at  $3\frac{1}{8}\%$ ,  $4\frac{1}{4}\%$  respectively.
16. £156. 10s.  $7\frac{1}{2}d.$  for 3 mos. (no grace) from May 12, June 13 at  $2\frac{1}{16}\%$ ,  $3\frac{3}{8}\%$  respectively.
17. £647. 15s.  $3\frac{3}{4}d.$  for 2 mos. (no grace) from Nov. 6, March 5 at  $1\frac{3}{16}\%$ ,  $2\frac{1}{8}\%$  respectively.
18. £318. 14s.  $5\frac{1}{2}d.$  for 4 mos. (no grace) from June 11, May 1 at  $3\frac{1}{8}\%$ ,  $4\frac{7}{16}\%$  respectively.
19. £2514. 7s.  $6\frac{3}{4}d.$  for 3 mos. (grace) from Nov. 1, 88, May 3 at  $5\frac{1}{8}\%$ ,  $2\frac{1}{16}\%$  respectively.
20. £768. 12s.  $9d.$  for 2 mos. (grace) from Jan. 1, 89, Feb. 3, 92 at  $7\frac{1}{4}\%$ ,  $3\frac{3}{8}\%$  respectively.

**9. Common Methods.**

A. Reckon 30 days to the month and 360 days to the year—used in America.

Rules based upon this reckoning.

(1) To find the time in years, months and days between two dates.

Reckon the months according to their rank in the year, and the dates as so many days.

Subtract as for a Table proceeding by 12 and 30.

*Ex. 1.* Ap. 6 to June 4 . 1889.

$$\begin{array}{r} 89.6.4 \\ 89.4.6 \\ \hline 1.28 \end{array}$$

*Ex. 2.* May 17 . 92 to Feb. 4 . 93.

$$\begin{array}{r} 93.2.4 \\ 92.5.17 \\ \hline 8.17 \end{array}$$

(2) Any p.c. for any number of years, months, days.  
Find interest for 1 year and take aliquot parts for the time, treating the years, months, days, separately.

*Example.* £863. 17s. 3½d. at 4 p.c. for 3 years 2 months 5 days.

$$\begin{array}{r} 8.63865 \\ \hline 4 \\ \hline 34.5546 \quad \text{Int. } \frac{1}{4} \text{ p.c. 1 yr.} \\ \hline 3 \\ \hline 103.664 \\ 2 \text{ mos. } \frac{1}{4} \text{ of 1 yr.} \quad 5.759 \\ 5 \text{ days } \frac{1}{12} \text{ of 2 mos.} \quad 0.479 \\ \hline \pounds 109.902 \quad (18s. 0\frac{1}{2}d.) \end{array}$$

B. Reckon exact no. of days to the month and 360 days to the year—used on the Continent.

Any p.c. for any no. of days.

Multiply  $\frac{1}{100}$  of principal by  $\frac{\text{days} \times \text{rate}}{360}$ .

Aliquotise the fraction  $\frac{\text{days} \times \text{rate}}{360}$ .

In this way we get the advantage of the factors of 360, but the interest is  $\frac{1}{73}$  too much.

$\frac{1}{73} = .01369863$ , hence divide by 73 or multiply by this decimal and subtract to get the absolute interest.

*Example.* Int. on £313. 8s. 4d. for 90 days at 3 p.c.  $\frac{90 \times 3}{360} = \frac{3}{4}$ .

$$\begin{array}{r} 3.13416 \\ 1.56708 \\ .78354 \\ \hline 2.35062 \\ 96 \ 310 \\ \hline 235 \\ 71 \\ 14 \\ \hline 2 \\ \hline 0.0322 \\ \hline \pounds 2.3184 \\ 6s. 4\frac{1}{2}d. \end{array} \quad \begin{array}{r} 3.13416 \\ 1.56708 \\ .78354 \\ \hline 2.35062 \\ 73 \mid 2.35062 \mid .0322 \\ \hline 160 \\ 146 \\ \hline \dots \\ \therefore \text{Int.} = 2.3506 \\ .0322 \\ \hline \pounds 2.3184 \end{array}$$

It is very often simpler to deduct at these rates:  
 $\frac{1}{4}d.$  in 1s. 6d., 1d. in 6s., 3s. in £11, £1 in £73.

Aliquotation is very simple to the base 360.

*Note.* The American Government and the U.S. Courts reckon 365 days to the year.

Many Continental Governments however reckon as in (B).

The British custom is to reckon 365 days.

### Compound Interest.

10. *Compound Interest* arises in reality when a sum of money is left to accumulate in banks or (with certain reservations) in government funds. But the actual process of calculation may be said never to be required in such cases, for the practice is at the end of the year to look up the interest on the sum for the year, add it to the principal and enter this as the new principal in the new year's account.

The calculation of compound interest does arise however in some very important cases—viz. the determination of annuities, and the calculation of Life Insurance Tables.

#### 11. Compound Interest and the method of Approximation.

$$\text{The Int. on £1 for 1 year} = \frac{\text{rate}}{100} \text{ (e.g. } \frac{4}{100} \text{)}$$

$$\therefore \text{Amt. of £1 for 1 year} = 1 + \frac{\text{rate}}{100} \text{ (e.g. } 1 + \frac{4}{100} \text{)}$$

$$= \frac{100 + \text{rate}}{100} \text{ (e.g. } \frac{104}{100} = 1.04 \text{)}.$$

Hence multiply principal by a similar expression to 1.04 to get the amount for 1 year and repeat the process according to the no. of years given—to get amount for given no. of years.

1°. To multiply by such an expression as  $1.04$ .

Put down principal in decimals to no. of places desirable for accuracy (4 in general to ensure exactness to farthings).

Add rate  $\times$  principal, commencing to multiply 2 places to left of final figure on right taken—make allowance in usual way and put the first figure in product under last taken on the right.

Repeat process to get amount for given no. of years.

*Example 1.* Compound amount of £729. 8s. 11½d. for 3 years at 4 p.c.

729.446875

29.1778

758.6247

30.3450

788.9697

31.5588

820.5285

£820. 10s. 7d.

Commencing at the 2nd 4 and making allowance for the 6.

2°. When the expression is of the form  $1.03\frac{1}{4}$ .

Use the same method, but the fraction must be allowed for in a third line each year, i.e. for the fraction divide the top line for the year by the denomr., placing answer two places to the right or divide the second line by the denominator of the fraction similar to  $\frac{1}{4} \div 3$ —then add as usual.

*Example 2.* Compound amount of £831. 9s. 7½d. for 2 years at 3½ p.c.

831.4822816

24.9445

4.1574

860.5842

25.8175

4.3029

890.7046

£890. 14s. 1d.

$\frac{1}{4}$  of 2nd line or  $\frac{1}{4}$  of 1st line moved two places.



3°. When the expression is of the form  $1.02\frac{1}{16}$ .

Aliquotise the fraction  $\frac{1}{16} = 0-242$ .

Then use the same method, but divide the first line for any year by the first aliquot and put answer two places to the right, then use remaining aliquots on this result.

Finally add as usual.

*Example 3.* Compound Amount of £642. 18s. 7½d. for 3 years at  $3\frac{1}{8}$  p.c.

	$3\frac{1}{8} = 3 - 224$	
3 - 224	642.9302083	
	19.2879	
	3.2147	
	1.6073	
	.4018	
3 - 224	667.4419	
	20.0232	
	3.3372	
	1.6686	
	.4172	
3 - 224	692.8881	
	20.7866	
	3.4644	
	1.7322	
	.4331	
	719.3044	
	£719. 6s. 1d.	

### EXAMPLES.

1. Compound amount of £732. 11s. 7½d. for 3 yrs. at 5 p.c. and for 2 yrs. at  $3\frac{1}{4}$  p.c.
2. Compound amount of £1182. 9s. 5½d. for 2 yrs. at  $3\frac{1}{2}$  p.c. and for 3 yrs. at  $2\frac{3}{4}$  p.c.
3. Compound amount of £651. 11s. 4¾d. for 4 yrs. at 4 p.c. and for 5 yrs at 1 p.c.
4. Compound amount of £105. 7s. 5½d. for 3 yrs. at  $2\frac{5}{8}$  p.c. and for 2 yrs. at  $3\frac{1}{2}$  p.c.
5. Compound amount of £312. 13s. 6¼d. for 2 yrs. at  $5\frac{5}{8}$  p.c. and for 4 yrs. at  $1\frac{1}{8}$  p.c.
6. Compound interest on £685 for 4 yrs. at  $1\frac{1}{2}$  p.c. and for 3 yrs. at  $3\frac{1}{8}$  p.c.

7. Compound interest on £1820 for 5 yrs. at  $\frac{5}{8}$  p.c. and for 4 yrs. at 3 p.c.

8. Compound interest on £970. 5s. 6d. for 6 yrs. at  $2\frac{1}{2}$  p.c. and for 4 yrs. at  $1\frac{3}{4}$  p.c.

9. Compound interest on £860. 4s. for 7 yrs. at 1 p.c. and for 3 yrs. at  $4\frac{1}{8}$  p.c.

10. Compound interest on £1000 for 4 yrs. at  $3\frac{1}{8}$  p.c. and for 3 yrs. at  $1\frac{1}{4}$  p.c.

## 12. The Formation of Tables.

Tables of the compound amounts of £1 at various rates for 1, 2, 3, 4, etc. years are of great use in Annuities and similar calculations. They are formed in this way:—

Find the compound amount for the 1st year. Then multiply it by given rate, putting unit figure two places to the right and adding in the preceding compound amount as you multiply. If the tables are limited to six places, when that number is reached begin to multiply at the third digit (adding in the overplus of the 2nd digit) from the right and add in the usual way.

yrs.	1 p.c.	yrs.	5 p.c.	yrs.	7 p.c.	yrs.	$1\frac{1}{2}$ p.c.
1	1·010000	1	1·050000	1	1·070000	1	1·015000
2	1·020100	2	1·102500	2	1·144900	2	1·030225
3	1·030301	3	1·157625	3	1·225043	3	1·045678
4	1·040604	4	1·215506	4	1·310796	4	1·061363
5	1·051010	5	1·276281	5	1·402551	5	1·077284
6	1·061520	6	1·340095	6	1·500730	6	1·093443
7	1·072135	7	1·407100	7	1·605781	7	1·109845
8	1·082856	8	1·477455	8	1·718186	8	1·126493
9	1·093685	9	1·551328	9	1·838459	9	1·143390
10	1·104622	10	1·628894	10	1·967151	10	1·160541

## 13. General Formula for Compound Interest.

Two principles enable us to arrive at a valuable formula for calculating compound amounts for many years.

1°. The compound amount of any principal = the principal  $\times$  compound amount of £1 for given rate and time.

2°. Any power (integral or fractional) of the amount of £1 for any rate and time is the amount of £1 at the given rate for the same multiple or fraction of the time.

Thus the square of amount of £1 for 2 years is the amount for 4 years; and the cube root of amount of £1 for 6 years is the amount for 2 years.

Hence the general rule is easily established:

The compound amount of any principal for  $n$  years at a given rate = the principal  $\times$  (amount of £1 for 1 year at given rate) <sup>$n$</sup> .

The convenience of this rule in connection with tables like those above is obvious.

Also its use with the method of approximation reduces the labour of finding the compound amount for many years to a minimum.

The principle involved is the squaring and re-squaring, etc. of successive powers of the compound amount for £1.

An example is given below.

#### **14. To find the compound amount for a large number of years.**

First find the compound amount of £1 for the no. of years at given rate correct to the number of places required by the size of the given principal to ensure the final answer correct to 3 places.

If principal is below 100—6 places are sufficient.

If principal is between 100 and 1000—7 places are sufficient, and so on.

The compound amount of £1 for  $n$  years = (amount of £1 for 1 year) <sup>$n$</sup> .

Thus the practical problem is to raise the amount of £1 for 1 year to a given power correct to 6, 7, or 8 decimal places, by means of the method of approximation for multiplication.

*Example.* Find compound amount of £640 for 20 years at 4 p.c.

Compound amount of £1 for 1 year = £1·04.

∴ we have to find  $(1·04)^{20}$  correct to 7 (or at any rate 6 places).

Multiply by itself	1·04	
	<u>416</u>	
.. ..	1·0816	square
	86528	
	10816	
	<u>6490</u>	
.. ..	1·1698586	4th power
	1169859	
	701915	
	105287	
	9358	
	585	
	93	
	<u>7</u>	
.. ..	1·3685690	8th power
	4105707	
	821141	
	109485	
	6843	
	821	
	<u>122</u>	
	1·8730809	16th power
	1·1698586	4th power
	<u>1·8730809</u>	
	1873081	
	1123848	
	168577	
	14984	
	937	
	150	
	<u>11</u>	
C. Amt. of £1 for 20 years	2·1912397	20th power
	<u>046</u>	
	13147438	
	<u>876496</u>	
	£1402·3934 (7s. 10½d.)	

*Note.* It has been discovered by observation that “ $70\frac{1}{2} \div$  by given rate” gives the no. of years during which any sum will double itself very nearly.

**15. Annuities.**

Annuities are calculated from tables which are made on these principles.

1°. The amount of £1 at compound interest is calculated for 1, 2, 3, etc. years at a given rate.

2°. The present worths of these amounts are placed in the second column.

3°. The amount of annuity, if allowed to accumulate at compound interest, is placed in the third column.

4°. The present worths of these accumulated annuities are placed in the 4th column.

**Model of Annuity Table, Annuity £1, Rate 5 p.c.**

Years	Amount	Present worth	Amount of annuity	Present worth of annuity
1	1·05	·95238095	1·	·95238095
2	1·1025	·90702947	2·05	1·85941042
3	1·157625	·86383759	3·1525	2·72324801
4	1·21550625	·82270247	4·310125	3·54595048
5	1·27628156	·78352616	5·52568125	4·32947664

The first column is formed by multiplying the previous row by 5, placing result two places to the right and adding in previous row as you proceed. When 8 places are reached commence to multiply at the 6th, making allowance for the 7th and adding result to 8th place of the previous row for new 8th place, and so on.

The 2nd column is formed by dividing the previous present-worth by 105, putting result two places to the left and at the end using division-approximation.

The 3rd column is formed by adding previous row to collinear amount (1st col.) of previous row.

The 4th column is formed by adding previous row to collinear present-worth (2nd col.) of row to be formed.

The chief questions are these:—

Q. 1. What annuity will a given sum buy at given rate for a given no. of years?

Find present-worth of annuity of £1 for given time at given rate.

Divide this into the given sum.

Q. 2. What sum will buy a given annuity for a given no. of years at a given rate?

Find present worth of annuity of £1 for given time at given rate.

Multiply this by the given sum.

Q. 3. What will a given annuity amount to, forborne for a given no. of years at a given rate p.c.?

Find amount of £1 annuity for given no. of years at given rate p.c.

Multiply by the given annuity.

Q. 4. What is the present worth of an annuity forborne for a given no. of years at a given rate p.c.?

Find present worth of £1 annuity for given no. of years at given rate p.c.

Multiply by the given annuity.

*Example 1.* What annuity will £1000 buy for 4 years at 5 p.c.?

3·5,4,5,9,5		1000·000		£282·012		1000·000		282·012
		290·810				50·000		14·101
		7·184				1050·000		296·113
		·42				52·500		282·012
		2				1102·500		578·125
						55·125		28·906
						1157·625		282·012
						57·881		889·043
						1215·506		44·452
								933·495
								282·012
								1215·507

*Example 2.* What sum will buy an annuity of £100 for 5 years at 5 p.c.?

Present worth of £1 ann. for 5 yrs. at 5 p.c. = £4·32947664.

∴ sum required = £432·948.

*Example 3.* What sum will buy an annuity of £282·012 for 4 yrs. at 5 p.c.?

Present worth of £1 ann. for 4 yrs. at 5 p.c. = £3·545950.

282·012
595·453
846·036
141·006
11·280
1·410
·254
14
£1000·000

*Example 4.* What will annuity of £100 amount to forborne for 5 yrs. at 5 p.c.?

Amount of annuity of £1 = £5·52563125.

∴ amount of annuity of £100 = £552·563 (11s. 3½d.).

*Example 5.* What will be present worth of annuity in Ex. 4?

Present worth of annuity of £1 = £4·32947664.

∴ present worth of annuity of £100 = £432·948 (18s. 11½d.).

### EXAMPLES.

1. Construct annuity tables for £1 at 3, 4, 3½, 4½ p.c. for 10 yrs.
2. Extend annuity table for £1 at 5 p.c. to 20 years.
3. What annuity can be bought for 20 yrs. at 5 p.c. with £1250?
4. What sum will buy £100 a year for 10 yrs. at 3½ p.c.?
5. What is present worth of an annuity surrendered now of £356 which has still 8 yrs. to run at 4 p.c.?
6. Find amount of annuity for £1 at 2 p.c. for 10 yrs.
7. Find amount of annuity for £240 at 5 p.c. for 18 yrs.
8. Find amount of annuity for £1 at 2½ p.c. for 5 yrs.
9. Find present worth of an annuity for £100 at 6 p.c. for 4 yrs.
10. Find present worth of an annuity for £250 at 3½ p.c. for 15 yrs.

**16. Discount for Time.**

*The present worth* of any sum is the principal which would amount to the given sum if put out to interest now at a given rate until the sum falls due. This is required in the calculation of annuities.

*Discount* is a percentage deducted from a debt or obligation of any kind paid before it is due.

*Interest* is a percentage added to a debt or obligation of any kind paid after it is due.

Firms doing business with each other always give discount or charge interest according to the time of payment.

Hence discount for time is interest at a given rate for a given time—deducted instead of added.

*Bankers* and *Bill Discounters* discount Bills of Exchange in exactly the same way. They deduct Interest on the Bill at the current rate for the number of days it has yet to run.

In discounting such bills interest is reckoned on the three days of grace allowed before payment can be demanded, in addition to the currency of the bill.

**17. Cash Discounts, Commission, Brokerage.**

*Discount for cash* is a percentage struck off a debt for prompt payment or ready money.

*Commission* and *Brokerage* are terms to denote a similar percentage charged by an agent on sales effected for another. The agent is often termed a broker, factor, or commission agent. He is said to act for a principal.

The question of time does not often enter into the calculation of such percentages.

*Brokers* are of many kinds, but they are all “middlemen” engaged in effecting sales between parties



directly or indirectly, and they all recoup themselves by charging a commission on the amount of sales.

Bill-brokers, discount brokers, and bullion merchants meet at the Royal Exchange (on 'Change) for the negotiation of bills of exchange and the sale of bullion.

Stock and share brokers, if members of the Stock Exchange, go there to buy of, or sell to, the stock-jobbers.

Colonial brokers engaged in the sale of tea, coffee, sugar and all colonial and foreign produce frequent the Commercial Sales Rooms in Mincing Lane.

Brokers for corn and grain have their head-quarters in Mark Lane.

Marine insurance and ship brokers frequent Lloyd's Room in the Royal Exchange to meet the underwriters.

The term "underwriters" arises from the way in which a broker gets a ship insured. He fills up a blank policy of insurance and passes it to various insurance representatives in the room, each of whom accepts a portion of the proposed insurance and "underwrites" it accordingly.

There is also a Coal Exchange, and in addition there are brokers of wool, fruit, fish, etc., each class having a recognised place of meeting.

### 18. Discount Questions.

1°. To find the present worth of a sum due in a given time, interest being at a given rate p.c.

Find the compound amount of £1 for given time at given rate.

Then the present worth is found by dividing the sum by this compound amount.

*Example.* Present worth of £500 due 5 years hence at 4 p.c.

C. Amt. on £1 for 5 years at 4 p.c. = £1.216653.

∴ present worth = £500 ÷ 1.216653

= £410.963 (19s. 3d.).

£1·000000
40000
1·040000
41600
1·081600
43264
1·124864
44995
1·169859
46794
1·216653
1·21,66,5,3   500·0000   £410·963
13 3388
1 1723
774
44
8

2°. To find the discount on a bill for any number of mos. from a given date and cashed before maturity.

Find date of maturity by adding 3 days of grace.

Calculate the number of days the bill has to run from the day on which it is cashed.

Reckon interest on bill for this number of days at current rate of discount.

Subtract interest from bill.

The answer is what the holder receives.

*Example 1.* Bill for £312. 18s. 6½d., 3 mos. date May 5, cashed June 3. Rate 2½%.

Currency = 92 + 3 = 95, date Aug. 8.

June 3 to Aug. 8 = 66 days.

$66 \times 2\frac{1}{2}\% = 132$   
 $33 \quad \left. \begin{array}{l} \\ \end{array} \right\} = 171\frac{1}{2}$   
 $8\frac{1}{2}$

3·12926
5 2171
3 1293
2 1904
313
62
16
3,6,5   535·88   1·468
170 8
24 8
2 9

Discount = £1. 9s. 4½d.

Cash for bill = £311. 9s. 1½d.

*Example 2.* Bill for £450. 8s. 6d., 2 mos. date March 4, cashed March 8. Rate  $2\frac{3}{4}$  p.c.

With grace time for discount = 60 days.	4·50425
$60 \times 2\frac{3}{4} = 165$	<u>561</u>
450·425	45043
Discount = $\frac{2·036}{2·036}$	27025
Cash for bill = £447·461 (9s. 2d.)	<u>2252</u>
	3,6,5   <u>743·20</u>   2·036
	<u>13 2</u>
	<u>2 2</u>
	..

3°. Discount for cash is merely a case of percentages.

Similarly any kind of brokerage or commission.

The percentage is always struck off the debt or amount of sales.

*Example 1.* Find net payment of bill for £651. 9s. 10½d. Discount  $2\frac{1}{2}$  p.c.

6·51493	651·494
<u>13·0299</u>	<u>16·287</u>
3·2575	£635·207 (4s. 1¾d.).
<u>16·287</u>	

*Example 2.* Commission on sales to amount of £2561. 9s. 4d. at  $3\frac{1}{4}$  p.c.

25·61466
<u>76·8440</u>
6·4037
£88·247 (4s. 11½d.).

*Example 3.* Brokerage  $\frac{1}{8}$  p.c. on sale of stock to value of £8645. 13s. 5d.

86·45670
<u>£10·807</u> (16s. 1¾d.).

### EXAMPLES.

(Grace is always added.)

1. Discount bill for 3 mos. date Feb. 20 on March 10. Amount £612. 13s. 8d., rate  $2\frac{5}{8}$  p.c.

2. Discount bill for 2 mos. date March 13 on March 20. Amount £715. 9s. 10d., rate  $2\frac{1}{6}$  p.c.

3. Banker discounts bill for 4 mos. date May 9 on June 1. Amount £1000, rate  $1\frac{1}{8}$  p.c.

4. Bill Discounter cashes bill for 3 mos. dated June 13 on July 9. Amount £579. 6s. 7d., rate  $3\frac{1}{4}$  p.c.

5. Banker discounts bill 2 mos. date July 4 on July 7. £651. 9s. 6d., rate 3 p.c.

6. Banker discounts bill 60 days sight, seen June 5, on July 10. £1000, rate  $1\frac{1}{8}$  p.c.

7. Merchant cashes bill 30 days date July 8 on July 10. £841. 7s. 6d., rate 2 p.c.

8. Money-changer discounts bill 90 days sight Nov. 6 on Nov. 20. £1000, rate  $1\frac{3}{4}$  p.c.

9. Banker discounts bill 3 mos. date Nov. 9 on Nov. 16. £541. 8s. 9d., rate  $2\frac{3}{4}$  p.c.

10. Banker discounts bill 4 mos. date April 6 on May 1. £852. 8s. 6d., rate  $2\frac{1}{2}$  p.c.

11. Commission at  $3\frac{3}{8}$  p.c. on sales £1512. 8s. 9d., £1756. 10s. 6d., £2124. 8s. 9d.

12. Brokerage at  $\frac{1}{8}$  p.c., 1 per mille on bills for £956. 8s. 3d., £1020. 7s. 6d.

13. Commission  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$  p.c. on inland bills for £712. 6s. 8d., £614. 9s. 10d., £10850. 14s. 6d.

14. Commission on sales at  $3\frac{1}{2}$ , 4,  $2\frac{3}{4}$  p.c. to amount of £1164. 8s. 5d., £3125. 10s. 6d., £5960. 12s. 8d.

15. Brokerage  $\frac{1}{10}$  p.c. plus commission  $\frac{1}{8}$  p.c. on bill for £1856. 9s. 4d.

## 19. Stocks and Shares.

A *Company* is formed to carry on a business requiring considerable capital, by the issue of shares, which may be bought by those who think it a good investment. They thus become shareholders, and are in fact the proprietors of the business. Such a company is called a joint stock company, and is usually managed by directors chosen from the shareholders.

At stated intervals (generally each half-year) the accounts are made up and the nett profit paid to the shareholders as dividends.

A *Firm* differs from a company only in having a

much smaller number of shareholders or partners who all jointly manage the concern.

Also a company is generally a *limited* one, i.e. one in which the liability of each shareholder is limited to the amount of his shares, whereas a firm is frequently unlimited.

Shares can only be purchased in whole numbers, but any quantity of stock may be purchased. For this reason many companies change their shares into stock.

A shareholder who desires to retire from a company can only do so by selling out at the market price of his stock or shares, usually through a stock broker, who charges a commission or brokerage.

The nominal value of a stock is by no means its market value. This depends on the credit and prosperity of the company. The market price is continually fluctuating.

Stocks are said to be *at a premium*, *at par*, or *at a discount* as the price of £100 stock is above, equal to, or below £100 sterling. Similarly shares are at a premium, at par, or at a discount as their price is above, equal to, or below their nominal or face value.

The two prices published each morning in the newspapers opposite any stock are the outside prices given or charged by a stock jobber. The jobber sells to the broker at the higher price and buys from him at the lower price. The broker sells to a customer at the higher price with brokerage added and buys from a customer at the lower price with brokerage deducted.

A broker's charge for Consols is 2s. 6d. per cent. ( $\frac{1}{2}$  p.c.), for other stocks from 5s. to 10s. per cent.

*Bonds* are particular classes of stocks or shares. They are in general certified documents issued by Governments, corporations or companies acknowledging their indebtedness, with the limits and conditions of payment and redemption of the debt. The interest is usually paid by coupons or warrants.

## 20. The National Debt.

The first item in the Public Debt of this country is the £664,263 repaid to the Goldsmiths of London at the accession of William III. as partial compensation for the loss they sustained at the hands of Charles II. when he closed the Mint and took the money stored there.

Another item is £11,015,100 owing to the Bank of England.

Wars however have been the chief cause of the immense growth of the Debt. William's campaigns added 16 millions, Marlborough's 38 millions, George II. increased it by 87 millions, the American War added 121 millions, and finally the Napoleonic struggle cost 600 millions. In 1816 the Debt was 900 millions, but happily since then great reductions have been made, and it now stands (March 31, 1903) at £759,943,282 (of which the funded portion is £695,645,762). It consists of three parts, (1) the Funded Debt, (2) the Unfunded Debt, (3) Terminable Annuities.

**The Funded Debt** consists of the unredeemable loans made by private persons to the British Government. They receive interest year by year, but can only recover the principal by selling out at the market price. The Government however has the right of paying off at par with due notice to the holders.

The chief kinds of stocks in the Funds were until recently

(1) The Consolidated Annuities or Consols at 3 p.c., an amalgamation of several loans into one of a uniform 3 p.c. interest.

(2) The Reduced Annuities or Consols, formerly paying higher than 3 p.c., but now reduced to 3 p.c. interest.

(3) The New 3 p.c. Annuities.

(4) A few  $2\frac{1}{2}$  p.c.,  $2\frac{3}{4}$  p.c.,  $3\frac{1}{2}$  p.c. and 5 p.c. Annuities.

Mr Goschen's Act for the Conversion of Consols (1888) made a new stock at  $2\frac{3}{4}$  p.c.

Holders of Consols, Reduced Consols, and New Threes were offered an equal amount of the New  $2\frac{3}{4}$ , with a small bonus for assenting by a certain date. The holders of all but 47 millions agreed; those who did not, have received the necessary 12 mos. notice that their stock will be paid off at par at the convenience of the Government. When this has been done the National Debt will consist almost wholly of  $2\frac{3}{4}$  p.c. stock. The Act also enacts that in 1903 the rate of interest will be reduced to  $2\frac{1}{2}$  p.c. and in 1923 the Government will have the right to pay off the debt at par.

The **Unfunded Debt** consists of securities bearing interest but redeemable by Government at certain (usually) short dates. The common form of these securities is that of **Exchequer Bills** or orders upon the Exchequer entitling the bearer to the specified sums together with interest at a stated rate p.c. for the time of the bill in days. The usual times of these bills are 1, 2, 3, 4, 6, 12 mos. from the date of issue. They are notified to be paid off or renewed on a given day, and if not sent in on that day the interest ceases. Their price depends on the credit of the nation. Large capitalists eagerly seek after these bills because they are readily convertible and are generally at a small premium to ensure easy borrowing on the part of the Government.

**Terminable Annuities** are the chief instrument by which the Funded debt is reduced. The Treasury is empowered under various Acts to give Annuities for a certain number of years in exchange for permanent stock. Thus £1000  $2\frac{3}{4}$  p.c. stock produces £27. 10s. as

yearly interest in perpetuity: the Treasury will give in exchange £55 a year for 20 years, at the end of which all payment ceases and thus in the end £1000 stock is struck off the debt.

The capitalised value of the present annuities paid by the Government stood at about  $68\frac{1}{2}$  millions on March 31, 1891, but a goodly number of them will fall in from 1901 to 1904.

**Sinking Funds** of various descriptions have been also employed to reduce the debt, and one is at present in operation: these would have been more successful if they had not been suspended so frequently on emergencies.

## 20. The Bank of England.

The Bank was originated in 1694, when 40 capitalists joined to lend £1,200,000 to the Government at 8 p.c. interest in return for a charter conferring certain privileges and powers to trade in bills of exchange and bullion.

The charter has been frequently renewed, and the debt of the Government to the Bank is now £11,015,100. This forms part of the subscribed capital, known as Bank Stock, on which 10 p.c. is usually paid.

In 1844 Sir Robert Peel's Bank Charter Act renewed the charter "until the debt of the public to the Bank be paid." It limited the note issue to £14,000,000 upon securities of which the above debt forms part; beyond 14 millions, notes can only be issued against coin or bullion stored in the Bank.

The principal functions of the Bank are:—

1°. For the Government: It keeps the Government banking account, superintends the transfers of stock (Consols, etc.) and pays the interest on the National Debt.



2°. For the coinage: It buys gold at £3. 17s. 9d. per oz. and sells sovereigns at £3. 17s. 10½d. per oz. Thus practically it is the only channel by which gold reaches the Mint.

3°. For the banks: It keeps the reserves of all the larger banks and discount houses.

4°. For the public: It issues notes which it must always cash in gold on presentation. If the issue exceed 14 millions, bullion must be kept to meet all notes beyond. When the supply of bullion is not much above the amount required to meet the notes, the Bank increases its rate of discount to attract gold to the Bank. When the supply is large the rate is lowered. The market rate of discount is largely influenced by the bank rate.

## 21. The Stock Exchange.

The Stock Exchange is regulated by a committee of 30, including the Chairman and Deputy-Chairman, chosen from among the members by ballot year by year. The members are 4000 in number, and consist of jobbers or brokers. Jobbers are dealers who buy and sell at market prices.

Brokers deal with the jobbers on behalf of the public and charge commission.

The Stock Exchange only recognises its own members, and all their dealings must follow the usages of "the House."

Settlements take place twice a month, at dates arranged by the committee.

Consols however are settled only once—at the beginning of the month.

Stock Exchange terms:—

1°. *Scrip* (Subscription) is applied to the certificates of payment of deposits or calls prior to the issue of the real bonds.

2°. *Time bargains* are speculative transactions without the payment or transfer of stock or shares. At the time appointed the parties receive or pay the difference between the then price and the price named.

3°. A *bull* buys stock, for which usually he cannot pay, with a view to selling again at a higher price.

A *bear* sells stock, which usually he does not possess, with a view to buying back at a lower price.

The Americans use the terms "long" and "short" for "bull" and "bear."

At the settlement the difference in the buying and selling prices alone changes hands.

4°. *Contango* is a sum paid per cent. or per share by a speculator for the rise for the privilege of deferring payment till the next settlement.

*Backwardation* is a sum paid per cent. or per share by a speculator for the fall to postpone delivery of shares or stock till the next settlement.

Both these are called "continuation rates." The postponement is called "carrying over."

5°. *Options* are transactions by which a speculator limits his losses by paying a fixed sum when the bargain is made. They may be "a put and call," "a put" or "a call."

A *put and call* enables an operator to sell or purchase at a fixed price on a certain day.

A *put* enables an operator to sell at a fixed price on a certain day.

A *call* enables an operator to buy at a fixed price on a certain day.

The Americans use the term "spread" for a "put" or "call."

A *straddle* is the same option at one price whether a put or call.

6°. *Arbitrage* occurs when identical securities are bought in one market and sold in another.

7°. *Hammering* is the declaration of a defaulter. The head waiter strikes three blows with a mallet.

8°. *Cornering* occurs when a scarcity of stock is created to prevent a dealer from obtaining what he has previously sold except at much higher prices. The dealer is said to be "cornered." This happens when the dealer undertakes to deliver stock or shares beyond his powers.

9°. *Slang* terms and contractions are used for nearly every stock known; e.g. Brums, i.e. L. and N. W. R. Stock; Goschens, i.e. 2½ Consols; Saras, i.e. Sheffield Deferred.

**Outside Brokers**, i.e. brokers who are not members of the Stock Exchange, supply their customers through the brokers who are members. The commission charged being halved between the outside broker and the member he employs.

## 22. Calculations.

Companies formed to carry on business divide their capital into stock or shares held by the members in varying proportions. These portions can be bought and sold without affecting the operations of the company. Hence the prices of stocks and shares are of importance. Their face-value is very different from the market value, which constantly fluctuates, being (1) at a *premium* above the face-value, (2) at *par* equal to the face-value, or (3) at a *discount* below the face-value.

Brokerage or Commission ( $\frac{1}{8}$ ,  $\frac{1}{4}$ ,  $\frac{1}{2}$  p.c. or more), Stamp, and Registration charges must be added or subtracted in all calculations.

**Form of Stock and Share Transaction.**

Contract Note Stamp Adhesive 1s.
---

KING STREET, LEICESTER,

April 26, 1904.

Dear Sir,

We beg to advise you of having *bought* on your account, subject to the rules and usages of the Leicester Stock Exchange, for the settlement of April 29, 10 shares £10 paid in the N. and S. Wales Bank, Limited.

	£	s.	d.
At £32½ per share . . . . .	323	15	0
Commission ½ p.c. . . . .	1	12	6
Stamp Duty and Registration Fee . . . . .	2	1	6
	<u>£327 9 0</u>		

We are

Yours faithfully,

ROBERTS &amp; JONES.

*E. Brown, Esq.*

If this had been *sold* on account of Mr Brown it would have stood thus:

	£	s.	d.	£	s.	d.
At £32½ per share . . . . .				323	15	0
Less Commission ½ p.c. . . . .	1	12	6			
„ Stamp and Fee . . . . .	2	1	6	3	14	0
				<u>£320 1 0</u>		

Amount due from Roberts &amp; Jones.

### Stock Exchange Contracts.

Bought by order and for account of Arthur Evans, Esq. (subject to the rules of the London Stock Exchange):

	£	s.	d.
£1000 Japanese 5 p.c. loan at $97\frac{1}{4}$ . . . . .	972	10	0
Plus Commission $\frac{1}{4}$ p.c. . . . .	2	9	0
„ Stamp 1s. . . . .		1	0
	<u>£975</u>	<u>0</u>	<u>0</u>

Sold by order and for account of Arthur Evans, Esq. (subject to the rules of the London Stock Exchange):

	£	s.	d.	£	s.	d.
10 Rand Mines Shares at $26\frac{7}{8}$				268	15	0
Less Commission $\frac{1}{2}$ p.c.	1	7	0			
„ Stamp 1s.		1	0	1	8	0
				<u>£267</u>	<u>7</u>	<u>0</u>

### Contango Contract.

Continued for Account of Arthur Evans, Esq.

Sold for this account May 15, 1904:  
 £1000 Russian 5 p.c. Stock at  $97\frac{1}{8}$ .  
 50 Chili Ord. at  $26\frac{1}{4}$ .

Bought for May 29, 1904 :  
 At  $97\frac{1}{8}$  } and 5 per cent.  
 At  $26\frac{1}{4}$  } Contango.  
 Relfe & Co.

This document requires a 4/- Contract Stamp.

**Account, May 15, 1904.**

Arthur Evans, Esq., in account with Relfe &amp; Co.

*Dr.*

			Comm.			
			£ s. d.	£	s.	d.
Ap. 30	To 30 Golden Hills brt. dn.	25½		765	0	0
	" Contango 1/- per share				1	10 0
May 4	" 1000 Russian 5 p.c.	97½	2 10 0	971	5	0
	" 50 Peruvians	6	2 10 0	300	0	0
	" 50 Chili Ord.	26½	4 10 0	1312	10	0
	" Commission on Sales		4 5 0	13	15	0
	" Contract Stamps				5	0
	" Balance				10	15 0
				3375	0	0
	To take up—					
	50 Peruvians					

*Cr.*

			Comm.			
			£ s. d.	£	s.	d.
May 1	By 10 Rand Mines	27	1 5 0	270	0	0
" 10	" 30 Golden Hills	26½	3 0 0	795	0	0
" 13	" £1000 Russian 5 p.c.					
	carried over	98½		985	0	0
" "	" 50 Chili Ord. carried over	26½		1325	0	0
				3375	0	0
	E. & O. E					
	By Balance				10	15 0
	To Deliver—					
	10 Rand Mines					

**EXAMPLES.**

1. Buy 18 shares at 34. Commission  $\frac{1}{2}$  p.c. Stamp £1. 7s. 6d. •  
Registration 3s. 6d. Contract Note 1s.
2. Sell 15 Rio Tintos at  $26\frac{1}{4}$ . Commission 1/- per share.  
Stamp duty 15s. Contract Note 1s.
3. £315 4 p.c. stock at  $111\frac{1}{4}$ . Commission  $\frac{1}{2}$  p.c. Stamps  
and Fee 18s. 6d. Contract 1s. Bought and sold.
4. £7245 Consols at  $89\frac{1}{4}$ . Commission  $\frac{1}{2}$  p.c. Stamp and  
Fee £2. 10s. 0d. Contract Note 1s. Bought and sold.
5. Sell out £8315 N.W. 4 p.c. stock at 171. Commission  
 $\frac{1}{2}$  p.c. Stamps and Fee £4. 0s. 0d. Purchase £3400 of Mid-  
land Railway 3 p.c. Debentures at  $149\frac{1}{2}$ . Commission  $\frac{1}{2}$  p.c.  
Stamp, Contract, Registration £3. 18s. 6d. Make out the  
Account.
6. Sell out £1000 District Preference 3 p.c. at 89 and buy  
£580 Midland  $4\frac{1}{2}$  at 149—charges in both transactions being  
Commission  $\frac{1}{4}$  p.c., Stamp £2, Registration and Contract 4s. 6d.  
Make out the Account.
7. Sell out £1000 District 3 p.c. stock at 90 and buy £500  
Great Western  $4\frac{1}{2}$  p.c. stock at 159. Commission for both  
transactions  $\frac{1}{2}$  p.c. Stamps etc. £3. 2s. 6d.
8. I have £10000 which I want to invest—my broker  
advises me to purchase £5000 Consols at  $85\frac{1}{2}$  and £5000 Deben-  
ture Corporation Stock at 110. Will there be any money over,  
Commission being  $\frac{1}{2}$  p.c. and Stamps, Contract and Registration  
£5 on each transaction?

9. Make out following account :

(1) Sold for me

Brt. down 10 Witwaterstands at 27, contango  
5 p.c.

30 Rio Tintos at 26. Commission  $\frac{1}{2}$  p.c.

£1000 Chinese 4 p.c. at 98 $\frac{1}{2}$ . Commission  $\frac{1}{2}$  p.c.

50 N. Staff. Bonds at 41. Commission  $\frac{1}{2}$  p.c.

Contract Stamps.

(2) Bought for me

25 Liptons at £11. 10s. 0d. Commission  $\frac{1}{2}$  p.c.

30 Rio Tintos carried over at 26 $\frac{1}{2}$ .

£1000 Waterworks at 95. Commission  $\frac{1}{2}$  p.c.

50 N. St. Bonds at 42, carried over.

10. Which will cost more to buy, 81 Chicago 5 p.c. bonds at 111 or 114 Greek 3 p.c. bonds at 79? Commission  $\frac{1}{2}$  p.c. and Stamps etc. £2. 2s. 6d. in each case.



**TABLE OF DAYS.**

This table is for use in all questions requiring the number of days between two dates, viz. Interest, Discount, Bills of Exchange, Equated Payments, Cash Balances, etc.

*Examples of its use.*

1. No. of days between Feb. 17 and Aug. 13.

$$\begin{array}{r} \{ \text{Aug. 13.} \quad 225 \\ \{ \text{Feb. 17.} \quad 48 \\ \hline \text{No.} = 177 \end{array}$$

2. No. of days between Nov. 6, 1888, and March 10, 1889.

$$\begin{array}{r} \{ \text{Dec. 31, 1888.} \quad 365 \\ \{ \text{Nov. 6, 1888.} \quad 310 \\ \hline 55 \end{array} \quad \begin{array}{r} \text{Mar. 10.} \quad 69 \\ \hline \text{No.} = 124 \end{array}$$

Jan.	1	1	Feb.	16	47	Apr.	3	93	May	19	139
	2	2		17	48		4	94		20	140
	3	3		18	49		5	95		21	141
	4	4		19	50		6	96		22	142
	5	5		20	51		7	97		23	143
	6	6		21	52		8	98		24	144
	7	7		22	53		9	99		25	145
	8	8		23	54		10	100		26	146
	9	9		24	55		11	101		27	147
	10	10		25	56		12	102		28	148
	11	11		26	57		13	103		29	149
	12	12		27	58		14	104		30	150
	13	13		28	59		15	105		31	151
	14	14	Mar.	1	60		16	106	June	1	152
	15	15		2	61		17	107		2	153
	16	16		3	62		18	108		3	154
	17	17		4	63		19	109		4	155
	18	18		5	64		20	110		5	156
	19	19		6	65		21	111		6	157
	20	20		7	66		22	112		7	158
	21	21		8	67		23	113		8	159
	22	22		9	68		24	114		9	160
	23	23		10	69		25	115		10	161
	24	24		11	70		26	116		11	162
	25	25		12	71		27	117		12	163
	26	26		13	72		28	118		13	164
	27	27		14	73		29	119		14	165
	28	28		15	74		30	120		15	166
	29	29		16	75	May	1	121		16	167
	30	30		17	76		2	122		17	168
	31	31		18	77		3	123		18	169
Feb.	1	32		19	78		4	124		19	170
	2	33		20	79		5	125		20	171
	3	34		21	80		6	126		21	172
	4	35		22	81		7	127		22	173
	5	36		23	82		8	128		23	174
	6	37		24	83		9	129		24	175
	7	38		25	84		10	130		25	176
	8	39		26	85		11	131		26	177
	9	40		27	86		12	132		27	178
	10	41		28	87		13	133		28	179
	11	42		29	88		14	134		29	180
	12	43		30	89		15	135		30	181
	13	44		31	90		16	136	July	1	182
	14	45	Apr.	1	91		17	137		2	183
	15	46		2	92		18	138		3	184

THE TABLE OF DAYS.

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July	4	185	Aug.	19	231	Oct.	4	277	Nov.	19	323
	5	186		20	232		5	278		20	324
	6	187		21	233		6	279		21	325
	7	188		22	234		7	280		22	326
	8	189		23	235		8	281		23	327
	9	190		24	236		9	282		24	328
	10	191		25	237		10	283		25	329
	11	192		26	238		11	284		26	330
	12	193		27	239		12	285		27	331
	13	194		28	240		13	286		28	332
	14	195		29	241		14	287		29	333
	15	196		30	242		15	288		30	334
	16	197		31	243		16	289	Dec.	1	335
	17	198	Sept.	1	244		17	290		2	336
	18	199		2	245		18	291		3	337
	19	200		3	246		19	292		4	338
	20	201		4	247		20	293		5	339
	21	202		5	248		21	294		6	340
	22	203		6	249		22	295		7	341
	23	204		7	250		23	296		8	342
	24	205		8	251		24	297		9	343
	25	206		9	252		25	298		10	344
	26	207		10	253		26	299		11	345
	27	208		11	254		27	300		12	346
	28	209		12	255		28	301		13	347
	29	210		13	256		29	302		14	348
	30	211		14	257		30	303		15	349
	31	212		15	258		31	304		16	350
Aug.	1	213		16	259	Nov.	1	305		17	351
	2	214		17	260		2	306		18	352
	3	215		18	261		3	307		19	353
	4	216		19	262		4	308		20	354
	5	217		20	263		5	309		21	355
	6	218		21	264		6	310		22	356
	7	219		22	265		7	311		23	357
	8	220		23	266		8	312		24	358
	9	221		24	267		9	313		25	359
	10	222		25	268		10	314		26	360
	11	223		26	269		11	315		27	361
	12	224		27	270		12	316		28	362
	13	225		28	271		13	317		29	363
	14	226		29	272		14	318		30	364
	15	227		30	273		15	319		31	365
	16	228	Oct.	1	274		16	320			
	17	229		2	275		17	321			
	18	230		3	276		18	322			

## SECTION VIII.

### ACCOUNTS AND BALANCES.

#### 1. Invoices and Bills.

A *Tradesman's Bill* is a list of goods bought with prices attached—which is presented for payment over the counter or sent in at fixed periods of the year to a customer who has an account with the tradesman—the term *Account* is used in the same sense as *Bill* when thus sent in.

*Invoices* are memoranda of goods with prices sent to a buyer and always accompanying the goods. When the time for payment comes an account embodying these invoices is sent to the customer.

Very frequently the details are not given but the goods are set down "as per Invoice" of a given date.

If the items are given the account is a detailed account.

Invoices are always detailed.

An *Account Sales* is a written account of goods sold on commission, their price, expenses, and net value—after deduction of the percentage.

1°. Instance of an *Invoice*.

Newark, 18th Feb. 1892.

Mr HENRY PARKER.

Bought of JOHN HARDING,  
Draper and Silk Mercer.

		£	s.	d.
Feb. 18	13 yds. black serge at 2s. 9½d.	1	16	3½
	3 yds. Honiton lace at 7s. 9d.	1	3	3
	18 yds. of twill at 3s. 4½d.	3	0	9
	10 dozen handkerchiefs at 6s. 6d.	3	5	0
	15 pairs tan gloves at 3s. 1d.	2	6	3
	14 yds. of silk at 4s. 6d.	3	3	0
		14	14	6½

Terms. 5 p.c. discount within one month.

2°. Instance of an *Account Sales*.

Account Sales.

Account sales of 1000 boxes of tea per S.S. San Salvador on account  
of Messrs. Hare and Blackett, Canton.

1892		£	s.	d.
June 1	250 boxes net 8250 lbs. at 4s. 2d.	1718	15	0
June 24	450 „ „ 10000 lbs. at 3s. 4d.	1666	13	4
July 7	200 „ „ 6000 lbs. at 3s. 6d.	1050	0	0
July 20	100 „ „ 2000 lbs. at 3s. 10d.	383	6	8
		4818	15	0
	Freight and charges £110. 9s. 6d.			
	Commission at 2½ p.c. £120. 9s. 4½d.	230	18	10½
	Net proceeds	4587	16	1½

London, July 31st, 1892. R. S. Budgett &amp; Co.

**EXAMPLES.**

## 1. Construct appropriate invoices for these transactions.

- (1) 84 yds. silk at 3s. 10½d.
- 91 yds. linen at 1s. 3½d.
- 102 yds. flannelette at 3½d.
- 20 pairs of taffeta silk gloves at 3s. 7d.
- 50 dozen handkerchiefs at 5s. 7d. per dozen.

- (2) 30 lbs. flour at  $3\frac{1}{4}d.$   
 84 bags oatmeal at  $25s. 6d.$   
 90 bags meal at  $27s. 4d.$   
 3 cwt. seeds at  $10s. 7d.$  per cwt.
- (3) 18 lbs. bacon at  $6\frac{1}{4}d.$   
 25 sultana cakes at  $9\frac{3}{4}d.$   
 30 Madeira cakes at  $1s. 1\frac{1}{4}d.$   
 19 lbs. lard at  $3\frac{3}{4}d.$   
 27 lbs. biscuits at  $6\frac{1}{4}d.$
- (4) 150 yds. blue serge at  $2s. 1\frac{1}{4}d.$   
 75 yds. calico at  $3\frac{3}{4}d.$   
 24 yds. silk at  $4s. 11d.$   
 10 doz. lengths cord at  $1s. 0\frac{3}{4}d.$   
 20 pairs silk gloves at  $2s. 3\frac{1}{4}d.$

2. Make out account sales on these transactions.

(1) 1000 lbs. of tea. 250 at  $3s. 5d.$  350 at  $4s. 1d.$  200 at  $3s. 4d.$  200 at  $3s. 7d.$  Commission  $2\frac{1}{2}$  p.c. Charges 7 per mille.

(2) 1000 bags of rice each containing 23 cwts. 225 at  $2\frac{1}{4}d.$  250 at  $3\frac{1}{2}d.$  500 at  $2\frac{3}{4}d.$  Remainder at  $4d.$  Commission 5 p.c. Charges 2 per mille.

(3) 8000 bags of maize. 2000 at  $25s. 9d.$  3000 at  $32s. 6d.$  1500 at  $34s. 7d.$  500 at  $41s. 3d.$  Commission  $4\frac{1}{2}$  p.c. Charges  $\frac{2}{3}$  p.c.

- (4) 7830 bales N.S. W. and Queensland wool at  $1s. 1\frac{1}{2}d.$  per lb.  
 2225 Victorian at  $1s. 3d.$  per lb.  
 4963 New Zealand at  $1s. 2\frac{3}{4}d.$  per lb.  
 503 Tasmanian at  $11\frac{1}{4}d.$  per lb.  
 Commission  $4\frac{3}{4}$  p.c. Charges  $\frac{1}{4}$  p.c.

### Book-keeping.

2. **Book-keeping** is the science of arranging commercial transactions in a series of books for the purpose of accounts.

The books which a merchant *theoretically* requires are three in number, but *practically* he uses others to reduce labour and simplify his accounts.

The three essential books are

1. *The Waste Book*, a rough record of all transactions in order of time.

2. *The Journal*, a technical arrangement of these transactions for transfer to the ledger.

3. *The Ledger*, a classified arrangement of the journal into various accounts.

Practically most merchants have separate books for all transactions of a similar character and use the journal to arrange only those transactions not included in the special subsidiary books. Such books are :—

4. *Bought Book*, a record of goods bought on credit.

5. *Sales Book*, a record of goods sold on credit.

6. *Cash Book*, a record of cash received and paid and discounts received and allowed.

7. *Petty Cash Book*, for small payments (subsidiary to the Cash Book).

8. *Bill Book*, a record of promises of payment to or by the merchant.

The Ledger will contain an abstract of the foregoing books.

The use of the *Waste Book* requires no explanation, it is ruled for one money column.

The use of the *Journal* is very important, it is ruled for two money columns—a debtor and creditor column—thus representing the two sides of an account in the ledger. Each transaction is at once assigned to two places, a personal account and some other.

*Example.*

I pay R. Brown £20. 15s. 4d. ∴ his account is debtor.

Cash account is obviously creditor.

I buy goods of S. Smith worth £15. 10s. 0d. ∴ goods account is debtor.

S. Smith's account is obviously creditor.

## The Journal.

			Dr.			Cr.		
	February.		£	s.	d.	£	s.	d.
4	R. Brown.	Dr.	20	15	4			
	To cash.	Cr.				20	15	4
7	Goods.	Dr.	15	10	0			
	To S. Smith.	Cr.				15	10	0

The *Ledger* is the most important book of all, it is ruled on each page in two sections, representing debtor and creditor sides. It contains a large number of accounts of all kinds—personal, real and fictitious. Each entry is placed to two of these accounts at least, thus embodying the principle of double entry, the principle always employed where accuracy is desired.

The most numerous accounts are the personal accounts.

Besides these there are usually the cash, the goods, the profit and loss, the balance, and the capital accounts.

Examples of these follow.



1°. **Personal Account.**

Mr Brown buys "goods" on 3 occasions and pays twice—the difference is the balance due. Each item of goods will appear in the goods account. Each cash item in the cash account.

WILLIAM BROWN, Cambridge.

Dr.				Cr.			
1892	To goods	£	s.	d.	By cash	£	s.
Jan. 5		4	10	6		4	10
" 12	"	12	7	5	"	10	0
" 26	"	7	18	0	Balance due	10	5
		24	15	11		24	15
							11

Frequently money is paid on account and thus there may be a balance in favour of the customer. Discount is reckoned or interest charged for the balance according as it is in favour of the debtor or creditor side.

It is customary among traders to charge the same rate of interest on both sides of the account.

**2°. Goods Account.**

The firm buys goods and, if possible, sells at a profit. Also the value of goods on hand must be inserted at a fair valuation to get the true profit or loss.

Dr.		Bought.	GOODS ACCOUNT.				Sold.	Cr.			
			£	s.	d.			£	s.	d.	
1892											
Jan. 1		To capital	250	0	0	Jan. 6	By J. Harvey	29	7	8	
" 5		To R. Sandol	25	12	6	" 19	By cash	18	5	0	
" 9		To cash	30	6	8	" 22	By S. Cornell	28	15	6	
" 28		To A. Jones	20	5	6	" 25	By cash	20	0	0	
" 31		To profit and loss—gain	50	3	6	" 31	Goods on hand	280	0	0	
			376	8	2			376	8	2	

### 3°. Profit and Loss Account.

This account collects the profits and losses; it is a fictitious account for the information of the Firm as to their position. The word "Capital" is used for "Balance" because the Firm's capital is increased by net profits or diminished by net losses.

Dr.		Losses.	PROFIT AND LOSS ACCOUNT.				Profits.	Cr.			
			£	s.	d.			£	s.	d.	
1892											
Jan. 8		To cash—stationery	5	0	0	Jan. 9	By T. Page—disc.				
" 13		" wages	15	15	0	" 15	By sales	80	5	6	
" 15		" rent	40	0	0	" 20	By goods	25	0	0	
" 26		To W. Brown—discount		7	6						
" 31		To capital—gain	44	18	6						
			106	1	0			106	1	0	

**4°. Balance Account.**

This account shows assets and liabilities and therefore the net capital of the Firm. A trial balance is frequently made to test the correct posting of the ledger.

Dr.		Assets.		BALANCE ACCOUNT.						Liabilities.		Cr.			
				£	s.	d.				£	s.	d.			
1892	To cash	150	10	6		Jan. 31	By R. Carson			50	18	6			
Jan. 31	To goods	400	0	0		"	By 3ms' bill less disct			198	0	0			
"	To W. Brown	10	5	5		"	By capital			410	17	5			
"	To 3 mos' bill less disct.	99	0	0		"									
"		659	15	11						659	15	11			

**5°. Capital Account.**

This shows the balance of the Firm's capital on Jan. 31, i.e. the net value of the estate. There might of course have been a loss—this would have occurred on Dr. side and the balance would have been so much less.

<i>Dr.</i>		CAPITAL ACCOUNT.						<i>Cr.</i>			
		£	s.	d.	Jan. 1 " 31	By cash By profit	£	s.	d.		
1892	To balance	418	10	3			350	0	0		
Jan. 31							68	10	3		
		418	10	3			418	10	3		

# 6°. Cash Account.

This shows receipts and payments. All receipts at office must be entered in office columns, and all cheques paid in or drawn to bank columns. Any transfer from bank to office or from office to bank must be entered as a debit to one and a credit to the other, or *vice versa*.

Dr. Cash

January

Cr.

1892	Receipts	Bank			Office			1892	Payments	Bank			Office		
		£	s.	d.	£	s.	d.			£	s.	d.	£	s.	d.
Jan. 1	To capital	300	0	0	£ 60	10	9	Jan. 6	By goods	47	10	0	£ 50	0	0
" 4	To goods	30	0	0	75	18	9	" 10	By J. Helby	20	0	0	12	13	6
" 15	To J. Spensley	50	0	0	20	0	0	" 18	By T. Rice	312	10	0	50	0	0
" 22	To bank							" 22	By office				48	16	0
" 28	To office							" 28	By bank						
		380	0	0	156	9	6	" 31	By balance	380	0	0	156	9	6

### 3. Equation of Payments.

A problem of great frequency and importance in business is to find *the equated time* or date when the sum of several debts due at different times may be paid.

The *term of credit* is the time to elapse before a debt is due.

The *equated term of credit* is the time to elapse before the sum of several debts due at different times is due—reckoned from some arbitrary *zero date*.

The *equated time* is found by adding the equated term to this convenient arbitrary *zero date*.

The *zero date* is generally either the first date of any bills due or the first day of the month in which the account begins.

The latter date is in general the more convenient.

The cases are three in number:—

“To find the equated time of bills bought (1) for unequal times at different dates, (2) for equal times at different dates, (3) for unequal times at the same date.”

(1) “For bills at different dates for unequal times,”

Convert times into days, using convenient zero date.

Multiply each item by its term of credit in days.

Add the items and these products.

Divide the sum of the products by the sum of the items.

The answer will be the equated term in days.

Add this to the zero date to obtain the equated time.

(2) “For bills at different dates for equal times,” we may proceed as in the above rule or find the equated time for the dates alone, adding on the term of credit afterwards.

(3) "For bills at the same date for unequal times," the given date should be taken as the zero date.

*Note.* Use the Table of Days throughout.

If months and fractions of a month (30 days to the month) are employed the work is less, but the results are not so accurate always.

*Example.* The equated time of the following bills.

Zero date Ap. 1.	1890 Ap. 4.	To merchandise 6 mos. credit	£320 × 186	2560 320
	May 20	„ 2 mos. „	£468 × 110	4680 1572
	June 1	„ 4 mos. „	£524 × 183	4192 524
	July 10	„ 3 mos. „	£760 × 192	1520 6840 760
				2072   352812   170
Equated term. 171 days.				14561
Equated time. Sept. 19, 1890.				...572

#### 4. The Averaging and Balancing of Accounts.

An *Account* is the statement of business transactions between debtor and creditor.

*Averaging Accounts* is the process of finding the date on which the balance is due (i.e. the equated term of the balance from some zero date).

A *Balance* is the difference of the two sides of an account.

A *Cash Balance* differs from the balance by the interest due or the discount claimable.

The creditor receives interest on the balance from the date on which it is due to the date of settlement.

The debtor is entitled to discount off the balance for the time he pays it before the date on which it is due.

**1°. Rule for Averaging Accounts.**

Multiply each sum on both sides by its term of credit in days—reckoned from a convenient zero date.

Add the amounts and the amounts multiplied by the days on each side—all four separately.

Find balance of the account and also the difference of the amounts  $\times$  the days on the two sides.

Divide this difference by the balance.

The answer will be the equated time in days from the zero date.

If the balance of the products is in favour of the debtor side—the equated term must be added to the zero date to get the date on which the balance is due, and if it is paid before this date discount is allowed.

If the balance of the products is in favour of the creditor side—the equated term must be subtracted from the zero date to get the date on which the balance was due, and interest is charged from this date to the date of payment.

**2°. Rule for finding the Cash Balance at any date.**

Multiply each sum on both sides by the difference in days between its date and the date of striking the cash balance.

Add the amounts and these products on both sides—all four separately.

Find balance and difference of products.

The difference of products  $\times \frac{\text{rate of interest}}{36500}$  gives interest due or discount claimable. Add to or subtract from balance—for the cash balance.

If the balance of products is in favour of the debtor side—interest must be added.

If the balance of products is in favour of the creditor side—discount may be subtracted.



3°. **Rule for monthly statements** (cash balances at the end of the month).

Multiply each item by the difference in days between the date and the end of the month.

Add the amounts and the products.

The products  $\times \frac{\text{rate of interest}}{36500}$  gives interest due.

Add to balance this interest for the cash balance.

*Note.* For all these operations use the Table of Days.

Less work is involved in working with months and decimals of a month (30 days), but the results are not so accurate.

*Example 1.* Averaging accounts.

To find the equated time of paying the balance of this account:—

1890	Dr.		1890	Cr.
		£ s. d.		£ s. d.
Mar. 12	Togoods. 3mos.	641 12 7½	May 10	By cash 350 0 0
Ap. 7	„ 4mos.	650 9 6	July 2	„ 372 8 9
May 10	„ 6mos.	1000 0 0	Aug. 12	„ 500 0 0

Zero date March 1.

		1924·890		24500·0
		64163·0		37243·7
		65047·5		7448·74
				1117·311
£641·630 × 103	}	32523·75	£350·000 × 70	50000·000
£650·475 × 159		5854·275	£372·437 × 123	30000·00
£1000·000 × 254		254000·000	£500·000 × 164	2000·00
£22292·105		423513·415	£1222·437	152309·751
£1222·437		152309·751		
Balance £1069·668		271205·664		

1,069,6|68 | 271205·664 | 253·7. Equated term.

573  
38  
7

The products are in favour of the debtor side.

∴ the equated term must be added to the zero date.

Thus the equated time is Nov. 10, 1890.

The work may be shortened by calling £641. 12s. 7½d. £642, and £650. 9s. 6d. £650, and £372. 8s. 9d. £372—the same result is obtained as to time here, but this is not always the case.

*Example 2. Averaging accounts.*

1889	Dr.		1889	Cr.	
		£ s. d.			£ s. d.
June 18	To goods	974 12 6	July 6	By note	150 0 0
Nov. 26	"	150 10 4	Dec. 16	By merchandise	232 8 11
1890			1890		
Feb. 20	"	124 7 3	Mar. 10	"	461 5 0

Zero date June 1.

	9746·25		4500·00	
	6822·375		750·00	
	15051·6		23244·50	
	10536·12		20920·05	
	1204·128		1859·560	
974·625 × 17	} = 24872·4	150·000 × 35	} = 92250·	
150·516 × 178		232·445 × 198		36900·0
124·362 × 264		461·250 × 282		922·50
<u>1249·503</u>	<u>76192·041</u>	<u>843·695</u>	<u>181346·610</u>	
843·695			76192·041	
Balance 405·808			105154·569	

4,05,8|08 | 105154·569 | 259. Equated term.

239  
36

The products are in favour of the credit side.

∴ the equated term must be subtracted from the zero date.

Thus the balance was due on Sept. 15, 1888.

*Note.* This may seem peculiar, but the debtor had 76192 days from June 1, 1889 to pay £1 by the terms of the transaction, whereas he actually took 181346 days to pay it (105154 days too much). Hence as compensation he should have paid the balance (£406) 259 days before June 1, 1889, since  $406 \times 259 = 105154$ .

*Example 3.* To find cash balance in *Example 1* on Aug. 12.

Discount at 5 p.c. p.a.

Aug. 12 to Nov. 10 is 90 days.

Discount on £1069·668 for 90 days at 5 p.c. = £13·187.

$$\begin{array}{r}
 10\cdot69668 \\
 \quad 09 \\
 73 \overline{) 962\cdot701} \quad | \quad 13\cdot187 \\
 \underline{232} \\
 13 \quad 7 \\
 \underline{6 \quad 40} \\
 56 \\
 \underline{5}
 \end{array}$$

$$\begin{array}{rcl}
 \therefore \text{Cash balance due if paid} & \} & \text{£1069·668} \\
 \text{on Aug. 12} & & \underline{13\cdot187} \\
 & & \text{£1056·481}
 \end{array}$$

*Example 4.* To find cash balance on March 10, 1890, in *Example 2*.

Interest at 5 p.c. p.a.

$$\begin{array}{rcl}
 974\cdot625 \times 265 & = & 258275\cdot625 \\
 150\cdot516 \times 104 & = & 15653\cdot664 \\
 124\cdot362 \times 18 & = & 2238\cdot516 \\
 \hline
 & & 276167\cdot805 \\
 & & 56575\cdot380 \\
 \hline
 & & 219592\cdot425
 \end{array}
 \qquad
 \begin{array}{rcl}
 150\cdot000 \times 247 & = & 37050\cdot000 \\
 232\cdot445 \times 84 & = & 19525\cdot380 \\
 461\cdot250 \times 0 & = & \phantom{00000000} \\
 \hline
 & & 56575\cdot380
 \end{array}$$

Balance of products is in favour of debtor side.  $\therefore$  Interest is added.

$$\begin{array}{r}
 73 \overline{) 2195\cdot92425} \quad | \quad 30\cdot081 \\
 \underline{5 \quad 92} \\
 84 \\
 \underline{11}
 \end{array}$$

$$\begin{array}{rcl}
 \therefore \text{Cash balance due if paid} & \} & \text{£405·808} \\
 \text{on March 10, 1890} & & \underline{30\cdot081} \\
 & & \text{£435·889}
 \end{array}$$

**Interest 5 p.c. p.a.**

[illegible]

- (1) To goods, May 9, 6 mos. £412. 17s. 6d.  
To goods, June 3, 4 mos. £351. 8s. 9d.  
To goods, July 4, 3 mos. £612. 13s. 7d.  
To goods, Aug. 7, 4 mos. £506. 8s. 8d.
- (2) To merchandise, Feb. 20, 5 mos. £364. 8s. 7d.  
To merchandise, March 11, 4 mos. £512. 7s. 6d.  
To merchandise, April 4, 3 mos. £416. 8s. 4d.  
To merchandise, June 6, 1 mo. £1000. 0s. 0d.
- (3) To goods, May 4, 4 mos. £5126.  
To goods, June 5, 4 mos. £7184.  
To goods, July 8, 4 mos. £9651.  
To goods, Aug. 9, 4 mos. £7364.
- (4) To goods, May 1, 5 mos. £764. 10s. 0d.  
To goods, May 1, 3 mos. £865. 5s. 0d.  
To goods, May 1, 4 mos. £900. 6s. 8d.  
To goods, May 1, 6 mos. £600. 0s. 0d.

- (5) To merchandise, Jan. 6, 3 mos. £651. 10s. 6d.  
 To merchandise, Feb. 10, 4 mos. £840. 0s. 0d.  
 To merchandise, Ap. 9, 2 mos. £712. 13s. 4d.  
 To merchandise, May 12, 3 mos. £1500. 0s. 0d.  
 To merchandise, June 10, 4 mos. £940. 12s. 6d.

2. Average and find the cash balances on the following accounts.

- (1) 1889. Dr.  
 Mar. 16, To goods, 3 mos. £580. 17s. 9½d.  
 Ap. 12, To goods, 4 mos. £708. 10s. 0d.  
 May 10, To goods, 4 mos. £952. 0s. 0d.

1889. Cr.  
 May 7, By cash, £640. 8s. 7d.  
 June 3, By cash, £312. 9s. 6d.  
 Aug. 5, By cash, £700. 0s. 0d.  
 Cash Balance due on Aug. 20.

- (2) 1890. Dr.  
 Jan. 19, To merchandise, 5 mos. £657.  
 Feb. 7, To merchandise, 3 mos. £840.  
 April 8, To merchandise, 4 mos. £1725.  
 May 19, To merchandise, 4 mos. £961.

1890. Cr.  
 Feb. 5, By cash, £500.  
 March 30, By cash, £700.  
 April 30, By cash, £1000.  
 June 6, By cash, £1200.  
 Cash Balance due on June 30.

- (3) 1892. Dr.  
 Jan. 18, To goods, 4 mos. £782. 6s. 6d.  
 Feb. 20, To goods, 2 mos. £814. 5s. 10d.  
 March 10, To goods, 3 mos. £900. 7s. 6d.  
 April 30, To goods, 6 mos. £654. 8s. 0d.  
 May 15, To goods, 3 mos. £500. 0s. 0d.

1892. Cr.  
 Feb. 1, By cash, £1000.  
 March 3, By cash, £500.  
 April 28, By cash, £700.  
 May 12, By cash, £630.  
 June 4, By cash, £400.  
 Cash Balance due on June 10.

(4) 1898. Dr.

				£	s.	d.
Mar. 7.	To Goods 3 mos.	...	...	640	12	6
Apr. 3.	To Goods 3 mos.	...	...	512	18	7
May 5.	To Goods 3 mos.	...	...	315	10	4
June 9.	To Goods 3 mos.	...	...	256	11	0

1898. Cr.

				£	s.	d.
Mar. 15.	By Bill 3 mos.	...	...	600	0	0
Apr. 10.	By Bill 3 mos.	...	...	500	0	0
May 30.	By Bill 2 mos.	...	...	350	0	0
July 20.	By Bill 1 mo.	...	...	250	0	0

Cash Balance on Sept. 30. Int. 5 p.c.

3. Draw up the monthly statements of these transactions.  
 "To goods."

(1) Jan. 3, £812. Jan. 5, £750. Jan. 9, £624. Jan. 13, £500. Jan. 23, £816. Jan. 27, £914.

Find cash balance, (interest at 5 p.c. p.a.) at end of month.

(2) May 1, £712. 10s. May 5, £847. May 12, £623. 7s. 6d. May 17, £500. May 22, £863. May 27, £914. 7s. 8d. Cash balance on May 31, at 5 p.c. p.a.

4. Average this account.

1890. Dr.	1890. Cr.
June 16, To goods, £970.	July 10, By note, £145.
Nov. 20, To goods, £156.	Dec. 12, By goods, £237.
1891.	1891.
Feb. 10, To goods, £125.	Mar. 6, By goods, £464.

## 5. Partial Payments or Payments on Account.

Partial payments are frequently made either before or after a bill or account is due—if before, discount is allowed; if after, interest is charged. In the case of bonds and other interest-bearing obligations the payments are usually indorsed on the back.

Bankers are constantly loaning money for profit—in partial payments they reckon amount of debt + interest to date of partial payment, then strike off the payment and difference is new principal. Similarly in

the case of banks which give interest on current account as money is drawn or put in the difference in sum becomes the new principal. Interest however is not added in but put into a further column for addition at the end of the half-year—this interest is calculated for the time that any temporary principal remains unchanged at the current rate of interest.

Merchants and traders charge and allow the same rate of interest on both sides of the account.

Commercial discount is a percentage struck off a debt and not off the money paid for cash or payment before the debt is due.

The terms of an agreement may include discount within certain days.

Hence in the case of an account when partial payments are made within the time or times on which discounts are allowed—the value of these partial payments is increased, i.e. a given payment will pay off more debt owing to the discount granted—this is always placed to the credit of the debtor (Example 4).

*Example 1.* A bank lends a customer £1000 on March 5, 1890—the customer pays £250 on June 3, £300 on Sept. 24—find cash balance on Dec. 31.

Interest 10 p.c. p.a.

	Dr.		Cr.		Dr. Bal.		
	£	s.	d.	£	s.	d.	
Mar. 5	To Cash	1000	0 0				
June 3				By Cash	250	0 0	90
Sept. 21				" "	300	0 0	113
				" Interest		59 19 2	98
						509 19 2	218850

Interest at 10 p.c. per annum on £218850 for one day = £59. 19s. 2d., found thus:

$$\begin{aligned} £218850 \times 10 \div 36500 &= £2188 \cdot 5 \times 2 \div 73 = £4377 \cdot 00 \div 73 \\ &= £59 \cdot 959 = £59. 19s. 2d. \end{aligned}$$

**Example 1a.** Balance this Bank Account up to Dec. 31, 1908—  
Interest being 1 p.c. p.a. and Commission on total Amount of Transactions  $\frac{1}{2}$  p.c.

July 1. Balance to Credit £251. 10s. 6d.  
 „ 7. Drawn £57. 5s. 0d.  
 „ 16. Paid in £104. 15s. 0d.  
 Aug. 19. Paid in £40. 0s. 0d.  
 „ 30. Drawn £71. 15s. 6d.  
 Sept. 15. Drawn £15. 0s. 0d.  
 Oct. 18. Drawn £79. 12s. 6d.  
 Nov. 17. Paid in £61. 5s. 0d.  
 Dec. 12. Paid in £115. 11s. 0d.  
 „ 24. Drawn £30. 0s. 0d.

	Dr.	Cr.	Bal.	Days	
	£ s. d.	£ s. d.	£ s. d.		
July 1	251 10 6		251 10 6	6	1512
„ 7		57 5 0	194 5 6	9	1746
„ 16	104 15 0		299 0 6	84	10166
Aug. 19	40 0 0		339 0 6	11	3729
„ 30		71 15 6	267 5 0	16	4272
Sept. 15		15 0 0	252 5 0	88	8316
Oct. 18		79 12 6	172 12 6	80	5190
Nov. 17	61 5 0		233 17 6	25	5850
Dec. 12	115 11 0		349 8 6	12	4188
„ 24		30 0 0	319 8 6	7	2233
„ 31	1 5 10	1 0 8	319 13 8		
	Interest	Commission	Balance		47202

Interest = £472.02  $\times$  1  $\div$  365 = £1. 5s. 10d.

Commission = £826.725  $\div$  800 = £1. 0s. 8d.

Balance = £319. 8s. 6d. + £1. 5s. 10d. - £1. 0s. 8d. = £319. 13s. 8d.

**Example 2.** A man begins a banking account on Jan. 18 by putting in £1000; on Feb. 9 he draws £89. 7s. 6d.; on Mar. 3 he puts in £100, and then on Mar. 5 he draws £96. 15s. 8d.; on Mar. 17 he draws £230. 3s. 7d., and on April 3 he draws £105; in May his



account remains unchanged, but on June 17, 18, 19 he draws £50, £76 and £90 respectively. Balance his account up to June 30. Interest 2 p.c. p.a.

## J. JONES.

Cr.		£	s.	d.	Days	Extending decimal.
Jan. 18	Cash	1000	0	0	22	£22000
Feb. 9	"	910	12	6	22	£20042
Mar. 3	"	1010	12	6	2	£ 2022
Mar. 5	"	913	17	3	12	£10968
Mar. 17	"	683	13	8	17	£11628
Apr. 3	"	578	13	8	75	£43425
June 17	"	528	13	8	1	£ 529
June 18	"	452	13	8	1	£ 453
June 19	"	362	13	8	11	£ 3993
June 30	Int.	6	6	1		£115060
	Balance	£368	19	9		2
						3,6,5   2301·20   6·304
						111 2
						. 1 7

*Example 3.* A merchant supplies a trader with goods to value of £1000 on March 5, 1890; the trader pays £250 on account on June 3, £300 on Sept. 24; find cash balance due on Dec. 31. Interest 10 p.c

Dr.		Cr.
March 5.	To goods £1000	June 3. By cash £250
		Sept. 24. " £300

$£1000 \times 301 = £301000$	$£250 \times 211 = £52750$
82150	$£300 \times 98 = £29400$
<u>£218850</u>	<u>£82150</u>

3,6,5   21885·0   59·959
3635
350 0
21 5
3 2

1000
550
450
59·959

∴ Balance due = £509.19s. 2½d.

*Example 4.* The terms of a transaction are 6 mos. credit, 10 p.c. discount for cash within 14 days, 5 p.c. discount within one month,  $2\frac{1}{2}$  p.c. within 3 mos.

Find balance due without discount on this account.

Dr. 1890.		Cr.	
Feb. 10.	To goods 6 mos. £2315·418	Feb. 24.	By cash £1200
			10 p.c. disc. 133·333
		Mar. 9.	By cash £ 300
			5 p.c. disc. 15·789
		May 8.	By cash £ 500
			$2\frac{1}{2}$ p.c. disc. 12·821
			<u>£2161·943</u>

∴ On Aug. 10 balance due = £153·475 without discount.

A discount of 10 p.c. on a debt enables £90 to pay £100. ∴ £1200 will pay £1333·333.

A discount of 5 p.c. on a debt enables £95 to pay £100. ∴ £300 will pay £315·789.

A discount of  $2\frac{1}{2}$  p.c. on a debt enables £97½ to pay £100. ∴ £500 will pay £512·821.

2315·418  
2161·943  
£153·475

### EXAMPLES.

1. Find cash balance due on Dec. 31, on a bond for £750 dated March 10, and endorsed £200, May 16; £350, Sept. 30. Interest 8 p.c. p.a.

2. A bank allows overdraft to £5000, viz. £3000 on Ap. 6, £2000 on May 20. The debtor pays in £1500 on June 30, £1750 on Aug. 1. What will be the balance against him on Sept. 1? Interest 6 p.c. p.a.

3. Balance R. Jones's account on June 30.

Cr.

Jan. 30, £3000, (By cash).  
Feb. 9, £750, Cheque.  
March 18, £364, Cheque.  
May 20, £7000, Bill of Exchange.

Dr.

Feb. 3, £325. 10s. 6d., (To cheque).  
March 5, £800. 10s. 0d., (To cheque).  
Ap. 1, £250. 0s. 0d., (To cheque).  
May 31, £5000. 0s. 0d., (Order to buy Stock).  
Interest  $2\frac{1}{2}$  p.c. p.a.

4. Find cash balance due on Sept. 1, of this account.  
Interest 5 p.c. p.a.

Dr.	Cr.
April 3, To goods, £1500.	June 18, By cash, £700.
May 9, To goods, £784.	Aug. 4, By cash, £650.

5. Find cash balance due on June 30, of this account.  
Interest 7 p.c. p.a.

Dr.	Cr.
Jan. 20, To goods, £632. 5s. 0d.	March 8, By cash, £1000.
Feb. 14, To goods, £845. 7s. 6d.	May 1, By cash, £500.
Ap. 3, To goods, £716. 10s. 6d.	June 20, By cash, £825.
May 26, To goods, £920. 15s. 4d.	

6. The terms of a transaction are 5 p.c. discount within 14 days, 2½ p.c. within 3 mos.

Find balance due without discount on this account.

Dr. 1892.	Cr. 1892.
Jan. 20, To goods, £2000.	Jan. 31, By cash, £1800.
Feb. 9, To goods, £1520.	May 8, By cash, £1200.

7. The terms are 10 p.c. for cash, 5 p.c. within one month, 3 p.c. within 3 mos., 1 p.c. within 6 mos.

Find balance due without discount on this account.

Dr. 1891.	Cr. 1891.
Feb. 17, To goods, £3500.	Feb. 17, By cash, £1000.
March 8, To goods, £1000.	May 7, By cash, £2000.
May 11, To goods, £1260.	Aug. 10, By cash, £800.
June 6, To goods, £1920.	Dec. 6, By cash, £1000.

## SECTION IX.

### **EXCHANGES AND EXCHANGE OPERATIONS.**

#### **1. Moneys and Bullion.**

The Moneys of all countries are of two kinds,  
(1) Metallic Currency, (2) Paper Currency.

*Metallic currency* consists of stamped pieces of metal (chiefly gold, silver, or bronze) of certain weights and fineness (purity) called the standard and regulated by law. The making and issuing of coin are usually in the hands of the Executive Government.

Private individuals frequently have the right to have any bullion coined into money on bringing it to the Mint.

In England this is done in the case of gold free of expense: in France, Germany, etc. a moderate charge is made to cover the mintage cost: in India a 2 p.c. seignorage is charged over and above the cost for coining silver.

*Paper currency* consists of Notes issued by the Government, or by public banks with the sanction of the Government. These notes are "payable on demand" metallic currency, and as long as they are regularly

paid they are simply a substitute for coins, and have no influence on the foreign exchanges. In many instances notes have been issued beyond the power of the Government to redeem the promise to pay on demand in coin. Hence specie payment is suspended and a law is passed, making the notes legal tender, thus establishing an inconvertible paper currency, shown by a premium on metallic money and a rise in the foreign exchanges.

## 2. **Weights and Measures.**

The standards or models for legal weights and measures in each country are fixed by the Government. There is a bewildering variety of these, as the models are entirely arbitrary.

The French metrical system shows promise of superseding the present chaos, but old usages die hard. Great Britain, the United States, Russia and Denmark, have not yet adopted any decimal system of weights and measures, though the metric system is contemplated in the latter two. China and Japan have a decimal system of their own. The great difficulty, however, lies in the fact that in spite of the adoption of the metric system, the old systems more or less remain, and the main units are still quoted for commercial purposes.

## 3. **Modern Coinages.**

The principal modern coinages are these:—

		Gold Mint value.
France	100 centimes = 1 franc	9½ <i>d.</i>
Belgium	100 centimes = 1 franc	"
Switzerland	100 centimes = 1 franc	"
Italy	100 centesimi = 1 lira	"
Greece	100 lepta = 1 drachma	"
Spain	100 centimos = 1 peseta	"

Servia	100 paras	= 1 dinar	9½d.
Bulgaria	100 stotinkis	= 1 leva	"
Roumania	100 banis	= 1 ley	"
Portugal	1000 reis	= 1 milreis	4s. 5½d.
Germany	100 pfennige	= 1 mark	11¾d.
Austria	100 heller	= 1 krone	10d.
Holland	100 cents	= 1 florin	1s. 8d.
U. States	100 cents	= 1 dollar	4s. 1½d.
Russia	100 kopecks	= 1 rouble	3s. 1½d.
Norway	100 öre	= 1 crown	1s. 1d.
Sweden	100 öre	= 1 crown	"
Denmark	100 öre	= 1 crown	"
Gt. Britain	240 pence	= 1£ sterling	"
Turkey	100 piastres	= 1£ Turkish	18s. 0¾d.
Egypt	100 piastres	= 1£ Egypt.	20s. 3½d.
India	16 annas	= 1 rupee	1s. 10½d.
Ceylon	100 cents.	= 1 rupee	"
China	1000 cash	= 1 tael	6s. 6d.
Japan	100 sen	= 1 yen	4s. 1d.
S. America	100 centavos	= 1 peso	3s. 9d.
Brazil	1000 reis	= 1 milreis	2s. 3d.
Mexico	100 centavos	= 1 peso	4s. 0½d.

In Gt. Britain the standard is gold, the oz. of gold = £3. 17s. 10½d. (Mint value).

In India the standard is silver.

In the United States the standards are gold and silver (bi-metallic).

In France the standard is gold, but the silver 5-franc circulates at the double valuation, i.e. gold is reckoned as  $15\frac{1}{2} \times$  silver, hence the oz. of pure silver =  $\frac{2}{31}$  of £3. 17s. 10½d. = 60⅞d. = 5s. 1d. nearly.

In Germany the standard is gold, but the old silver thaler circulates at 3 marks, and thus the double valuation is slightly in force.

France, Belgium, Switzerland, Italy, and Greece form the Latin Union with coins of the same weight

and fineness. Spain, Servia, Bulgaria, and Roumania have adopted the same system of coins but they have not joined the Union.

The column of Mint values must be read with the caution that it gives the gold values. The silver values depend either on the  $15\frac{1}{2} : 1$  ratio or on the market price of silver. The Mint value of any silver coin on the  $15\frac{1}{2} : 1$  ratio, or when taken as a token, is very different from its market value.

This varies from day to day, and depends on the supply of coins and the price of silver. The market value is usually very near its intrinsic value, but supply and demand may raise or depress the price.

Gold is very stable in value, especially in countries with a purely gold valuation. In bi-metallic countries the fluctuations of silver derange the gold values a little.

#### 4. Exchanges.

1°. An "Exchange" in commerce occurs when a person pays his creditor by transferring to him a debt due to himself from someone else.

The object of these "exchanges" is the avoidance of bullion-transmission, which is costly.

If A in London owes a debt to B in Paris, and C in Paris owes a debt of equal amount to D in London, it is obvious that if A were to pay D, and C were to pay B, the debts would be settled without any transfer of money.

B and C might be one and the same person and thus for an exchange four persons and two debts are usual, three persons and two debts are necessary.

In practice bills of exchange are used as the medium of exchange.

2°. "Inland Exchange" is the remittance of bills to places in the same country by which means debts are discharged without cash transfers.

If one place owes more than another it requires bills on it, and this accounts for the demand for London bills—London always being a large creditor to the rest of the country. This demand causes a premium on London bills, generally commuted for time in the case of inland bills, e.g., the premium between Edinburgh and London is 1s. p.c. or 4 days' interest.

3°. "Foreign Exchange" is the remittance of bills to foreign places in payment of debts, in order to avoid the transmission of money or specie abroad.

The condition of the coinages as well as the balance of trade influences the rate of exchange.

A rate of exchange is the value of the money of one country in that of another.

It is the custom to quote the rate between two countries by keeping one equivalent fixed and leaving the other to fluctuate.

Thus Paris *gives* to London a varying number of francs and centimes for £1 sterling. London *gives* to Lisbon so many pence per milreis.

The place with the fixed price is said to *receive* the fluctuating price.

In quotations the fixed price is usually omitted.

The equivalence between the currencies of two countries (i.e. the rate of exchange arising out of the state of the coinage) is called the Mint par of exchange (nominal exchange).

The rate arising from the balance of trade is called the commercial par (real exchange).

The short rate of exchange will agree with the nominal par only as long as there is no balance of indebtedness between the two countries. If there is a balance of indebtedness the short exchanges will deviate from the par until the difference would cover the expense of shipping bullion. These limits of the



exchanges are called specie points, and when one or other is reached bullion begins to flow in or out.

The long rate of exchange is based upon the short rate, and is equal to the short exchange plus or minus the interest for the long-bill's time, and the cost of stamps.

In the case of long bills the specie points may be temporarily exceeded from other considerations, viz. the position of the acceptor, the need to realise etc.

4°. The course of exchange depends on the state of *direct* trade between two places.

Hence it may be advantageous to send money by a circuitous route.

The course of exchange is then said to be determined by the arbitration of exchange.

5°. The real exchange in London as centre is influenced by seven causes.

1. The balance of payments due or owing.
2. The state of foreign exchanges.
3. The state of the currency.
4. Remittances to London as commercial centre of the world to meet payments due to other countries.
5. Political security.
6. The state of the money market (esp. the comparative rates of discount).
7. Free or prohibitive tariffs.

6°. The calculation of the *pars* of exchange involves two difficulties,

(1) Coins are only approximately of the weight and fineness laid down in the Mint regulations, and there is wear and tear besides. This difficulty is got over by assuming them to be in accordance with the statements of the various Mints.

(2) Countries have different standards of value, some taking gold, others silver.

A fixed par of exchange only exists between countries using the same standard. If one use gold, the other silver, the par of exchange will vary in the ratio of the relative value of gold to silver; for when gold is the standard, silver is merchandise, and coined only as a subsidiary currency at an artificially high value, and when silver is the standard, gold is merchandise with a variable premium.

Since Germany adopted the gold standard in 1871, most of the chief states of Europe and America have tended towards the single gold valuation, established in England since 1816. This has caused silver to depreciate and disorganised the exchanges based upon the silver standard (i.e. exchanges between gold-using and silver-using countries). The relation between a gold- and a silver-using country is really one of barter, but the disadvantages of such a condition of things were formerly minimised by certain countries using a double valuation, i.e. using gold and silver at a fixed ratio. But when France largely abolished this valuation, this safety-valve was lost, and there have been rapid fluctuations in silver and the silver exchanges.

### 5. **The Foreign Exchanges.**

The foreign exchanges are the rates at which the money of one country is exchanged into that of others. The intrinsic value of £1 in French money is 25·22½ francs and is called the Mint Par of Exchange, but this is very different from the market value of £1 in francs. This value is constantly fluctuating up and down, and the study of exchanges involves the study of the causes of these and similar fluctuations.

If A owes 25,000 francs in Paris, he can transmit gold pounds and exchange it there for French money or he can buy francs in London and transmit them,

in either case there will be the cost of transmission, and it is obvious that a cheque or bill could be sent much more cheaply. Hence a debtor always tries to buy bills or cheques on the country or place in which his creditor resides. When they are plentiful or the demand is small, he can get these bills more cheaply than when they are scarce or the demand is brisk. In the former case he will not have to pay for cheques so much as the Mint par, but in the latter he will have to pay more. There are however limits to the fall and rise of exchanges, determined by the cost of transmitting bullion. It costs about 10 centimes per 25 francs to send gold to Paris; thus no man would buy cheques at 25·12 francs per £ because by transmitting gold he would get  $(25·22\frac{1}{2} - 0·10 =) 25·12\frac{1}{2}$  francs per £. In the same way there is a limit to the rise (in the case of Paris 25·32 $\frac{1}{2}$ ). The two limits are called "Specie Points." When the rate of exchange falls or rises to a specie point, transmission of bullion is imminent except in special cases and for temporary causes.

Another point of great importance is that the price in London of cheques on Paris, and the price in Paris of cheques on London, always tend to become identical as being in reality the price of the same thing, viz. a fixed quantity of gold. In practice of course these prices are constantly differing by a small number of centimes owing to the difference of information in the two cities—news reaching one hours before the other perhaps. As a result people speculate in exchanges like stocks. Another result is that the important question constantly arises whether it is better in paying debts to buy cheques or bills and remit them, or to get our agents to draw upon us and sell the cheques and bills in their own market. The principle which governs the answer is that whatever will cost us less in sterling, or get us more in foreign money, is to be preferred.

It is not so customary to deal in cheques (demand bills) as in bills payable at various dates. This introduces two new elements—the credit of the drawer and acceptor of the bill, and the discount due for present payment. If the credit of the parties is good, the difference of the long and short rates represents the discount due or owing for the time the long bill has to run.

It is to be observed that the short rates govern the bullion movements, the long rates fluctuating too much on the question of credit to be reliable guides.

There are two classes of exchanges quoted in London—(1) those in which countries give variable rates for the £ sterling, (2) those which receive from London variable rates in pence for fixed foreign coins. In the former case a low exchange is unfavourable to us, and a high one favourable, and vice versa for the latter case.

The chief points to be remembered are:—

(1) The prices of cheques and bills fluctuate according to the laws of supply and demand.

(2) These fluctuations in cheques (not long bills) cannot rise above or fall below the Mint par beyond the specie points determined by the cost of transmitting bullion.

(3) Theoretically the short exchanges between two centres tend to become identical, practically they differ slightly.

(4) Long bills involve the additional elements of the credit of the parties, and discount.

## 6. **Bills of Exchange.**

Legal definition.—A Bill of Exchange is a letter from a creditor to his debtor ordering him to pay (1st) a certain sum of money, (2nd) to a certain person, (3rd) at a certain event.

More practically.—A Bill of Exchange is a written order for the payment of a specified sum of money at a stated time.

Strictly a *draft* differs from a *bill* in being addressed to an agent and not to a debtor, but the term is frequently used for a bill, especially before acceptance.

The usual forms of Bills of Exchange are these :—

#### I. INLAND BILL.

£287. 15s. 8d.

London, May 10, 1873.

Three months after date pay to myself (or A. B.), or order, the sum of two hundred and eighty-seven pounds fifteen shillings and eight pence, value received.

WILLIAM SMITH.

To Mr ARTHUR HASSELL,  
Draper, 330 Strand, London.

#### II. FOREIGN BILL.

£394. 17s. 4d.

London, June 30, 1891.

Two months after date pay this our First of Exchange (Second and Third of the same tenor and date not paid) to Messrs Hill and Son, or order, three hundred and ninety-four pounds seventeen shillings and four pence, value received, and place the same to our account.

J. WEBSTER & Co.

Messrs DIMSDALE & Co.  
Quebec, Canada.

#### III. FOREIGN BILL.

No. 510. F18570-75.

Manchester, Ap. 3, 1890.

Fourteen days after date, pay this bill of Exchange to our order, the sum of eighteen thousand five hundred and seventy francs, seventy-five centimes, value received, which place to account as per advice.

H. SCHWANN.

To Messrs MERCIER FRÈRES,  
Lyons.

## IV. BILL FROM ABROAD.

Payable in London.

No. 1800. £480. 10s. 0d. (Date when due.)  
 Reichsbank, Hamburg,  
 July 1, 1891.

Ninety days after sight, of this our First of Exchange (Second and Third of same tenor and date being unpaid) pay to the order of Messrs Brown and Co. the sum of four hundred and eighty pounds ten shillings sterling, value received, which place to account of this bank as advised.

EMIL SCHMIDT, Manager.

To Messrs CHILD &amp; Co., London.

## V. FOREIGN BILL.

Drawn in sterling.

No. 612. £412. 6s. 8d. Nottingham,  
 Nov. 18, 1892.

Two months after date pay to the order of Messrs Garnier the sum of four hundred and twelve pounds, six shillings, and eight-pence sterling, exchange as per endorsement, value received, and place the same to account as advised.

T. &amp; H. WELLS &amp; Co.

To Messrs KOHN & Co., Paris.  
 (25.18½.)

Forms II. III. IV. are strict drafts, as they are addressed to agents.

The creditor who addresses the letter is called the *drawer*.

The debtor or agent to whom it is addressed is termed the *drawee*.

The person to whom it is to be paid is termed the *payee*.

If the creditor sells it before it is sent he is called the *seller* and he must *indorse* it before parting with it.

When it reaches the payee he must at once get the drawee to write his name across it with "*accepted*,"

he is then called the *acceptor*. The payee may then sell it after indorsing, and it may pass from hand to hand until it is due, each person parting with it indorsing it. The person actually possessing it at any time is called the *holder*.

Bills of Exchange are drawn at sight or on demand : others at periods after date or after sight, and some at usance.

If the bill is after sight, the acceptor must put on the date of his acceptance.

Banks sell Bills of Exchange on their agents or correspondents, to their customers.

*Days of Grace* are allowed, but these are reckoned in discounting.

**A Promissory Note** is an absolute promise to pay (1st) a certain sum of money, (2nd) to a certain person, (3rd) at a certain event.

Its usual form is :—

£143. 4s. 9d.

London, May 4th, 1875.

Three months after date I promise to pay John Stiles, or order, the sum of one hundred and forty-three pounds four shillings and ninepence for value received.

TIMOTHY GIBBONS.

The same principles govern their use and sale, but they are not so common as Bills of Exchange.

To be valid and regular, a Bill of Exchange must give the names of the drawer, drawee and payee, the sum of money for which, the place where and the day on which it is drawn, and the time it is intended to run.

A Bill of Exchange passes by indorsement of the *transferrer*.

This indorsement is *special* or *general*.

It is *special* when it is made payable to the order

of the transferee [Pay to the order of Messrs Findlay & Co., Willesden], and this order must be given by him when he parts with the bill.

(If the bill is originally payable to the order of the payee, he must give the order before parting with it by indorsement.)

The indorsement is called *general*, when the signature only of the transferrer is given [William Brown]. The former is safer, especially for transmission by post.

In foreign indorsements the place and date are often stated as well as the time of negotiation and whether for value received or only on account.

The drawer also commonly states whether the bill is to be accepted "with or without advice" and whether he too draws it on account or not, stating to what account it is to be carried.

Bills of Exchange are generally drawn in sets of two or more, any one of which being paid discharges the rest. They must be stamped, as a rule, to become legal.

The first of a set is usually sent forward unindorsed to a correspondent in the place upon which it is drawn, so that he may get it accepted and keep it till demanded by the holder of one of the other bills. The correspondent's address being put on the bill or bills in circulation, thus, "first with Messrs Reed & Co., to whom in case of need."

The words "in case of need" are added to show that the correspondent is to be applied to in the event of non-acceptance or non-payment to save the honour of the drawer and to prevent the expense of the bill's return through the intervening indorsers.

The endorsed second and accepted first are fastened together and form one bill.

Bills are drawn for various periods according to the usages of places, but more commonly the time is fixed with regard to the purposes of the bills.



Bills at sight are drawn for reimbursement on account of returned or dishonoured bills.

Sight drafts are also drawn for returns on bullion and Foreign Stocks.

A *foreign* bill is properly one payable in a foreign place in regard to the drawer or any subsequent holder. Thus a bill drawn upon Paris is always a foreign bill to London, wherever it was originally drawn.

But a bill drawn in Paris upon London is also sometimes called a foreign bill. Strictly it should be termed a bill *receivable* by the holder, or *payable* by the payer.

The amount of a foreign bill is usually given in the currency of the country where it is to be paid, but occasionally in the currency of the drawer's country.

Thus mercantile bills for shipments or for settlements, especially when remitted to London for negotiation, are frequently drawn in sterling.

But such bills generally bear the words "exchange as per indorsement" and are paid abroad at the rate of exchange notified in the indorsement.

Without such an indication, bills in sterling on foreign countries are paid at the Short Rate of Exchange on London.

### 7. Bills of Lading, etc.

*Invoice.* An account of goods sent by merchants to their correspondents at home or abroad in which the peculiar marks, the numbers, the value and contents of such packages are set forth: also the freight, insurance and other charges.

*Manifest.* A document signed by the master containing

(1) The names of places where the goods on board have been laden.

(2) The places for which they are destined.

- (3) The name and tonnage of the vessel.
  - (4) The name of the master and the name of place to which the vessel belongs.
  - (5) A particular account and description of all the packages on board with marks and numbers.
  - (6) The particulars of the goods in the packages or of any loose goods with the names of the respective shippers and consignees as far as known to the master.
- The manifest must be made out, dated and signed by the master, at the place or places where the goods or any part of them are taken on board.

*Bills of Lading.* An acknowledgment, signed usually by the master of a ship, certifying the receipt of goods on board, and engaging under certain exceptions to deliver the said goods safely at the port to which the ship is bound, either to the shipper or to such other person as he may signify by written assignment upon the bills of lading on the payment of the stipulated freight.

The exceptions are—the Act of God, the Queen's enemies, fire, all accidents and dangers of seas, rivers, and navigation—with many others.

The bills of lading determine the contents of the cargo of a ship.

A bill of lading differs from a charter-party in being evidence of and title to the particular goods *shipped for conveyance* under the contract made by the charter-party.

On delivery of the goods on board the master or mate signs a common receipt, which must be returned or cancelled on the bills of lading being handed over.

Three stamped bills are usually made out—one for the merchant or shipper—one (sent by post) for his agent or consignee—a third for the master as security and for guidance in delivering the goods.

If the goods inside the packages are unknown or the goods liable to deteriorate "contents unknown" or "liable to deterioration" is endorsed on the bill.

When the ship is hired by a charter-party the bills are delivered to the charterer, but in a "general" ship each person sending goods receives bills of lading for them.

Upon delivery of the goods at the port of destination to the shipper's factors or assignees the giving up of the bills of lading (sent by post to the factors) is a sufficient discharge, but the master may insist on a receipt.

Bills of lading are *transferable* either by blank or special endorsement like bills of exchange, and the master is bound to deliver the goods to the holder producing the endorsed bill who has acquired a legal right to it.

If several parties claim the goods, or if there is any doubt as to whom the goods should be delivered, or if no bill of lading is produced to him during the *lay* or *demurrage days*—the master should lodge his goods with a wharfinger so as to preserve his lien on the freight. In the last case he should apply for judicial authority to sell as much of the cargo as will pay the freight and charges.

*Charter-party.* A contract in writing, between the owner or master of a ship and the freighter, by which the former lets the ship or part of the ship under certain specified conditions for the conveyance of the goods of the freighter to some place or places. Generally, however, a charter-party is a contract for the use of the whole ship.

No precise form of words or set of stipulations is necessary.

Forms are made to suit special cases.

A charter-party specifies the nature of the voyage and the terms on which the cargo is carried, the

stanchness, full-complement, expeditiousness, being chiefly insisted on.

The vessel is to be ready on a certain day to receive the cargo.

A certain number of days called *lay days* are allowed for loading and discharging the cargo, and the freighter may detain the vessel for a further specified time on payment of a daily sum for *demurrage*.

If the vessel is retained beyond these two periods the freighter is liable to an action though the delay be unavoidable.

The lay days may be running days (every day) or working days (except Sundays and C.-H. holidays) or weather working days.

*Freight.* The sum paid by the merchant or other person hiring a ship or part of a ship for its use during a specified voyage or time.

The rate is usually fixed by the charter-party or bill of lading.

The master may retain the freight (at a wharfinger's) until freight and other charges are paid, but not for *demurrage*.

*Bottomry* is a pledge (as security) of the *ship itself* for the repayment of money advanced to the master or mate to enable him to make the voyage. If the ship is lost the lender loses his money, but at the end of the voyage the borrower and ship become liable for the money lent (with interest) however great.

*Respondentia* is a pledge (as security) of the *lading* for the same purpose exactly.

These two bonds have no settled or precise form.

### 8. Quotations and newspaper reports.

The extracts following are from the *Daily News*.

The first is a type of the daily quotations for rates of discount, loans and bullion operations.

The fine rates are for Bankers' bills of a good class.  
There is also evidence in the extract of the serious depreciation of the South American exchanges, and the extreme lowness of the price of silver.

Bank rate (discount) .....	2 $\frac{1}{2}$ p.c.
Market disct., 60 days' (fine) 1 p.c. (trade) —	p.c.
Do. 3 mnths. (fine) 1—1 $\frac{1}{8}$ p.c. (trade) 1 $\frac{1}{2}$	p.c.
Do. 4 mnths. (fine) 1 $\frac{1}{8}$ —1 $\frac{3}{8}$ p.c. (trade) 1 $\frac{3}{4}$	p.c.
Do. 6 mnths. (fine) 1 $\frac{7}{8}$ p.c. (trade) 2	p.c.
Loans to bill-brokers, day to day .....	1 p.c.
Do. do. for one week .....	1 p.c.
Rate for fortnight, loans to stockbrokers .....	3 to 3 $\frac{1}{2}$ p.c.
Deposit allowance (banks) .....	1 p.c.
Do. do. (discount houses) at call .....	$\frac{3}{4}$ p.c.
Do. do. do. at notice .....	1 p.c.

At the Bank to-day the sum of 25,000*l.* in bar gold was bought, and 9,000*l.* in gold was received from Bombay. The silver market has been quiet, and prices have further fallen to 39 $\frac{1}{4}$ d. per oz. for bars. The tone was fairly firm in the morning at this quotation, but a further drop in Eastern exchanges produced weakness at the close. Mexican dollars are quoted 38 $\frac{1}{4}$ d. per oz. A decline of  $\frac{1}{8}$ d. has taken place in the Brazilian exchange, 11 $\frac{1}{2}$ d. (Bank paper) being the last quotation.

The second gives the daily quotations at the various places as telegraphed to London for rates of exchange on London.

Below also are the London rates on 'Change for bills.

These latter are quoted on Tuesdays and Thursdays.

The final statement further shows the state of the South American credit.

The variable prices are alone given—the fixed prices being understood.

T.T.'s are Telegraphic Transfers.

*Example 1.* "Petersburg, 3 mos. 24 $\frac{1}{2}$  to 24 $\frac{1}{2}$ " denotes so many pence given for the rouble.

*Example 2.* "Copenhagen, 3 mos. 18·36 to 18·40" denotes so many kroner and öre per £.

EXCHANGES ON LONDON (telegraphed from)—Paris, 25.17; Berlin, eight days, 20.40½; Frankfort, 20.41; Vienna, 118.75; Hamburg, three months, 20.85; Amsterdam, sight, 12.08½; Bombay, T.T's, 1s. 3 1-32d.; Calcutta, T.T.'s, 1s. 3d.; Hong Kong, 2s. 9½d.; Shanghai, 8s. 10½d.; St Petersburg, three months, 97.90; Rio de Janeiro, 11½d. (bank); Portugal, 39½d.; Italy, 26.82; Spain, 29.20.

On 'Change bills were in small supply, and the tendency of the exchanges was against this country. French and Belgian remittances were in special request. Of the paper exchanges Spanish currencies improved in value. Cheques on Paris were dealt in at 25.16½ to 25.17½, and on Germany at 20.42 to 20.44.

Amsterdam and Rotterdam, 12.1½ to 12.2½; ditto, three months, 12.8½ to 12.8½; Antwerp and Brussels, three months, 25.38½ to 25.38½; Hamburg, 20.52 to 20.54; Berlin and German Bank Places, 20.52 to 20.54; Paris, cheques, 25.15 to 25.20; Paris, three months, 25.28½ to 25.35½; Marseilles, three months, 25.28½ to 25.33½; Austria, three months, 12.00 to 12.05; Petersburg, three months, 24½ to 24½; Moscow, three months, 24½ to 24½; Genoa, Leghorn, Naples, etc., three months, 26.58½ to 26.58½; Spain, three months, 40½ to 48½; Lisbon, three months, 39½ to 39½; Oporto, three months, 39½ to 39½; Copenhagen, three months, 18.86 to 18.40; Christiania, three months, 18.88 to 18.42; Stockholm, three months, 18.39 to 18.43.

BUENOS AYRES, APRIL 6. The premium on gold has advanced to 245 per cent.—*Reuter*.

In the calculation of direct bills of exchange (remittances or drafts) this second extract is of prime importance and should constantly be consulted.

The explanation of the exchanges on London will be found each under its country.

The London rates will be found under Great Britain (London course of exchange).

The other leading Daily Papers print similar lists, and most financial houses issue full lists of the kind on Tuesdays and Thursdays.

Through kind permission of Messrs S. Montagu and Co., the well-known bullion merchants, a portion of their bi-weekly list is inserted to show the prices of bullion and coins.

MESSRS S. MONTAGU & CO.  
60, OLD BROAD STREET, LONDON,  
December 8th, 1892.

PRICES CURRENT OF BULLION, &c.

Bar Gold	... .. per oz. standard	77/10 $\frac{3}{4}$
"	containing 1 oz. of Silver in the lb.	77/11 $\frac{1}{2}$
American Eagles	... .. per oz.	76/6
Napoleons	... .. per piece	15/10
New Russian Gold	... .. per oz.	76/7
Brazilian Gold Coin	... .. do.	77/8
Turkish Sovereigns	... .. do.	77/5 $\frac{1}{2}$
Spanish Doubloons	... .. do.	78/10
South American Doubloons	... .. do.	78/9
Bar Silver, fine	... .. per oz. standard	88 $\frac{1}{2}$
"	containing 5 grains Gold per lb.	88 $\frac{1}{2}$
5-Franc Pieces	... .. per piece	8/11 $\frac{1}{2}$
Mexican Dollars	... .. per oz.	} 37 $\frac{3}{16}$
"	New Coinage ... .. do.	
Bolivian, 400 Grain and Chilian Dollars	...	} —
"	500 Grain and Peruvian Sols	
Bolivian Half \$ base	... ..	—
Austrian Florins	... ..	—

## 9. Methods of Payment.

When a merchant has a debtor or creditor in a foreign place there are several ways in which the obligation between them may be met.

1. The creditor may enclose with his account or send shortly after it advice of his having drawn upon the debtor a bill of exchange payable at a certain date.

This bill of exchange or one of a set will be presented for acceptance to the debtor by the creditor's agent or by some creditor of the original creditor, i.e. by the payee named in the bill.

The creditor may have named himself or order as the payee, and in that case he will sell the bill in the place or country where he resides.

After the merchant has accepted the bill it may pass from holder to holder by sale until the date of

payment arrives when the then holder will present it for payment.

These bills if drawn in England when the drawer has to receive the amount in full and at once are drawn in £ sterling, with the words inserted "exchange as per endorsement."

The drawer then receives the full amount less brokerage, and in the country less banker's commission.

2. If no advice of the drawing of a bill upon him arrives speedily—the debtor will send a remittance (bill of exchange) or instruct his agent in the foreign place to draw upon him. He may obtain the bill of exchange (for remittance) through his bankers, who will draw upon their correspondent, and the debtor will send the bill or one of the set to the creditor, who will get the bank's agent to accept it—the creditor then may sell the bill, and it will pass as before till the time of currency expires when the last holder will be paid by the agent.

A debtor of the merchant may take the place of the bank's agent, the bill will be drawn upon him, and he must accept it.

Also the merchant may purchase a bill on the foreign place in the open market either on account or for the exact amount—this will be sent to the creditor in the same way as before and the drawee must accept it—the bill then passing in the usual way by sale.

If the merchant instructs his own or his banker's agent to draw upon himself—the agent will do so and offer the bill for sale in the foreign place—with the proceeds paying the creditor. In due time this bill will be presented for acceptance to the debtor, and finally it must be paid at the expiration of its currency.

3. Either of the above classes of ways may be adopted also by means of indirect bills, i.e. by buying



or selling bills on other places to be transmitted to or from the foreign place for the purpose of being sold or bought therein or in the country of the debtor.

4. A merchant may also pay his debt abroad by sending gold to gold-using countries, or silver to India, China, Japan, &c. He can also send international securities to be sold abroad.

### 10. **Exchange operations.**

These consist of direct exchanges, cross exchanges, and indirect or arbitrated exchanges.

Direct exchanges are operations between two countries based on their rates of exchange without the intervention of a third place.

These are naturally the most common operations between any two places.

Cross exchanges are the direct exchanges between two places considered with regard to a third place.

Indirect exchanges or arbitrations of exchange are operations between two countries through the medium of one or more countries, and the rates are spoken of as arbitrated rates of exchange. These rates are called simple or compound according as there is one place intervening or more than one and the operations are called simple or compound arbitrations. This may be expressed by saying that arbitrations of exchange are simple or compound according as they are based on one or more cross exchanges.

The distinction between simple and compound arbitrations is sometimes stated thus—A simple arbitrated rate is one calculated through a place intervening with direct paper, whereas a compound arbitrated rate is one calculated through a place intervening with indirect paper.

Compound arbitrations are also called circuitous when three or more places intervene—a very rare case.

The object of an indirect exchange is to obtain a better rate for remitting or drawing than the direct rates give, and the object of comparing several arbitrated rates is to find the best rate of all.

A foreign rate is one expressed in foreign money—a sterling rate is one expressed in sterling money.

A short price is the price of a bill at sight, or at short sight, which is generally 3 days' sight.

In speaking of operations with indirect paper they are said to be made *through* the intervening place. Two courses are open, (1) to remit direct paper to the intervening place with instructions to forward the proceeds to the final place, or (2) to remit direct paper to the intervening place and send instructions to the correspondent at the final place to draw upon the intervening place. The same rate is produced but the second is the less complicated mode, and it is also attended with fewer charges.

### 11. Direct Remittances.

The most natural and therefore the most usual way of paying any foreign debt is to obtain a bill of exchange from a banker or in the market, and directly remit it to the creditor abroad.

Such bills are frequently drawn in the currency of the foreign country, but if in sterling they must bear the words "exchange as per endorsement"—being endorsed with the current rate of exchange of the date of the bill.

The cost of such a bill is easily calculated.

*Example.* 18752·64 fl. (Austrians) due in Vienna at 12·07, 3 mos., rate of discount 2 p.c.

18752·64 fl. at 12·07 3 mos. will cost	£1553·657
Int. at 2 p.c. 3 mos.	7·768
	£1561·425
Brokerage 1 p.m.	1·561
Stamp $\frac{1}{2}$ p.m.	·781
	£1563·767

### EXAMPLES.

(Brokerage 1 p.m., Bill-stamp  $\frac{1}{2}$  p.m.)

1. 15000 francs at 25·21 $\frac{1}{4}$ .
2. 7865 marks at 20·53, 3 mos., discount 2 p.c.
3. 8645 Dutch florins at 12·3 $\frac{3}{8}$ , 3 mos., discount 3 p.c.
4. 25000 rupees at 1s. 3 $\frac{1}{2}$ d., 4 mos., discount 2 $\frac{1}{2}$  p.c.
5. 3785·6 roubles at 24 $\frac{1}{8}$ , 3 mos., discount 3 $\frac{1}{4}$  p.c.

Further remittances are given under each country.

### 12. Direct Drafts.

Another method of payment almost as common as that of direct remittances is for the creditor to draw a bill of exchange upon the debtor—sending one of the set to him for acceptance and negotiating the other at once.

Such bills are sometimes in the currency of the creditor's country and thus often bear the words "exchange as per endorsement," especially in the case of bills drawn in £ sterling.

The value of such a bill is easily calculated, as also the cost to the debtor.

*Example.* £824. 7s. 10d., 3 mos., Discount 2 $\frac{1}{2}$  p.c. Endorsed 25·18.

Value in sterling	= £824·391
Disct. 3 mos., 2 $\frac{1}{2}$ p.c.	5·152
	£819·239
Brokerage (1 p.m.)	£0·819
Bill-stamp ( $\frac{1}{2}$ p.m.)	£0·409
	1·228
	£818·011
Cost in francs = £824·391 $\times$ 25·18	
	= 20758·17 francs.

### EXAMPLES.

(Brokerage 1 p.m., Bill-stamp  $\frac{1}{2}$  p.m.)

1. £712. 18s. 4d., 3 mos., 2 $\frac{1}{4}$  p.c., 25·19 $\frac{1}{2}$  (France).
2. £320. 7s. 8d., 3 wks., 4 $\frac{1}{2}$  p.c., 20·40 (Germany).
3. £1000, 3 mos., 3 $\frac{1}{2}$  p.c., 97·60 (Russia).
4. £415. 12s. 8d., 60 days, 3 p.c., 4·87 $\frac{1}{4}$  (U. S.).
5. £1826. 9s. 7d., 4 mos., 2 $\frac{1}{2}$  p.c., 3/10 $\frac{1}{8}$  (China).

Further drafts are given under each country.

**13. Exchanges and Multiples.**

The method of multiples lends itself specially well to the calculation of exchanges, as decimals are always employed.

The variations in the quoted exchanges are within such narrow limits that it is a very simple matter to construct tables which may cover all the rates arising for each country.

*Example of a foreign rate.*

20·38½ (Germany).

M.	
1 = £	·049055697817
2	·098111395634
3	·147167093451
4	·196222791268
5	·245278489085
6	·294334186902
7	·343389884719
8	·392445582536
9	·441501280353

*Example of a sterling rate.*

40½ (Portugal).

Mr.	
1 = £	·1671875
2	·3343750
3	·5015625
4	·6687500
5	·8359375
6	1·0031250
7	1·1703125
8	1·3375000
9	1·5046875

*Ex. Find cost of 8654·48 marks.*

£147·167
29·433
2·453
·196
·020
·004
£179·273

B. 1 p.m. ·179  
S. ½ p.m. ·089

£179·541 (10s. 10d.)

*Ex. Find cost of 7329 milreis.*

£1170·318
50·156
3·344
1·505
£1225·318

B. 1 p.m. 1·225  
S. ½ p.m. ·613

£1227·156 (3s. 1d.)

**EXAMPLES.**

Construct the multiples corresponding to the following rates :

- (1) 25·29½, 3 mos., (France), (2) 20·48, 3 mos., (Berlin),  
(3) 12·1½, (Amsterdam), (4) 11·98, (Vienna),  
(5) 39½, (Spain), (6) 41½, (Portugal), 3 mos.,

and use them to obtain cost of bills for 8560 francs, 12150 marks, 3575·30 Dutch florins, 4560 Aust. florins, 3560 pesetas, 8000 milreis, respectively.

Discount is to be taken at 4 p.c. where necessary.

#### 14. Comparison of Direct Rates.

Between any two places there are in general always two distinct rates.

Thus between London and Paris there is the London rate on Paris and the Paris rate on London. These may be short or long rates: if short they are always tending to equality, but if they are long rates their difference increases with the time and the rate of discount. Also the two rates differ frequently in the method of quotation, and therefore in order to compare them a calculation has to be made.

The necessity for comparing the two direct rates between two places is constantly arising in business. For the payment of any debt the question whether it will be cheaper to remit bills or to instruct one's agent abroad to draw upon us is always of importance.

##### Rules and Principles.

1°. Reduce the direct rates to short prices (if necessary) by adding or subtracting interest for the times of the quotations at current rate of discount allowed at each place—afterwards changing them so as to be quotations of the same kind.

There are two reasons for this. (1) The interest for the time a bill has to run affects the price in direct operations—acting in opposite ways upon the two rates when of the same kind or in the same way when of opposite kinds. (2) In indirect operations the arbitrated rates are for present money, as bills are never sold for credit.

It will be seen how closely direct and indirect rates are connected as we proceed.

The interest is added or subtracted in accordance with these laws:—

(1) If *A* have a foreign rate and *B* a sterling rate—the interest must be taken from *A*'s rate and added to *B*'s.

(2) If *A* have a sterling rate and *B* a foreign rate—the interest must be added to *A*'s rate and taken from *B*'s.

(3) If *A* and *B* have both sterling rates—the interest must be added to both rates.

Ex. (1) London on Paris, 3 mos.,  $25\cdot55$  } Rate 4 p.c.  
           Paris on London, 3 mos.,  $25\cdot10$  }  
 for both places,  $\therefore$  interest for 3 mos. = 25 c.  
 Thus London short rate =  $25\cdot30$ . Paris short rate =  $25\cdot35$ .

Ex. (2) London on Lisbon, 3 mos.,  $52\frac{1}{4}$  } Rate 5 p.c.  
           Lisbon on London, 3 mos.,  $53\frac{1}{4}$  }  
 in Lisbon, 3 p.c. in London,  $\therefore$  interest on 50 at 5 p.c. =  $\frac{5}{8}$ , at 3 p.c. =  $\frac{3}{8}$ .  
 Thus London short rate =  $52\frac{7}{8}$  and Lisbon short rate =  $52\frac{7}{8}$ .

Ex. (3) London on Petersburg, 3 mos.,  $24\frac{3}{16}$  } Rate  
           Petersburg on London, 3 mos.,  $97\cdot90$  }  
 4 p.c. in each place. Interest on  $24\cdot1875 = 24\cdot1875$   
 and on  $97\cdot90 = 9790$ .

Thus London short rate =  $24\cdot429$  and Petersburg short rate =  $98\cdot879$ .

The principle underlying these laws is that a bill sent to Paris now would have interest deducted for the time it has to run, while a bill drawn on London would have interest added for the same time—and *vice versa* for London and Lisbon. In the case of London and S. Petersburg both such bills would have the interest added.

2°. Compare the short rates so obtained and—after observing whether the operation is in reality a remittance or a return and whether the rate is foreign or sterling—decide whether a remittance or draft is to be preferred.

The following laws will be a guide to the operator:—

(1) For remittances from *A* to *B*.

(a) If *A* has a higher foreign rate or lower sterling rate than *B*—*A* should remit to *B*.

(b) If *A* has a lower foreign rate or higher sterling rate than *B*—*B* should draw upon *A*.

(2) For returns from *B* to *A*.

(a) If *A* has a higher foreign rate or lower sterling rate than *B*—*B* should remit to *A*.

(b) If *A* has a lower foreign rate or higher sterling rate than *B*—*A* should draw upon *B*.

The principle underlying these rules is that for remittances abroad a high foreign rate or low sterling rate requires less money from the payer, and for returns to this country a low foreign rate or high sterling rate will produce more money for the payee.

3°. Find the actual cost of the remittance or draft, making allowance for brokerage, stamps, interest or discount, and commission if any.

These charges should be studied under each country.

Multiples may be constructed for frequently-occurring rates.

The methods of approximation apply.

*Example.* A debt of 20000 roubles is owed by a London merchant to one in S. Petersburg. The London rate is 3 mos.  $24\frac{1}{8}$ , the S. Petersburg rate is 3 mos. 97·90. The rate of discount for each place is 4 p.c.; find whether he should remit or instruct his agent to draw and show the difference.

(1) Reduction to short rates of the same kind.

London.		S. Petersburg.	
	24·1875		9·79
Int. (1 p.c.)	·241875	Int. (1 p.c.)	·0979
	<u>24·429375</u>		<u>9·8879</u>
9·8879   240	24·272 pence per rouble.		
	42242		
	2690		
	<u>713</u>		
	21		

## (2) Comparison.

London 24.429375 }  $\therefore$  S. Petersburg should draw at  
 S. Petersburg 24.272 } 9.79 3 mos. per £.

## (3) Routine of the difference in paying on account.

## (a) Remittance.

£2000 invested in Russian bills at  $24\frac{1}{8}$  3 mos. will buy 19844.96 Ro.  
 Disc't. at 1 p.c. (3 mos. at 4 p.c.) 198.45  
19646.51  
 Less brokerage at 1 p.m. 19.65  
 Stamp at  $\frac{1}{2}$  p.m. 9.82 29.47  
19617.04 Ro.

## (b) Draft.

£2000 draft on London offered in S. Petersburg at  
 97.90 3 mos. will produce 19580.00 Ro.  
 Interest at 1 p.c. 195.80  
19775.80  
 Less brokerage at 1 p.m. 19.78  
 Stamp  $\frac{1}{2}$  p.m. 9.89 29.67  
19746.13 Ro.  
 Less commission  $\frac{1}{8}$  p.c. 24.68  
19721.45 Ro.

*Note.* Unless the debtor has an agent in the foreign place or unless he banks with a bank which has a correspondent he would find a difficulty in getting a draft upon himself. Banks have a large network of agents for this purpose. Large houses of bill-brokers also have correspondents or branches all over Europe. Further, if two firms have frequent dealings with each other, there is no difficulty in paying an account or getting drafts.

Isolated debts are usually settled by direct remittance or draft.

London merchants have greater facilities for buying bills on 'Change than those in the country. The purchase of bills lends itself best to payments on account, but occasionally the exact bills required may be bought.

## (4) Routine of the exact difference.

## (a) Cost of a bill for 20000 roubles.

20000 roubles at 3 mos.  $24\frac{1}{8}$  will cost £2015.625  
 Interest at 1 p.c. 20.156  
2035.781  
 Plus brokerage at 1 p.m. 2.036  
 Stamp at  $\frac{1}{2}$  p.m. 1.018  
£2038.835



(b) Draft required for 20000 roubles.

Preliminary Determination — given rate 9·79 3 mos.

Discount 4 p.c. Brokerage 1 p.m. Stamp  $\frac{1}{2}$  p.m. Com.  
 $\frac{1}{4}$  p.c. Draft required

$$= \frac{2000}{9 \cdot 79 \times 1 \cdot 01 \times \cdot 99725} = £2028 \cdot 2519$$

£2028·2519 offered in S.P. at 9·79 3 mos. will produce 19856·586 Ro.  
 Interest 1 p.c. 198·566

20055·152

Brokerage 1 p.m. 20·055

Stamp  $\frac{1}{2}$  p.m. 10·028

Commission  $\frac{1}{4}$  p.c. 25·069

55·152

20000·00 Ro.

Similarly the comparison of any other pair of rates is made.

If a rate is quoted short it will require no change.

If both are short and of the same kind—no calculation is required and it will be obvious on inspection whether we should remit or draw.

If one rate is long and the other short but of a different kind the simpler plan is to reduce the long rate to short and convert the other into the same kind.

e.g. London—Paris. Cheques 25·18—25·17 $\frac{1}{2}$ . London should remit.

London—Berlin. London, 3 mos. 20·53. Berlin, 8 days, 20—40. (Reduce London rate to short.)

London—Russia. London, 24 $\frac{3}{16}$ , 3 mos. Russia, 97·80 short. (Reduce London rate to short and convert Russian.)

### EXAMPLES.

(Consult *Daily News* extract, pp. 230—32.)

*Notes.* If one rate is given, it is rate of remitting place.

If two rates are given, they are rates of first upon second, and second upon first place.

The rates of discount also are in same order as the places.

The bills also are debts in the country of the given currency.

Brokerage, commission, stamps, postage, must always be considered. The rates will be found under each country. The interest (discount) rates are generally the bank rates, where payable, and of course vary from time to time. Some places reckon at the market rate of discount.

Decide on the advantage of remitting or drawing, and find minimum cost of payment.

1. London—Paris, 25000 francs, rates, 3 mos. 25·19, 3 mos. 25·15½, discount 3 p.c., 3½ p.c.

2. London—Amsterdam, £865. 12s. 6d., rates, 3 mos. 12·3½, 3 mos. 12·08½, discount 2 p.c., 2½ p.c.

3. London—Berlin, 8375 marks, rate, 8 days 20·40½.

4. London—Paris, £567. 9s. 7d., rate, cheques 25·17.

5. London—Calcutta, 15000 rupees, rate, T.T.'s 1s. 3½d.

6. London—Lisbon, 23156·2 milreis, rates, 3 mos. 39½, 3 mos. 39½, discount, 3 p.c., 2½ p.c.

7. London—Vienna, £641. 17s. 6d., rates, 3 mos. 12·04, 3 mos. 118·75, discount, 1½ p.c., 3 p.c.

8. London—Petersburg, £7000, rates, 3 mos. 24½, 3 mos. 99·65, discount, 3½ p.c., 2 p.c.

9. London—Genoa, 12516 lire, rates, 3 mos. 26·53½, 3 mos. 26·32½, discount, 2 p.c., 3 p.c.

10. London—Shanghai, £864. 13s. 4d., rate, 6 mos. 3s. 10½d., discount, 3 p.c.

Further direct exchanges are given under each country.

### 15. Indirect or Arbitrated Exchanges.

It is at times advantageous to pay a debt or to receive a remittance in an indirect way, e.g. by purchasing bills on other places and sending them for sale in the place where a debt is owing.

A merchant can only purchase such bills as are in the market and therefore most firms have agents or correspondents abroad to whom they send bills on account—the agents actually paying their debts. Bankers, for this reason amongst others, have a large number of

agents in different places on whom they draw for the convenience of their customers.

Rules and principles.

1°. Find the arbitrated rates of the indirect exchanges which seem likely to give better rates than the more advantageous of the direct rates (found as above)—using the chain rule and adding in or subtracting the extra brokerages, commissions, bill stamps, and postage involved.

Bills at short dates are seldom used for any but direct operations.

Bills at long dates are specially suitable for arbitrations because there is time to wait for an improvement of the rate of exchange.

No reductions are necessary for time as in the case of direct rates because all the operations are for present money.

For instance, if I buy Hamburg bills at 3 mos.—from the price would be deducted the interest for 3 mos. at the Hamburg rate—I should send them at once to Paris (say) to be sold and their interest for 3 mos. at the Hamburg rate would be deducted—thus what I gained in London I should lose in Paris, and the ultimate arbitrated rate would be unaffected by the time.

Simple and compound arbitrated rates are equally unaffected by the time—both are determined by the chain rule.

The charges involved in these indirect operations (brokerages, stamps, agents' commission, postage) seriously restrict the profit arising from them—in fact indirect exchanges even of the simple kind are only used to a large extent by banks or houses with branches in different places or for joint account.

Compound and circuitous arbitrations especially require a large network of agents to make them possible or profitable.

The usual charges are  $\frac{1}{8}$ — $\frac{1}{10}$  p.c. for brokerage :  $\frac{1}{10}$ — $\frac{1}{20}$  p.c. for stamps :  $\frac{1}{4}$ ,  $\frac{1}{8}$  or  $\frac{1}{16}$  p.c. for commission as agreed.

Whether these charges are to be added or subtracted will be clear on considering their effect in increasing the price or in diminishing the return.

2°. From a comparison of the direct short rates and the arbitrated rates (after observing whether the operation is in reality a remittance or a return and whether the rate is foreign or sterling)—decide whether any arbitrated rate is more advantageous than the more favourable direct-rate.

The following rules will be a guide to the operator :

(1) For remittances from *A* to *B* or drafts upon *A* by *B*.

(a) If *A* has a foreign rate any greater arbitrated rate will be better than the direct rate.

(b) If *A* has a sterling rate any less arbitrated rate will be better than the direct rate.

(2) For returns to *A* from *B*, or drafts by *A* upon *B*.

(a) If *A* has a foreign rate any less arbitrated rate will be better than the direct rate.

(b) If *A* has a sterling rate any greater arbitrated rate will be better than the direct rate.

The principle underlying these rules is that for remittances the highest foreign rate or lowest sterling rate requires least money from the payer, and for returns the least foreign rate or highest sterling rate will produce most money for the payee.

3°. With the best arbitrated rate, go through the routine necessary to obtain the value of any bill of exchange required with due regard to interest, broker-

ages, commission, stamps, postage, etc.—employing tables or actual calculation for the various steps.

Multiples may be constructed for frequently-occurring rates.

The methods of approximation apply.

*Example.* Find arbitrated rates between London and Paris from the formulæ.

Amsterdam.  
 $? = 1\text{£}.$   
 $1 = 12\cdot1 \text{ Flor.}$   
 $100 = 210\frac{1}{4} \text{ Francs.}$

Madrid.  
 $? = 240d.$   
 $49\frac{1}{2} = 1 \text{ Duro.}$   
 $1 = 5\cdot15 \text{ Francs.}$

Frankfort.  
 $? = 1\text{£}.$   
 $1 = 20\cdot60 \text{ M.}$   
 $100 = 123 \text{ Francs.}$

Vienna.  
 $? = 1\text{£}.$   
 $1 = 12 \text{ Flor.}$   
 $100 = 212 \text{ Francs.}$

The rates for direct bills being at London 3 mos. 25·60, at Paris 3 mos. 25·12 $\frac{1}{2}$ , and discount rate at both places being 4 p.c.

Decide on the relative advantages of the direct rates and the arbitrated rates for remittances and returns.

- (1) To reduce direct rates to short prices.

3 mos. at 4 p.c. = 1 p.c.

$$\begin{array}{r} 25\cdot60 \\ \cdot 256 \\ \hline \text{London } 25\cdot344 \end{array}$$

$$\begin{array}{r} 25\cdot12\frac{1}{2} \\ 2512\frac{1}{2} \\ \hline \text{Paris } 25\cdot376 \end{array}$$

- (2) Arbitrated rates.

Amsterdam.	Frankfort.
12·05	20·60
2·1025	1·23
24·10	20·60
1·205	4·12
2410	·618
6025	25·338
25·335125	25338
1 p. mille extra brokerage	25·312
25·319	

	Vienna.		Madrid.	
	12·00		240	
	<u>2·12</u>		5·15	
	24·00		1200	
	1·2		24·0	
	<u>·24</u>		12·00	
	25·44	4,9·5	1236	24·969
1 p. mille extra brokerage	2544		<u>2460</u>	<u>24969</u>
	25·415		4800	24·993
			<u>345</u>	
			48	
			<u>4</u>	

## (3) Results of comparison.

For direct remittances Paris should draw—difference about  $\frac{1}{4}$  p.c.For direct returns London should draw—difference about  $\frac{1}{4}$  p.c.For indirect remittances through Vienna—advantage over direct rate (Paris drawing)  $\frac{5}{8}$  p.c.For indirect returns through Madrid—advantage over direct rate (London drawing)  $1\frac{1}{2}$  p.c.

Direct remittance or return.

25·376 P. on L.

25·344 L. on P.

32

Indirect remittance.

25·415 Vienna.

25·376 Paris.

4

3·2 cents on 2500 =  $\frac{1}{4}$  p.c.4 cents on 2500 =  $\frac{1}{4}$  p.c. or  $\frac{1}{4}$  p.c.

Indirect return.

25·344 London.

24·993 Madrid.

35

35 cents on 2500 =  $1\frac{1}{2}$  p.c.

(4) The routine of the work is shown thus for payments on account—(indirect operations are almost always through agents).

A. To pay a debt in Paris (**Remittance**).

## (1) By direct remittance.

£1000 invested in buying Paris 3 mos. bills at

25·60 will purchase

25600·00 Fr.

Disc't. for 3 mos. at 4 p.c.

256·00

25344·00 Fr.

Brokerage at 1 p.m.

25·34

French stamp ( $\frac{1}{2}$  p.m.)12·67

38·01

25305·99 Fr.

(2) By direct draft on London.

£1000 draft offered in Paris at 25-12½	3 mos.	
	would produce	25125-00 Fr.
Int. for 3 mos. at 4 p.c.		251-25
		<u>25376-25 Fr.</u>
Brokerage 1 p.m.	25-38	
English stamp (½ p.m.)	12-69	38-07
		<u>25338-18 Fr.</u>

(3) By indirect remittance through Vienna.

£1000 invested in Vienna bills at 12 will purchase	12000·00 Fl.
Brokerage 1 p.m.	12·00
	<u>11988·00 Fl.</u>
11988 Fl. sold in Paris at 212 will produce	25414·56 Fr.
Brokerage 1 p.m.	25·41
Commission $\frac{1}{2}$ p.c.	31·77
(Vienna bill) French stamp ( $\frac{1}{4}$ p.m.)	6·35
	<u>63·53</u>
	25351·03 Fr.

**B. To receive a debt from Paris (Return).**

(1) By direct remittance.

25000 Fr. invested in 3 mos. London bills at 25·12½	
will buy	£995·025
3 mos. interest at 4 p.c.	<u>9·950</u>
	£985·075
Brokerage 1 p.m.	<u>·995</u>
	£984·070
English stamp (½ p.m.)	<u>·498</u>
	£983·572

(2) By direct draft on Paris.

25000 Fr. draft offered in London at 25·60	3 mos.
	would purchase £976·563
3 mos. interest at 4 p.c.	9·766
	<hr/> £986·329
Brokerage 1 p.m.	·977
	<hr/> £985·352
French stamp ( $\frac{1}{2}$ p.m.)	·488
	<hr/> <u>£984·864</u>

(3) By indirect return through Madrid.

25000 Fr. invested in Madrid bills at 5·15	
would purchase	4854·37 Piastres.
Brokerage 1 p.m.	4·85
	<hr/> 4849·52 Piastres.

4849·52 Spanish Dollars sold in London at 49½	
will purchase	£1000·213
Brokerage 1 p.m.	1·000
Commission ¼ p.c.	1·250
(Madrid bill) English stamp (½ p.m.)	·500
	<hr/> 2·750
	<hr/> £997·463

Examples for practice are given under the principal countries.

The above Example is proposed for solution in the Modern Cambist but the method adopted here is slightly more rigorous than in the example actually worked out in that valuable work.

## 16. Compound Arbitrations.

The rate of exchange between two places produced by remittances through an intervening place in any other than direct paper of that place is called a compound arbitrated rate.

These compound arbitrations are called circuitous when more than three places intervene or when the proceeds of the bills return to the original place after passing through two or more places.

The increased charges are the great restrictions upon these operations, and even compound arbitrations (much less circuitous operations) are rarely used.

The chain rule solves the question as before, but great attention to the various charges is necessary.



*Example 1.* I buy in London a sight bill upon Amsterdam at 12·2, sending it to Amsterdam, where the proceeds buy 3 mos. Hamburg bills at 58·75. These are forwarded to Paris and sold at 123. What is the rate between London and Paris?

	? = 1£.
	1 = 12·10 Florins short.
	58·75 = 100 Marks 3 mos.
	100 = 123 Francs.
	<hr/> = 25·33 Francs.
3 brokerages (1 per mille)	7599
2 commission ( $\frac{1}{2}$ p.c.)	·633
Dutch stamp ( $\frac{1}{10}$ p.m.)	1·7731
French stamp ( $\frac{1}{2}$ p.m.)	0633
	<hr/> ·163351
	<hr/> 25·166

*Example 2.* I buy in London a 3 mos. Hamburg bill at 20·50, and it is sold in Amsterdam at 58·75. The proceeds are used to buy 3 days' sight bills on Genoa at 42·25 Florins per 100 Lire, and these are sold in Paris at 11 p.c. discount—find rate between London and Paris.

	? = 1£.
	1 = 20·50 Marks 3 mos.
	100 = 58·75 Florins.
	42·25 = 100 Lire short.
	100 = 89 Francs.
	<hr/> = 25·37
4 brokerages (1 per mille)	·10148
4 commissions ( $\frac{1}{2}$ p.c.)	·12685
Dutch stamp ( $\frac{1}{10}$ p.m.) on Hamburg bill	·00888
French stamp ( $\frac{1}{2}$ p.m.) on Genoa bill	·00634
	<hr/> ·24355
	<hr/> 25·13

Examples are given under certain countries.

The rates must always be corrected for brokerages, commissions, stamps—strictly also postage cannot be neglected.

*Example 3.* Simple Circuit.

I buy £1000's worth of Hamburg bills at 20·50, and send them to be sold in Amsterdam at 59 $\frac{1}{2}$ . In return I receive bills of 2 mos' date at 11·90. Is the operation profitable if the time is 67 days and bank interest available 5 p.c.?

Commission  $\frac{1}{2}$  p.c. London brokerage 1 p.m. Amsterdam  $\frac{2}{3}$  p.m.  
 Dutch stamp  $\frac{7}{10}$  p.m. English stamp  $\frac{1}{2}$  p.m.

£1000 will buy bills on Hamburg at 20·50 to amount of 20500·00 M.  
 Less brokerage 1 p.m. 20·50  
 20479·50 M.

20479·50 M. will sell in Amsterdam at 56 $\frac{1}{2}$  for 12185·30 Fl.  
 Less commission  $\frac{1}{2}$  p.c. 15·23  
 brokerage  $\frac{2}{3}$  p.m. 9·15  
 (Hamburg bill) Dutch stamp  $\frac{7}{10}$  p.m. 4·26 28·64 Fl.  
 12156·66 Fl.

12156·66 Fl. will buy bills on London at 11·90 to amount of 1022·569 £  
 Less commission  $\frac{1}{2}$  p.c. 1·278  
 brokerage  $\frac{2}{3}$  p.m. 0·767 2·045  
 1020·524 £

These bills will be cashed in London for £1020·524  
 Less brokerage 1 p.m. = 1·021  
 Discount 2 mos. 3 p.c. = 5·108  
 English stamp  $\frac{1}{2}$  p.m. = ·510 6·634  
 £1013·890

Now interest on £1000 for 67 days at 5 p.c. = £9·178.  
 Hence profit = £1013·890  
 £1009·178  
 £4·712 (14/3)

### EXAMPLES.

(Consult the countries for the charges, Section X.)

1. Bought in London, £1000 Paris bills, 3 mos. 25·32 $\frac{1}{2}$ , transmitted to Hamburg to be sold at 83. In return, bills of 3 mos. date at 20·35 are sent. What is the profit if time is 40 days and bank interest be 2 $\frac{1}{2}$  p.c. ? Discount 3 p.c.

2. Bought in Paris, 20000 francs London bills, 3 mos. 25·13, transmitted to Amsterdam to be sold at 12·06. In return, bills of 2 mos. date at 47·61 are sent to Paris. What is profit if time is 20 days and rate of interest available 3 p.c. ? Discount 1 p.c.

3. Bought in Berlin 25000 marks Russian bills, 3 mos. 217, transmitted to London to be sold at 24 $\frac{1}{2}$ . In return, bills of 3 mos. at 20·52 $\frac{1}{2}$  are sent to Berlin. What is profit if time is 50 days and rate of interest available 4 p.c. ? Discount 2 p.c.

*Example 4. Circuitous Arbitration.*

I buy 3 mos. bills upon Hamburg at 20·50 and sell them in Amsterdam at 58·75. The proceeds are invested in 3 days' sight bills upon Genoa at 42 to be sent to Paris and sold at 11 p.c. discount—the proceeds being laid out in buying 3 mos. bills upon Madrid at 5 Francs per Peso. These bills are sent to me in London and sold here at 48*d.* per Peso. What gain or loss p.c. arises? (Tate.)

? = 100£.  
 1 = 20·50 M. 3 mos.  
 100 = 58·75 Amst. Fl.  
 42 = 100 Lire short.  
 100 = 89 Francs.  
 5 = 1 Piastre (3 mos.).  
 1 = 48*d.*  
 240 = £1.  
£102·085

6 brokerages (1 p.m.)	£·613	
4 commissions ( $\frac{1}{2}$ p.c.)	£·510	
Int. 3 weeks 4 p.c.	£·236	
Eng. stamp on Madrid bill ( $\frac{1}{2}$ p.m.)	£·051	
French stamp on Genoa bill ( $\frac{1}{2}$ p.m.)	£·026	
Dutch stamp on Hamburg bill ( $\frac{7}{10}$ p.m.)	£·036	£ 1·472
		<u>£100·613</u>

∴ Net profit is about  $\frac{1}{2}$  p.c. with postage deducted.

The routine upon £1000 is shown thus:—

£1000 will buy bills upon Hamburg at 20·50 to	amount of	20500·00 M.
Brokerage 1 per mille	20·50	
		<u>20479·50 M.</u>
20479·50 M. will sell in Amsterdam at 58·75 for		12031·71 Fl.
Commission $\frac{1}{2}$ p.c.	15·04	
Brokerage 1 p.m.	12·03	
(Hamburg bill) Dutch stamp $\frac{7}{10}$ p.m.	4·21	31·28
		<u>12000·43 Fl.</u>
12000·43 Fl. will buy bills on Genoa at 42 to	amount of	28572·45 Lire.
Commission $\frac{1}{2}$ p.c.	35·72	
Brokerage 1 p.m.	28·57	64·29
		<u>28508·16 Lire.</u>

28508·16 Lire will sell in Paris at 11 p.c. disct. for	25372·26 Francs.
Commission $\frac{1}{2}$ p.c.	31·72
Brokerage 1 p.m.	25·37
(Genoa bill) French stamp $\frac{1}{2}$ p.m.	6·34
	63·43
	<u>25308·83 Francs.</u>
25308·83 Francs will buy bills on Madrid at 500	
to amount of	5061·76 Dollars.
Commission $\frac{1}{2}$ p.c.	6·33
Brokerage 1 p.m.	5·06
	11·39
	<u>5050·37 Dollars.</u>
5050·37 Dollars will sell in London at 48d. per	
dollar for	1010·074 £.
Brokerage 1 p.m.	1·010
(Madrid bill) English stamp $\frac{1}{2}$ p.m.	·505
	1·515
Int. on £1000 for 3 weeks at 4 p.c.	1008·559 £.
	2·308
	<u>1006·251 £.</u>
Postage at 1 per mille	1·006
	<u>1005·245 £.</u>
∴ Net profit = $5\frac{1}{2}$ per mille or $\frac{1}{2}$ p.c.	

## 17. Banking and Exchanges.

### 1°. Bank-notes.

Bank of England notes are legal tender so long as the Bank pay them in gold, but no change need be given.

Bank of England notes are not legal tender at the Bank or its branches, where sovereigns may be demanded.

Private Bank-notes are not legal tender—in case of failure of the issuer the receiver cannot recover—in case of forgery the receiver can recover.

Foreign Bank-notes should be sold through a broker as foreign bills.

The “stopping” of bank-notes does not give a bank the right to refuse payment to the presenter if his replies (as to how he got possession of the note) are satisfactory. They can only inform the loser of the circumstances under which it was paid.

## 2°. Cheques.

A cheque is an order *on a banker* for the payment on demand of a stated sum of sterling money.

Endorsement signatures are necessary on the back of all cheques payable to order but not to bearer.

Every alteration in a cheque should be initialled.

No titles of courtesy should be used in endorsements—a cheque endorsed “Mr J. Smith” would be refused payment.

Dividend warrants must always be signed by the actual person to whom they are payable.

Procuration endorsements are usually accepted by banks (there is no legal compulsion) in the form “per pro H. Smith, partner” [or secretary, etc.]. The signer must put his full ordinary signature, not his mere initials.

All cheques, except those of poor law guardians and Government departments, require to be stamped.

“Crossed cheques” are not recognised abroad.

They are cheques crossed on the face by two parallel transverse lines with or without the words “and Co.,” “Not Negotiable,” “Under—pounds,” or the name of a banker with no lines.

The effect of the crossing is to limit the cashing of the cheque to a particular banker or through a bank at any rate.

To cancel a crossing the drawer (alone) must write across the cheque “Pay Cash” with signature.

“Not negotiable” cheques are crossed cheques which are cashed at holder’s risk, the rightful owner being able to reclaim the amount if there is any flaw in the title to it.

The drawer of a cheque is the proper person to stop payment of it.

After the banker hears of a customer’s death he must return his cheques unpaid, marked “drawer deceased.” The death of a partner does not stop

payment of cheques signed by him in the firm's name, nor the death of any official of a society stop cheques drawn by him officially.

A bankrupt's cheques must not be paid but only after an "act of bankruptcy" in the eyes of the law.

Care must be taken to ascertain the exact state of any account before refusing payment on the ground of want of funds; when a cheque is returned for this reason it is usually endorsed "Refer to Drawer" (R/D) or "Not sufficient funds" (N/S)—on no account is the deficiency to be stated. Smaller cheques may still be paid in England, but in Scotland the cheque first presented establishes a lien on the balance in favour of the holder.

In all cases of "dishonoured" cheques (for whatever reason dishonoured) notice should be given to the customer from whom received. They should be returned to the bank or person from whom received unless for special reasons it is desirable to keep them.

All paid cheques should be punched (to show cancellation) before being given up. They are the legal property of the drawer, but the paying banker is entitled to a receipt for their correctness and correct debiting.

3°. *Cash orders* are inland drafts on demand drawn by traders on traders (difference from cheques). They are subject to the laws regulating bills of exchange. Many bankers look upon them with extreme disfavour.

#### 4°. *Bills of Exchange.*

Bankers have to exercise great judgment in distinguishing between bills arising properly from mercantile dealings and those which are merely mortgages on property not realisable or disguised accommodation bills.

Inland bills are those drawn and payable within the British Isles, including the Channel Islands.

Such bills are generally single (unlike foreign bills).

Every alteration in a bill should be initialled by all the parties.

Every bill should be dated.

If the date is accidentally omitted the holder may insert a date as near the true date as he can gather to be the case, provided he inform the acceptor.

The currency of a bill is the period before it is due.

If this is expressed in months, calendar months are meant and three days' grace is added.

Treasury Bills and B. of E. "Bank Post Bills" are the only English bills which take no grace.

Days of grace are granted on bills of exchange and promissory notes.

If they are payable by instalments grace is allowed on each payment.

*At sight* drafts are payable on demand without grace.

*After sight* drafts are reckoned from the date of the drawee receiving the bill.

If a bill falls due on a Bank Holiday it is paid on the day after—if it falls due on any other public Holiday or on Sunday, Good Friday or Christmas Day it is paid the day before.

Bank Holidays in England and Ireland are Easter Monday, Whitsun Monday, the 1st Monday in August and the 26th of December, or the 27th if the 26th is a Sunday.

Bank Holidays in Scotland are New Year's Day and Christmas (if these fall on a Sunday the Monday following), Good Friday, and the first Mondays in May and August.

The words denoting the amount decide the value of the bill—the figures are merely a memorandum.

A bill expressed in foreign money is calculated at

the rate of exchange at the place and on the day it is payable.

5°. *Acceptance.* No claim can be made upon the drawee until he has accepted the bill. In the case of "sets" of bills from abroad—an unaccepted bill will generally indicate in whose hands the accepted bill is—the holder of the unaccepted bill can demand the accepted bill, and if the two agree the acceptor must give it up.

A bill must be presented for acceptance without delay—at latest on the business day following the day of receipt.

The bill must be transmitted to the drawee for acceptance—he is not required to call to see it. If it be sent by post a stamped envelope should be included for its return.

Presentation should be during the drawee's business hours, and if the bill be left after such hours he may date his acceptance a day later. Also he has till the close of the next business day to decide upon the course he will adopt.

All documents attached to a bill must be shown for examination, but should not be left except with a banker.

If the drawee is dead it should be left with his personal representatives or at his last residence. If he is bankrupt the bill may be presented to him or his trustee.

The drawee signifies acceptance by his mere signature (even on the back).

A bill after sight must also have the date of sighting. The drawee cannot make the bill payable out of the town in which he resides.

He must on no account accept more than one of a "set" or he may be called upon to pay twice.

The acceptor may cancel his acceptance as long as



he holds the bill, but afterwards he cannot revoke it or dispute the drawer's signature.

A "general" or "clean" acceptance consists simply of a signature with or without an indication of the place of payment.

A "qualified" acceptance occurs when the equivalent of these words "and there only and not elsewhere" is attached.

The drawee must accept the bill personally or by his agent.

Minors cannot give acceptances—they are invalid.

Acceptances by a partner in a firm which does not trade only binds the signer, but in trading firms such acceptances bind the firm.

This is the case even with two distinct firms having common partners and trading under the same style.

6°. *Acceptance for honour.* If a bill is refused acceptance by the drawee or if protested "for better security" owing to his failure after acceptance it may be protested for honour. After protest it should be presented to any "case of need" upon it, and if the case of need intervenes he should accept thus :

"Accepted for honour and account of A., B. & Co. (drawer or endorser) with £ s d notarial charges, and will be paid if regularly presented when due.

X., Y. & Co."

7°. *Non-acceptance.* Bills refused are to be promptly returned to the owner.

An inland bill need not be noted or protested, but a foreign bill must always be protested unless there are express instructions to the contrary.

In the United Kingdom and United States the holder can *at once* recover the amount of an unaccepted bill from the prior endorsers, but in other countries he can only demand better security for payment when the bill matures.

8°. *Endorsements.* In the United Kingdom a bill may be to bearer or order.

If to order it must be endorsed. All bills paid to credit must be endorsed by the customer. A bill payable to several persons not partners must be endorsed by all. A minor does not bind himself by his signature, but he thereby gives a good title to another.

An executor or trustee may endorse usually with a guarantee from his bankers.

A forged or unauthorised endorsement gives the holder no right to the bill.

A restrictive endorsement does not bind the payer as to the correct application of the amount.

An endorser "*sans recours*" is only liable for previous forgeries, not for failure of payments.

An endorser to whom the bill is re-endorsed cannot sue the intermediate parties.

A bankrupt endorser gives a good title if the taker of the bill acts in good faith.

9°. *Stamps.* All bills drawn in the United Kingdom must be drawn on stamped paper. If drawn abroad the proper *adhesive* stamps must be affixed before it is circulated (not necessary on acceptance) and cancelled by the initials or name of the issuing party, and true date written across. Penalty for not cancelling the stamps £10.

Of a set *one* bill (only) must have a stamp for the full duty.

10°. *Presentation for payment* should be made on the exact date of maturity at the place indicated under all circumstances within the control of the holder.

If it is not presented on the date the endorsers are discharged, but the acceptor is still liable with compensation for loss due to the delay.

An endorser paying the bill on the acceptor's failure

can realise the acceptor's estate and use any securities deposited with the holder to cover the bill in order to recoup himself.

The bill should not be surrendered until cheques or any articles not legal tender offered in payment are realised.

Orders to "Retire Acceptances" should be signed just like a cheque.

The death or bankruptcy of the acceptor if known to his bankers cancels their authority to pay.

Payment for honour should only be made after the bill has been presented to the drawer and protested for non-payment.

11°. *Non-payment.* A foreign bill not paid on the due date should be noted on the same day with a view to future protest or it may be protested at once. After the due date such a protest is useless. An inland bill need not be noted on dishonour but it may be if considered desirable.

Where no notary is available any responsible person may give a certificate signed by two witnesses—this certificate is equivalent to a protest and must bear 1/- stamp (postage-stamps are available).

A dishonoured bill should be sent on the next day back to the customer unless the banker wishes to retain it for special reasons—in this case he must send notice to all the endorsers.

12°. *Short bills* or bills for collection are bills which are not sold to the banker but which he merely collects for his customer.

Lost bills and notes are subject to the laws of lost cheques.

13°. *Promissory notes* are subject to the laws of bills of exchange.

(1) They must be stamped.

(2) They should state the consideration (for value received, etc.).

(3) They may be promised jointly, severally, or jointly and severally.

A joint note is good against all the makers together.

A several note is good against all the makers severally.

A joint and several note is good against all the makers together and severally.

A note in the form "I promise to pay" signed by several makers is considered a joint and several note.

14°. *Shipping bills* should always be accompanied with a full set of bills of lading, insurance policy and (for India bills to be sold in London at any rate) with a receipt for freight—they should be to order of the shipper.

15°. *Foreign bills*—usually sold through exchange brokers in London on Tuesdays and Thursdays.

Crossed cheques and drafts are unknown abroad.

Bills upon Germany, Austria, Italy, Switzerland, Russia must bear the words "bill of exchange," otherwise acceptance cannot be demanded.

All foreign bills above £50 should be drawn in sets of at least two—one being sent forward for acceptance—the other being negotiated.

All bills on Portugal, Spain or India of *any* amount must be in sets of two at least.

Cheques on France must have the date written fully in words.

Russian bills have now commonly the double date (old and new style) 16/28 Feb.

In France, Spain, Russia, and Sweden the equivalent for "accepted" must form part of an acceptance.

All foreign bills (except French, which may be "to bearer" and Italian, which may be "to Mr —") should be "to order."

The course of exchange indicates the daily quotations for foreign bills—the better quotation, is for fine banker's bills, the worse, for fair mercantile bills. London gives to some countries a variable number of pence for a standard coin, but more usually receives a variable amount of foreign money for the pound sterling. (A list is given elsewhere.) It is obvious that in the former case the fewer pence required and in the latter case the larger number of the foreign standard coins obtainable for the sovereign, the more favourable is the exchange to this country.

In the case of Russian bills there should be no endorsements on the copy destined for acceptance—German "acceptance" bills may have endorsements.

Protests should not be levied till the day after maturity.

16°. *Days of grace abroad*—with accepted bills in Russia 10 days' grace may be taken, but firms of a high class do not avail themselves of it. There is no grace on unaccepted bills.

It is not quite accurate to say there are no days of grace in some continental countries, e.g. Holland and Belgium allow *two days* for protests, and interest is reckoned in discounting as if there were two days' grace; so in Germany, Italy, and elsewhere.

## 18. **Banking Operations.**

Bankers generally receive orders for remittances or drafts within certain limits (i.e. they are not to go outside certain stated prices for bills on different places), and as in general these limits and the current prices do not agree it often becomes necessary to determine whether the order should be executed or which rate should be preferred.

The principles guiding the choice are the same as those in exchanges proper:—

For remittances, the highest rate in foreign money or the lowest rate in sterling.

For drafts, the lowest rate in foreign money or the highest rate in sterling.

Certain formulæ are used in these calculations.

Assuming London to be the place of operation :

Let  $S$  = a sterling rate.  $F$  = a foreign rate.  $g$  = the given price or limit.  $p$  = the present price.

Then the above principles may be expressed algebraically thus :—

(1) For remittances

(a) with  $F$  rates, the rate improves as  $\frac{p}{g}$  increases.

(b) with  $S$  rates, the rate improves as  $\frac{g}{p}$  increases.

(2) For drafts

(a) with  $F$  rates, the rate improves as  $\frac{g}{p}$  increases.

(b) with  $S$  rates, the rate improves as  $\frac{p}{g}$  increases.

It is worth noticing that a fraction increases as its numerator increases if the denominator is fixed, or as the denominator decreases if the numerator is fixed.

The fractions should usually be converted into decimals for ease in comparing them.

The usual questions arising for bankers are :—

(1) The comparison of given prices with present rates for remittances to choose the best rate.

(2) The comparison of given prices with present rates for drafts to choose the best rate.

(3) The comparison of given prices with present rates, remittances and drafts to see if an order should be executed or not.

(4) To find the second present rate (when only one is given) so as to fix a limit above or below which an order must not be executed.

In the practical working the terms cash and bills are used in England instead of the terms remittances and drafts—the terms *argent, papier: geld, briefe: denaro, lettera*, etc. are similarly used abroad.

The same terms are employed to express the improvement in the rate and the state of the market—as a rate improves it is “better for cash” or “worse for bills,” or *vice versa*—the bill market is quoted “paper” when the supply is greater than the demand, and “cash” when bills are scarce.

*Example 1.* I have an order to remit bills upon either Paris at 25·60, Amsterdam at 12·2, Hamburg at 20·60 or the nearest rate. The present rates are Paris 25·50, Amsterdam 12·0½, Hamburg 20·58. Which should I choose?

F. Paris	$\frac{p}{g} \frac{25\cdot50}{25\cdot60} = \cdot996.$		25,60   $\frac{25\cdot500}{2\ 460}$   ·996 156
F. Hamburg	$\frac{p}{g} \frac{20\cdot58}{20\cdot60} = \cdot999.$	Use (1 per mille) worse	20,60   $\frac{20\cdot580}{2\ 040}$   ·999 186 1
F. Amst.	$\frac{p}{g} \frac{12\cdot025}{12\cdot1} = \cdot993.$		12·1,0,0   $\frac{12\cdot0250}{1\ 1350}$   ·993 ·460

*Example 2.* I have an order to draw upon Paris at 25·40 or Amsterdam at 12·1, Cadiz 49½, Lisbon 51½. Which is best to use when present rates are 25·50, 12·2½, 49½, 50½?

F. Paris	$\frac{g}{p} \frac{25\cdot40}{25\cdot50} = \cdot996.$	Use (4 per mille) worse	25,5   $\frac{25\cdot40}{2\cdot45}$   ·996 15
F. Amst.	$\frac{g}{p} \frac{12\cdot1}{12\cdot2\frac{1}{2}} = \cdot992.$		12·13,7,5   $\frac{12\cdot05000}{112625}$   ·992 3387
S. Cadiz	$\frac{p}{g} \frac{49\frac{1}{2}}{49\frac{1}{2}} = \cdot990.$		39,7   $\frac{3930}{357}$   ·990 6
S. Lisbon	$\frac{p}{g} \frac{50\frac{1}{2}}{51\frac{1}{2}} = \cdot987.$		20,5   $\frac{2020}{175}$   ·987 14

*Example 3. Remittances and drafts.*

I have an order to remit to Amsterdam at 12·4 and draw upon Hamburg at 20·40 or at equivalent rates. Should I execute it—the present rates being 12, 3½, 20, 42?

$$\left. \begin{array}{l} \text{F. Amsterdam} \quad \frac{p}{g} \frac{12 \cdot 195}{12 \cdot 2} \\ \text{F. Hamburg} \quad \frac{g}{p} \frac{20 \cdot 40}{20 \cdot 42} \end{array} \right\} \frac{12 \cdot 195}{12 \cdot 200} \times \frac{20 \cdot 40}{20 \cdot 42} = \frac{248 \cdot 778}{249 \cdot 124} < 1.$$

Rates unfavourable.

*Example 4. Remittances and drafts.*

I have an order to remit bills upon Cadiz at 49½ and to draw on Lisbon at 53 or at equivalent rates. Should I execute it at the present prices 49½, 53½?

$$\left. \begin{array}{l} \text{S. Cadiz} \quad \frac{g}{p} \frac{49 \frac{1}{2}}{49 \frac{1}{2}} \\ \text{S. Lisbon} \quad \frac{p}{g} \frac{53 \frac{1}{2}}{53} \end{array} \right\} \frac{197}{199} \times \frac{215}{212} = \frac{42355}{42188} > 1.$$

Rates favourable.

*Example 5. Equivalent rates.*

I have an order to remit upon Paris at 25·55 and to draw upon Hamburg at 20·40. If the Hamburg rate improves to 20·36, to what Paris rate am I limited?

$$\left. \begin{array}{l} \text{F. Paris} \quad \frac{p}{g} \frac{x}{25 \cdot 55} \\ \text{F. Hamburg} \quad \frac{g}{p} \frac{20 \cdot 40}{20 \cdot 36} \end{array} \right\} \frac{x}{25 \cdot 55} \times \frac{20 \cdot 40}{20 \cdot 36} = 1.$$

$$\therefore x = \frac{20 \cdot 36 \times 25 \cdot 55}{20 \cdot 40} = 25 \cdot 49 \frac{1}{2}.$$

*Example 6. Equivalent rates.*

If I have to remit upon Paris at 25·65 and draw upon Lisbon at 56 at what rate may I draw upon Lisbon if the Paris rate falls to 25·55?

$$\left. \begin{array}{l} \text{F. Paris} \quad \frac{p}{g} = \frac{25 \cdot 55}{25 \cdot 65} \\ \text{S. Lisbon} \quad \frac{p}{g} = \frac{x}{56} \end{array} \right\} \frac{x}{56} \times \frac{25 \cdot 55}{25 \cdot 65} = 1.$$

$$\therefore x = \frac{56 \times 5 \cdot 13}{5 \cdot 11} = 56 \cdot 21 \text{ pence.}$$

The above examples are similar to those in Tate, but the solution of the questions on remittances and drafts is different.

The London Course of Exchange should constantly be consulted.

The methods of approximation apply—especially the methods of prediction for equivalent rates.



**EXAMPLES.**

1. Choose between these rates for remitting—the banker being ordered to remit upon Paris at 25·40, Amsterdam 12·2, Hamburg 20·50, Copenhagen 18·50, Vienna 12·06, Italy 28·30, Spain 40½*d.*, Lisbon 47½*d.*, Berlin 20·48, or the nearest rate.

- (a) Paris 25·38, Amsterdam 12·1½, Vienna 11·98.
- (b) Hamburg 20·43½, Copenhagen 18·47, Spain 40*d.*
- (c) Italy 28·12, Lisbon 46½*d.*, Berlin 20·44.

2. Choose between these rates for drawing—the banker being ordered to draw upon Paris at 25·25, Amsterdam 12·1, Hamburg 20·35, Copenhagen 18·30, Vienna 12·05, Italy 28·10, Spain 28·10, Lisbon 51, Berlin 20·36, or nearest rate.

- (a) Paris 25·41½, Amsterdam 12·2½, Vienna 12·04.
- (b) Berlin 20·40, Lisbon 52½*d.*, Copenhagen 18·33.
- (c) Italy 28·15, Spain 28·07, Hamburg 20·38.

3. Remittances and drafts. Are these orders to be executed at the given present rates?

- (a) Remit to Amsterdam 12·3½ at 12·2.  
Draw upon Hamburg 20·41½ at 20·43.
- (b) Remit to Berlin 20·38½ at 20·37.  
Draw upon Paris 25·21 at 25·22½.
- (c) Remit to Spain 42½*d.* at 43.  
Draw upon Lisbon 49½ at 50.
- (d) Remit to Spain 42 at 42½*d.*  
Draw upon Paris 25·20 at 25·22.

4. Equivalent rates.

- (a) My order is—Remit Paris 25·50.  
Draw Hamburg 20·40.  
The Hamburg rate improves to 20·35.  
To what Paris rate am I limited?
- (b) My order is—Remit Paris 25·55.  
Draw Lisbon 50½*d.*  
The Paris rate falls to 25·50.  
To what Lisbon rate may I go?
- (c) My order is—Remit Vienna 12·075.  
Draw Berlin 20·42.  
The Vienna rate falls to 12·05.  
To what Berlin rate may I go?
- (d) My order is—Remit Hamburg 20·52.  
Draw Paris 25·18.  
The Paris rate improves to 25·21.  
To what Hamburg rate am I limited?

### 19. Bullion Operations.

Bullion—i.e. gold and silver in bars, coins, or other forms—is imported and exported to create funds for exchanges.

The two chief questions which arise are :—

1°. To find the par of exchange between two countries from the prices of bullion in them.

2°. To find the arbitrated equivalent price of bullion in one country from the rate of exchange and the price of bullion in the other country.

Gold and silver are valued according to their purity—using a certain standard as the basis. The purity is determined by an assay.

Thus two preliminary questions arise before the above two questions can be properly answered.

1°. Assaying or the valuation of bullion.

2°. Reports or the method of reporting assays.

The standard weight and fine weight of gold and silver are found from an assay report of a certain weight of bullion.

The first is called “standarding” and the second may be termed “fining.”

The assay report is expressed in carats, dwts, or millièmes.

Bullion (especially in bars and ingots) is the chief regulator of exchanges, since when the arbitrated par obtained from the prices of bullion differs from the rate of exchange for bills by an amount exceeding the charges of purchase, transport and sale bullion begins to flow into the country offering the better price.

Bars or ingots are preferred to coins for transport except for places where there is no facility for mintage.

Besides the par of exchange arbitrated from the prices of bullion there exists between two countries with the same standard (e.g. between this country and all the countries completely or virtually of the gold valuation) a par of exchange based upon the quantities of pure gold contained in coins of full weight and fineness. This par ( $\equiv$  the number of foreign unit-coins which contain the same quantity of pure gold as the sovereign) is called the mint par or theoretical par—it is fixed by law.

The price of bar gold in a country with a gold standard cannot differ much from its mint par, i.e. cannot be at a considerable premium or discount—thus the mint par and the par arbitrated from gold prices will be almost always identical but in reality they are distinct.

The mint pars or more strictly the arbitrated gold pars govern the short exchanges between England and the countries of the gold valuation.

The two limits between which these short exchanges can fluctuate are found by adding to or subtracting from the above par of exchange the cost of shipping gold from one place to the other. These limits are called the “specie points” or “bullion points” and exist for every one with some slight variations due to facilities in business, etc.

The specie points are of great importance to the commercial community, because if the short exchanges rise or fall to them, gold will be exported or imported and any large movement of gold affects the rate of discount.

The mint pars and specie points are given under all the countries which have them. It is also to be noted that the arbitrated pars are based upon the facts given for each country under the word bullion, and these should be continually consulted.

20. **Assaying. English Assays.**

The value of bullion depends upon the quantity of pure metal it contains as found in the process of assaying.

The value of the alloy is disregarded as a compensation for the expense of refining.

1°. **Gold assays** were formerly made in carats of  $\frac{1}{4}$  carat grains each.

1 carat weighs  $\frac{1}{24}$  lb. Troy = 10 dwts. = 240 grs.,  
 $\therefore$  1 ct. grain = 60 grains Troy.

The proportions however are alone essential.

Purity would be 24 carats fine.

British standard gold is 22 carats fine ( $\frac{11}{12}$  or  $916\frac{2}{3}$  in millièmes).

The reports were made by comparing the ascertained weights with this standard—the difference being called the “betterness” (B) or “worseness” (W) of the metal.

The assay report was given in this form:—

Russia—ducat (gold) assay B  $1\frac{1}{2}$ , weight  $5\frac{1}{2}$  gr.

This means that the Russian ducat is 1 carat  $2\frac{1}{2}$  ct. grams above the standard, and  $\therefore$  its fineness is  $23\frac{1}{2}$ .

The full particulars of the assay of a given weight of gold would thus be set out:—

Given the weight and assay.

Fineness = standard  $\pm$  assay report.

$$\text{Fine weight} = \frac{\text{fineness}}{\text{absolute fineness (24)}} \times \text{weight.}$$

$$\text{Standard weight} = \frac{\text{fineness}}{\text{standard fineness (22)}} \times \text{weight}$$

or fine weight +  $\frac{1}{11}$  of fine weight.

Value in sterling = fine weight in oz.  $\times$  price per oz. fine.

Value in sterling = standard weight in oz.  $\times$  price per oz. standard (the usual method).

Value in sterling = full weight in oz.  $\times$  price per oz. weight (according to purity).

*Example.* Russian ducat assay B  $1\cdot2\frac{1}{2}$ , weight 54 gr., at £3.17s.10½d. per oz. standard.

$$\text{Fineness} = 22 + 1\cdot2\frac{1}{2} = 23\cdot2\frac{1}{2}.$$

$$\text{Fine weight} = \frac{23\cdot625}{24} \times 54 \text{ gr.} = 53\cdot15 \text{ grains.}$$

$$\text{Standard weight} = \frac{23\cdot625}{22} \times 54 \text{ gr.} = 57\cdot98 \text{ grains (53\cdot15 + 4\cdot93).}$$

$$\text{Value in sterling} = \frac{57\cdot98}{480} \times 77\cdot875s. = 10s. 3d.$$

2°. **Gold assays** are now expressed in millièmes and thirds—the weights being in oz. and decimals.

The fineness is thus expressed directly and the other particulars are set out in the same way.

Given the weight and fineness of a quantity of gold bullion.

$$\text{Fine weight} = \text{fineness} \times \text{weight} \div 1000.$$

$$\text{Standard weight} = \text{fine weight} + \frac{1}{11} \text{ of fine weight.}$$

$$\text{Value in sterling} = \text{fine weight in oz.} \times \text{price per oz. fine.}$$

$$\text{Value in sterling} = \text{standard weight in oz.} \times \text{price per oz. standard (usual for bars).}$$

$$\text{Value in sterling} = \text{full weight in oz.} \times \text{price per oz. weight (according to purity).}$$

*Example.* 140·375 oz. gold, 896 fine, at 77s. 6d. per oz. standard.

	140·375
	698
Fine wt. = 140·375 × 896 = 125·776 oz. fine	112 8000
	12 6338
	8422
	<hr/>
	125·7760
Standard wt. = fine wt. + $\frac{1}{11}$ fine wt. = 137·210 oz. st.	11·434
	<hr/>
	137·210
	5783
	<hr/>
	411 6300
Value in sterling = standard wt. in oz. × 77s. 6d. = £531.13s.9d.	109 7680
	9 6047
	6861
	<hr/>
	531·6888

3°. **Silver assays** are made in ozs., dwts., and  $\frac{1}{8}$  dwts.—12 oz. or 240 dwts. being purity.

The British standard silver is 11 oz. 2 dwts. fine or 222 dwts. fine ( $\frac{37}{40}$  or 925 in millièmes).

The reports are made from a comparison of the given weights with this standard—the difference as before being called the “betterness” (B) or “worse-ness” (W) of the metal.

The assay report is given in this form:—

France: franc (silver) assay, W 0·7. Weight, 3 dwts.  $5\frac{1}{2}$  gr.

This means that the franc is 7 dwts. below the standard and  $\therefore$  its fineness is 215.

Given the weight and assay of silver bullion—the particulars are thus found.

Fineness = standard  $\pm$  assay report.

Fine weight =  $\frac{\text{fineness}}{240} \times \text{weight (aliquotise) or full weight} \pm \frac{\text{assay report}}{240}$  of full weight.

Standard weight =  $\frac{\text{fineness}}{222} \times \text{weight or fine weight} + \frac{3}{37}$  of fine weight or full weight  $\pm \frac{\text{assay report}}{222}$  of full weight.

Value in sterling = fine weight in oz.  $\times$  price per oz. fine (cake silver).

Value in sterling = standard weight in oz.  $\times$  price per oz. standard (bars).

Value in sterling = full weight in oz.  $\times$  price per oz. weight (coins).

*Example.* Russian rouble of 13 dwts. 8 grs. W 14 at 5s. per oz. st.  
Fineness = 222 - 14 = 208.

Fine wt. =  $\frac{208}{240} \times 320$  grs. =  $\frac{11}{3} \times \frac{2}{3}$  oz. =  $\frac{22}{9}$  oz.

Standard wt. =  $\frac{208}{222} \times \text{fine wt.} = \frac{40 \times 26}{37 \times 45} = \frac{104}{99}$  oz.

Value in sterling =  $\frac{104}{99} \times 5s. = \frac{1040}{99} = 3\frac{4}{9}s. = 3s. 1\cdot477d.$

4°. **Silver assays** might also very conveniently be expressed in millièmes—with the weights in oz. and decimals.

For instance in the above examples,

W 14 denotes fineness  $866\frac{2}{3}$ .

$$\therefore \text{fine weight} = .866666 \times \frac{2}{3} = \frac{1.733333}{3} = .57777 \text{ oz.}$$

$$\text{Standard weight} = \frac{.57777 \times 40}{37} = .624624 \text{ oz.}$$

$$\text{Value in sterling} = .624624 \times 5s. = 3.123123s. = 3s. 1.477d.$$

In the case of the franc,

W 0.7 denotes  $\frac{215}{40} = \frac{43}{8} = 895.8$  fineness.

$$\text{Fine weight} = 77\frac{1}{2} \times .8958 \text{ grains. Standard weight} = \frac{40}{37} \times 77\frac{1}{2} \times .8958 \text{ grains.}$$

$$\text{Value in sterling at } 5s. \text{ per oz.} = (\frac{5}{480} \times \frac{40}{37} \times 77\frac{1}{2} \times .8958)s.$$

## 21. Continental Assays.

These are always expressed in millièmes and tenths for both gold and silver.

Their use will be obvious for all from a French example.

Given 3.071 kilogrammes gold 917 fine—premium 2 per mille on 3437 fr. per kilog. fine.

$$\text{Fine weight} = 3.071 \times .917 = 2.816 \text{ kilog.}$$

$$\text{Value in frcs.} = 2.816 \times 3437 \text{ fr.} = 9678.59 \text{ francs.}$$

$$2 \text{ per mille} = \frac{19.35}{1000}$$

$$9697.94 \text{ francs.}$$

Given 42.117 kilog. silver 925 fine—discount 10 p.c. on 218.89 francs per kilog. fine.

$$\text{Fine weight} = 42.117 \times .925 = 38.958 \text{ kilog.}$$

$$\text{Value in frcs.} = 38.958 \times 218.89 = 8527.516 \text{ francs.}$$

$$10 \text{ p.c.} = \frac{852.751}{100}$$

$$7674.765 \text{ francs.}$$

American assays are in millièmes and halves.

The Chinese reckon in percentages and call it the touch (toques).

The Russians reckon in zolotniks, and  $\therefore$  96 represents fine metal.

## 22. Reduction of English Reports to the Decimal Form, and vice versa.

1°. To reduce a decimal gold report to the British standard (22 carats).

Multiply the report by  $24 \div 1000$ .

Find difference of this and 22. Mark B or W accordingly.

*Example.* 900 fine.  $\begin{array}{r} 21.6 \\ 22 \\ \hline \end{array}$   $\cdot 4 = 1\frac{1}{2}$  ct. grains. W  $0\ 1\frac{1}{2} = 0\ 1\frac{1}{2}$  nearly.

2°. To bring a British gold report to a decimal form.

Add or subtract the report to 22, and divide by 24 to three (or 4) figures.

*Example.* B 1.  $1\frac{1}{2}$ .  $23\ 1\frac{1}{2}$ .  $24 \mid \begin{array}{r} 23.375 \\ 177 \\ \hline 95 \end{array} \mid \cdot 974$  raide.

Br. stand. gold =  $916\frac{2}{3}$ .  $12 \mid \begin{array}{r} 110 \\ 20 \\ \hline 80 \\ \hline 8 \end{array} \mid \cdot 916$

3°. To bring a silver millième report into a British form.

Multiply the report by 240, mark off 3 places of decimals.

Find difference of result and 222, mark B or W accordingly.



<i>Example.</i>	938.	938
		240
		<hr/>
		37·520
		187·6
		<hr/>
		225·120
		222
		<hr/>
		B 3·120

4°. To turn an English silver report into a millième form.

Add or subtract given report to or from 222.

Divide result by 240 to 3 figures—answer will be French report.

*Example.* W 7.  $222 - 7 = 215$  |  $2150 \div 240 = 8958 = 896$  raide.

$$\begin{array}{r} 2300 \\ \underline{140} \\ 20 \\ 1 \end{array}$$

Br. standard silver = 925 F.  $240 \mid 2220 \mid 925$

$$\begin{array}{r} .600 \\ \underline{1200} \\ \dots \end{array}$$

### EXAMPLES.

- Find all the particulars from these Assay Reports.
  - Russian Imperial (1801), 981 fineness,  $185\frac{1}{2}$  grains wt.
  - Spanish Doubloon (1772), W 0.  $2\frac{1}{4}$ ,  $416\frac{1}{2}$  grains wt.
  - Sicca Rupee, B 0. 13, weight 7 dwts.  $11\frac{1}{2}$  grs.
  - U. S. Dollar, W 0.  $8\frac{1}{2}$ , weight 17 dwts. 8 grs.
  - Mexican Dollar, 898 fine, weight  $416\frac{1}{2}$  grains.
  - Turkish Piastre, fine, weight 7·216 grammes.
- Find prices at 77s. 9d. per oz. st. of
  - 673·829 oz. gold, 910 fine.
  - 712·56 oz. gold, 876 fine.
  - 210 lb. 9 oz. 17 dwts. 20 grs., 916 fine.
- Find prices at  $43\frac{5}{8}$ d. per oz. st. of
  - 874·612 oz. silver, 912 fine.
  - 77000 grains silver, 892 fine.
  - 913·215 oz. silver, 984 fine.

4. Transform these reports.

- (1) 961 fine to old gold report.
- (2) 835 fine to English silver report.
- (3) B 1  $1\frac{1}{4}$  to millièmes.
- (4) 864 fine to zolotniks.
- (5) W 0  $7\frac{1}{2}$  to Chinese report.
- (6) W 0  $9\frac{1}{4}$  to millièmes.

### 23. Arbitrations of Bullion.

These consist of arbitrated pars of exchange and arbitrated prices of bullion.

Pars of exchange are the rates calculated for operations in bullion—arbitrated from the prices of bullion in the two countries.

An arbitrated price of bullion is the price of gold or silver in one country arbitrated from its price in another and the rate of exchange between the two.

The difference then is simply that in arbitrating pars the prices are given and we have to find the rate of exchange, whereas in arbitrating prices one price and the rate of exchange are given and we have to find the other price.

The method of arbitration is exactly the same as for bills of exchange but two additional points must be considered—the relations between the gold and silver weights of the two countries and the degrees of fineness at which the prices are rated.

These particulars will be found under each country and should be frequently consulted.

The chain rule is employed but the calculations are much simplified by the use of “fixed numbers.”

In England bullion is rated at standard fineness but some coins (doubloons, dollars, etc.) in large quantities are valued per oz. weight in accordance with their estimated fineness.

In other countries gold and silver bars are quoted per weight fine and coins per piece.

Thus to compare English and foreign rates for coins we must know the average weights.

1000 Sovereigns	weigh	256·50 oz.	7·978 kilog.
„ Napoleons	„	207·00 oz.	6·438 „
„ German 20-marks	„	256·00 oz.	7·962 „
„ Half imperials	„	210·40 oz.	6·544 „
„ Half eagles	„	268·56 oz.	8·353 „
„ Spanish doubloons	„	867·00 oz.	26·967 „
„ Mexican dollars	„	868·00 oz.	26·998 „
„ Spanish pillar dollars	„	866·00 oz.	26·936 „
„ Austrian florins	„	396·50 oz.	12·333 „

Also 1 oz. Troy = 31·10349552 grammes.

Pars and prices are given for practice under some of the chief countries and fixed numbers or tables are supplied to facilitate the calculations. For coins the above average weights are of great importance.

## SECTION X.

### ON 'CHANGE AT HOME AND ABROAD.

#### GREAT BRITAIN.

##### Money and Bullion.

4 farthings = 1 penny.  
12 pence = 1 shilling.  
20 shillings = 1 pound.

Accounts are kept in sterling (£. s. d., librae, solidi, denarii).

"Sterling" (easterling) abroad is used for "English money."

Farthings (fourthings) are written as fractions.

Since 1816 gold has been the sole standard of value.

The gold coins are the sovereign and half-sovereign.

Amounts are sometimes stated in guineas (21s.) and half-guineas (10s. 6d.), but the coins are not now made.

Legally "1869 sovereigns are to be coined out of 40 Troy-pounds weight of gold  $\frac{11}{12}$ ths fine,"  $\therefore$  1 sovereign should weigh 123·27445 grains (7·98805 grammes), and the half-sovereign the half of this.

The Master of the Mint must not issue a sovereign of less than 123·07445 grains or more than 123·47445 grs. (the remedy allowance is thus 0·2 grain).

Sovereigns cease to be legal tender when less than  $122\frac{1}{2}$  grains, half-sovereigns when less than  $61\frac{1}{8}$  grains.

The sovereign and half-sovereign of the Sydney and Melbourne mints are legal tender here.

The common silver coins are the half-crown, florin, shilling, sixpence, threepence.

Legally "60 shillings are to be coined out of 1 Troy-pound weight of silver  $\frac{37}{40}$  fine."

$\therefore$  1 shilling should weigh  $87\frac{3}{4}$  grains (5.65518 grammes) and the other coins in proportion.

The silver coins are legal tender for 40s. only. Their real value is about 60 % of their nominal value.

The bronze coins (95 parts copper, 4 tin, 1 zinc) are the penny, half-penny, and farthing.

Legally "40 pence must weigh 1 lb. Avoir. of bronze."

$\therefore$  1 penny should weigh 175 grains, and the other coins in proportion.

Pence and half-pence are legal tender for 12d., and farthings for 6d.

Their real value is just  $\frac{1}{4}$  of their nominal value.

The silver and copper coins being only tokens, pass for more than their intrinsic value, and when much worn are received back at the Mint (or Bank) for their nominal value.

Gold coins are only taken at their actual value except under a special Act of Parliament passed for the purpose of improving the coinage.

The promissory notes of the Bank of England are legal tender for sums above £5 except at the Bank's own branches.

The gold coins of the Sydney and Melbourne mints are legal tender here.

"Seigniorage" is the profit on our silver and bronze coinage.

Thus an oz. of silver whose market price is about

3s. 6d. is actually coined into five shillings and one sixpence—giving a profit of 2s. per oz.

‘Bullion’ consists of bars, dust, or “groups” (mixed coins, medals, pieces of gold).

A bullion-broker will assay and weigh any bullion for the owner and buy it if desired.

Railways charge 1s. per cent. between any two stations in Great Britain above the usual charge for freight for carrying bullion.

Mixtures of gold and silver with alloy or not are called *partings*—gold or silver partings as the gold or silver predominates.

In a gold parting—each is reported in millièmes: in a silver parting, the silver is reported in dwts. (B or W), the gold in grains per lb. Troy of the bullion.

The London charges for “assaying” are these: fine gold, 3s. per bar: fine silver, 1s. 6d.: parting gold, 4s.: parting silver, 3s. 6d. For “refining” the charges are 20 dwts. fine silver per lb. of bullion in parting gold, and 5 grains fine gold per lb. of bullion in parting silver.

The British standard of purity for gold is  $\frac{11}{12}$  or 916 $\frac{2}{3}$  fine in millièmes.

Assays of the fineness of gold are reported in thousandths and thirds.

Thus an assay 900 fine means that in 1000 parts, 900 are pure gold, 100 alloy.

[Formerly gold was reported by the “carat” ( $\frac{1}{24}$  of the Troy pound)—24 being purity.

The carat was divided into 4 carat grains, and these again into eighths.

Assays were reported as so many carats, carat grains and eighths “better” (B) or “worse” (W) than the standard 22 ( $\frac{1}{4}$ ). Pure gold would thus be B 20, and gold 900 fine W 0.1 $\frac{5}{8}$  nearly.

This method is still used in marking jewellery, the standard fineness of which is 18 carats or  $\frac{3}{4}$ .

The British standard of purity for silver is 11 oz. 2 dwts. of fine silver out of 12 oz. Troy or  $\frac{37}{40}$  ( $\frac{222}{240}$ )—expressed in millièmes 925.

In assays oz. dwts. and  $\frac{1}{2}$  dwts are used or oz. and decimals.

The fineness of silver is reported as so many dwts. “better” (B) or “worse” (W) than the standard 222.

Thus pure silver = B 18. Silver 900 fine = W 6.

There is some tendency towards the use of millièmes, as in the case of gold.

The bullion weight is the Troy oz. and decimals.

The Mint coins gold for private account free of charge if the value be £20,000 or over—the only expense being that of assaying, and the price being £3. 17s. 10½d. per oz. standard. Practically however the Bank of England buys gold at £3. 17s. 9d. per oz. standard and is now almost the sole medium by which bullion is imported to the Mint—the difference in price being more than compensated by the speedy realisation and the consequent gain in interest.

The Bank buys gold bars under these conditions and charges:

(1) The gold is melted by the Bank melters into bars of not more than 204 oz. Troy weight at a charge of  $\frac{1}{4}$ d. per oz.

(2) The bars are assayed by the Bank assayers at a charge of 4s. 6d. per oz.

The Bank buys gold of standard fineness at 77s. 9d. per oz. and sells bars of 900 or 916 fineness at 77s. 10½d. and bars of 990 or greater fineness at 77s. 11d.

It receives light gold coins by weight at 77s. 9d. per oz. standard.

It also buys and sells other gold coins (Napoleons, Russian Imperials, United States Eagles, German Reichsmarks) at about these prices:—

French coins	76/2½	per oz. (buying)	76/7	per oz. (selling).
German "	76/3	" "	76/7	" "
Russian "	77/7½	" "	77/11	" "
U. S. "	76/3½	" "	76/7	" "

The Bank delivers gold at its provincial branches at 9s. per mille (4s. per mille for delivery and 6d. per cent. for advice of the money).

The Crown alone issues silver and copper coins—it authorises the Bank to select and send to the Mint deteriorated pieces.

Bar silver is usually quoted per oz. standard.

Coins are usually quoted per oz. weight.

Cake silver per oz. fine.

### **Bills and Exchanges.**

Bills between London and the Continent are generally at 3 months' date.

Those from India or China on London at 4 or 6 months' date.

Bills drawn from London usually specify the time—if the time is not stated on any bill *Usance* is reckoned—this varies with the place:—

France, Genoa, Malta, 30 days' date. Spain and Portugal, 2 months' date. Germany and Holland, 1 month's date. Italy, 3 months' date. New York, 60 days' sight. South America, 90 days' sight.

Continental bills are by no means so frequently at *usance* as formerly.

Occasionally bills are marked double or half-*usance*.



The days of grace are three—there is no grace on demand bills or at Bank of England on Bank Post bills.

All bills due on Bank holidays are paid the day after, but bills due on a Sunday, Christmas Day, or Good Friday are paid the day before.

Bills due on the 31st of a month of 30 days are due on the 30th plus the 3 days of grace.

Bills payable in the country are charged  $\frac{1}{16}$ — $\frac{1}{4}$  p.c. commission.

Inland bills on London are at a premium, but this is usually commuted for time, e.g. the time par of bills from Edinburgh on London is 4 days (equivalent to a premium of 1/- p.c.).

The market for foreign bills is on Tuesday and Thursday.

If the time-currency of a foreign bill does not correspond with that of the quoted Course of Exchange the difference of interest is based upon the rate of discount of the place where the bill is payable.

Interest is calculated by taking the exact number of days and the year at 365 days.

Bill-brokers are a distinct class from bill-discounters, although some bill-brokers act as bill-discounters or English bill-brokers.

Brokerage (1 per mille =  $1 \frac{1}{1000}$ ) is only charged on bills payable abroad and is reckoned on the net price at which the foreign bill is sold. It is always paid by the seller and often by the buyer of the bill.

Bankers and bill-discounters recoup themselves by a commission added to the rate of discount or by a charge to their customers.

Discount-brokers are content with  $\frac{1}{16}$  or even  $\frac{1}{32}$  p.c. p.a. as discount commission for bringing two parties to an agreement, but they are only responsible for the genuineness of the names on the bills. They

do not endorse them or guarantee them for so small a profit.

Payments are generally made by cheques—cashied through the London Clearing House, an exclusive body of joint stock and private banks.

An unpaid bill must be “noted” the day it falls due with a view to future “protest.” The charge for “noting” is 1s. 6d. in London—the charge for a “protest” is about 5s. plus 6d. per £100 up to £2000, and then 1s. per £1000 or fraction in addition.

The bill stamps for all bills drawn, negotiated or payable in the United Kingdom are:—Demand or up to £5, 1d.; £5—£10, 2d.; £10—£25, 3d., then 3d. per £25 up to £100, then 1s. per £100 or fraction.

Bank of England or Ireland bills, bankers’ drafts for settlement, letters of credit, and Government department bills are exempt from stamps.

Bills in foreign money are stamped at the current rate of exchange.

Bills drawn within the United Kingdom must bear an impressed stamp.

Bills drawn out of the United Kingdom must bear adhesive stamps to be affixed before negotiation, but not before acceptance.

## London. Course of Exchange.

Amsterdam and Rotterdam.	short.	12 $\frac{1}{4}$ to 12 $\frac{1}{4}$	Florins and Stivers for £1.
"	3 mos.	12 $\frac{3}{8}$ to 12 $\frac{3}{8}$	Florins and Stivers for £1.
Antwerp and Brussels.	3 mos.	25 $\frac{3}{4}$ to 25 $\frac{3}{4}$	Francs and Centimes for £1.
Hamburg.	3 mos.	20 $\frac{4}{9}$ to 20 $\frac{4}{9}$	Marks and Pfennige for £1.
Berlin and German Bank places.	3 mos.	20 $\frac{4}{9}$ to 20 $\frac{4}{9}$	Marks and Pfennige for £1.
Paris.	cheques.	25 $\frac{1}{4}$ to 25 $\frac{1}{4}$	Francs and Centimes for £1.
"	3 mos.	25 $\frac{1}{4}$ to 25 $\frac{1}{4}$	Francs and Centimes for £1.
Marseilles.	3 mos.	25 $\frac{1}{4}$ to 25 $\frac{1}{4}$	Francs and Centimes for £1.
Austria.	3 mos.	25 $\frac{1}{4}$ to 25 $\frac{1}{4}$	Francs and Centimes for £1.
Petersburg.	3 mos.	12 $\frac{1}{2}$ to 12 $\frac{1}{2}$	Florins and Kreuzers for £1.
Moscow.	3 mos.	25 $\frac{1}{4}$ to 25 $\frac{1}{4}$	Florins and Kreuzers for £1.
Genoa, Leghorn, Naples etc.	3 mos.	25 to 25 $\frac{1}{4}$	Pence and Fractions for 1 Rouble.
New York.	60 days' sight.	26 $\frac{1}{4}$ to 26 $\frac{1}{4}$	Pence and Fractions for 1 Rouble.
Spain (Madrid etc.).	3 mos.	4 $\frac{1}{16}$ to 4 $\frac{1}{16}$	Lire and Centesimi for £1.
Lisbon.	3 mos.	42 to 42 $\frac{1}{2}$	Pence and Fractions for 1 U. S. Dollar.
Oporto.	3 mos.	41 $\frac{1}{2}$ to 41 $\frac{1}{2}$	Pence and Fractions for 1 Piastre.
Copenhagen.	3 mos.	41 $\frac{1}{2}$ to 41 $\frac{1}{2}$	Pence and Fractions for 1 Milreis.
Christiania.	3 mos.	18 $\frac{3}{4}$ to 18 $\frac{3}{4}$	Pence and Fractions for 1 Milreis.
Stockholm.	3 mos.	18 $\frac{3}{4}$ to 18 $\frac{3}{4}$	Kronas and Oere for £1.
Calcutta.	demand.	18 $\frac{3}{4}$ to 18 $\frac{3}{4}$	Kronas and Oere for £1.
"	30 days' sight.	1 $\frac{1}{2}$ to 1 $\frac{1}{2}$	Pence and Fractions for 1 Rupee.
Madras.	sight.	1 $\frac{1}{2}$ to 1 $\frac{1}{2}$	Pence and Fractions for 1 Rupee.
Bombay.	sight.	1 $\frac{1}{2}$ to 1 $\frac{1}{2}$	Pence and Fractions for 1 Rupee.
Hong Kong.	sight.	2 $\frac{1}{10}$ to 2 $\frac{1}{10}$	Pence and Fractions for 1 Dollar.
Shanghai.	sight.	3 $\frac{1}{16}$ to 3 $\frac{1}{16}$	Pence and Fractions for 1 Tael.

**EXAMPLES.**

Decide on the best way of paying these debts and find the actual cost, including brokerage and stamps.

(1) London—Austria, £356. 8*d.* 4*d.*, rates 12·05, 118·75, 3 mos. Discount rates 3 and 4 p.c.

(2) London—France, 10,000 Fr., rates 25·18, 25·19½ cheques.

(3) London—Russia, 20,000 Ro., rates 24½, 101·40, 3 mos. Discount rates 4 p.c. each.

(4) London—New York, £975. 10*s.* 8*d.*, rates 49½, 4·85½. 60 days' sight. Discount 2½ and 3 p.c.

(5) London—Calcutta, 8075 Rs., rates 1*s.* 2½*d.*, 1*s.* 2½½*d.* sight.

**Weights and Measures.**

The bullion weight is Troy weight, but the Bank always uses oz. and decimals.

In weighing diamonds the oz. Troy is divided into 151 carats, ∴ 1 diamond carat = 19 grs. Troy.

In weighing pearls the oz. Troy is divided into 600 grains, ∴ 5 pearl grains = 4 grs. Troy.

In standarding gold the lb. Troy is divided into 24 carats of 4 carat grains, ∴ 1 gold carat = 240 grs. Tr.

The lb., oz., gr. in Apothecaries' weight are those of Troy weight.

The commercial weight is Avoirdupois weight, 1 lb. Av. = 7000 grs. Tr., and 1 lb. Tr. = 5760 grs. Tr.

A great many things nominally sold by measure are really sold by weight—the measures being taken to be of a given weight, e.g. a firkin of butter = 56 lb.

At Mark Lane wheat is nominally sold by the quarter, but really by weight, 1 qr. being taken to be 504 lb. and 1 bush. to be 63 lb.

Foreign wool is always sold by the lb. British wool occasionally by the pack of 240 lbs. and by the sack of 2 weys of 2 tods of 2 stones (14 lbs.) of 2 cloves.

The ordinary liquid measure is the gallon—filled with distilled water it must weigh 10 lbs. Avoir. at 62° Fahr. and 30 inches barometer.

1 gallon = 277·274 cub. inches, ∴ 1 cub. ft. of water weighs 62·321 lbs.

The dry measures are the peck, bushel and quarter.

1 qr. = 8 bush. = 32 pks. = 64 galls. 10 qrs. = 1 last.

The tun of 2 pipes, 3 puncheons, 4 hhds, or 6 tierces is chiefly used for oil. 1 tun of oil is estimated at 20 cwt. 1 qr.

For beer the butt of 3 barrels of 2 kilderkins of 2 firkins of 9 gallons is used.

For wines pipes of varying quantities of gallons are divided into hhds, quarter casks and octaves.

The commercial length is the yard of 3 feet of 12 inches.

The inch is divided into halves, quarters, eighths, or into tenths or twelfths.

1760 yds. = 1 mile. 1 geog. mile =  $\frac{1}{80}$  of a deg. =  $\frac{1}{80}$  of 69½ miles.

For land the perch of 5½ yds. is used, or the chain of 100 links. 1 chain = 4 perches.

For cloth the yard is divided into qrs. eighths and sixteenths.

1 sq. of flooring is 100 sq. ft. 1 rod of brickwork (1½ thick) = 272½ sq. ft.

For land, acres, roods, sq. poles or acres, sq. chains, sq. links are used. 10 sq. chains = 1 acre.

1 load of earth is a cubic yd. 1 load of timber = 50 cubic feet.

Deals and battens are usually sold by the S. Petersburg standard.

The old wine gallon (231 cub. ins.), and the Winchester bushel (2150·42 cub. ins.) are still used in the U.S. and some Colonies.

1 imp. bush. = 1·031557 Win. bush.  
 1 Win. bush. = ·9694472 imp. bush.  
 1 imp. gall. = 1·20032 o. w. gallon  
 1 o. w. gall. = ·833109 imp. gallon.

The old beer gallon of 282 cub. inches is not now used.

## FRANCE.

### Moneys and Bullion.

100 centimes = 1 franc.  
 (10 centimes = 1 decime.)

The gold coins are 5, 10, 20, 50, 100 franc pieces, all 900 fine.

155 20-francs weigh 1 kilogramme,  $\therefore$  3010 francs are the equivalent of 1 kilogramme gold 900 fine.

The 20-franc pieces are called Louis or Napoleons.

The silver coins are 5, 2, 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$  franc pieces, each 835 fine, except the 5-franc which is 900 fine.

40 5-franc pieces weigh 1 kilogramme,  $\therefore$  200 francs are the equivalent of 1 kilog. silver 900 fine.

The nominal value of the silver coins (other than the 5-franc) is  $7\frac{1}{2}$  p.c. above their real value—owing to their having been issued originally at 900 fine.

The bronze coins are 10, 5, 2, 1-centimes—they weigh the same number of grains.

France and the Latin Union at first adopted the double standard or valuation.

After 1867 the 5-franc piece was alone legal tender in silver, but it was still coined and  $\therefore$  the double standard remained in force at 1 :  $15\frac{1}{2}$ . The adoption

by Germany in 1871 of the gold standard caused such a depreciation in silver that this legal relation was absurdly high, and to prevent the export of gold France and the Latin Union had finally to suspend the coinage of the 5-franc piece.

Thus the French monetary system is a gold standard conjointly with the circulation of the existing 5-franc silver pieces as legal tender; this is called the *étalon boiteux* and gives to the silver 5-franc in all markets a value higher than that of the silver it contains by about 15 p.c. The number of these 5-franc pieces still in circulation is from 4 to 5 hundred millions.

If the balance of trade had been largely against France this *étalon boiteux* would have caused a serious derangement in the foreign exchanges, but this has not been the case yet, and the exchanges in France and the States of the Latin Union have almost always adjusted themselves as if they had had a purely gold valuation.

French reports of gold and silver assays are in millièmes and tenths: they are thus more accurate than British reports.

The French add *raide* if the fineness is barely up to the report, *franc* or *bien franc* if fully up to the report.

The bullion weight is the kilogramme.

1 kilog. = 32·151 oz. Tr. 1 oz. Tr. = ·0311035 kilog.  
 ∴ 1 oz. Brit. stand. gold = ·0285115 kilog. fine gold, and  
 1 oz. Brit. stand. silver = ·0287707 kilog. fine silver.

The value of 1 kilogramme gold 900 fine is 3010 francs (this is the Mint rate of coinage).

The Mint buys gold at a discount of Fr. 6·70 per kilogramme gold 900 fine.

Thus it gives  $(3100 - 6·70) = 3093·30$  Fr. per kilog. gold 900 fine, i.e. 3437 per kilog. fine.

Hence gold (in bars) is negotiated at a premium or discount on Fr. 3437 per kilog. fine (new tariff).

Silver (in bars) is negotiated at a premium or discount on Fr. 218·89 per kilog. fine (old tariff).

To find tariff rate for any other fineness.

Multiply (for gold) Fr. 3437 by fineness and mark off 3 places.

Multiply (for silver) Fr. 218·89 by fineness and mark off 3 more places.

*Example 1.* Standard gold 916 $\frac{2}{3}$ .  $\cdot 916\frac{2}{3} \times 3437 = 3150\cdot 58$  Fr.

*Example 2.* Standard silver 925.  $\cdot 925 \times 218\cdot 89 = 202\cdot 47$  Fr.

N.B. The rate for gold (old tariff) is Fr. 3434·44, and the rate for silver (new tariff) is Fr. 220·55, but these are not used practically.

At present gold is about at par, but silver is at a large discount.

The above charges include the *rétenue* or charge for importation at the mints.

The Bank of France receives gold in bars of about 200 oz. and not under 994 fine at 3437 Fr. per kilog. fine, and it pays cash with or without a discount of 12 days' interest at 3 p.c. p.a. according to the scale of importation (1 per mille).

Bars of gold under 994 fine are only taken by the Mint at 3437 Fr. per kilog. fine, and are paid for by a *bon de monnaie* of about 40 days—discountable as *papier haute banque* (fine paper).

Foreign gold coins are quoted per piece usually, but the Mint and the Bank of France buy them by weight at 3437 Fr. per kilog. fine, reckoning sovereigns at 916 fine, imperials at 916, American eagles at 900, German 20-marks at 899 $\frac{1}{2}$ , Austrian 8-florins as Napoleons, and paying cash with or without a discount of 12 days' interest at 3 p.c. (1 per mille).



The Bank of France sells these coins at the above rates plus a premium based on the stock and demand (now 4—5 per mille).

The charge for assaying is Fr. 3·90 per bar: for refining Fr. 5 per kilog.

Mint Par.

? = £1.

£1869 = 480 oz. Troy ( $\frac{11}{12}$  fine).

12 oz. st. = 11 oz. fine.

1 oz. f. = 31·10349552 grammes f.

900. gr. fine = 1000 grammes Fr. stand. coin.

1000 gr. st. = 3100 francs.

£1 = Fr. 25·2215.

Coins. 20-Francis = 15s. 10·3d.

10-Francis = 7s. 11·15d.

Specie points Fr. 25·11. Fr. 25·33.

At former gold would be shipped from London to Paris.

At latter gold would be shipped from Paris to London were it not that the banks in France can pay in silver coins to some extent and also take measures to prevent the export of gold by placing a premium upon it.

### **London—Paris. Arbitrated Pairs and Prices.**

1. Bar gold. Chain for par of exchange.

? = 20s.

London price in s. = 1 oz. st.

12 = 11 oz. fine.

1 = 31·1035 grammes fine.

1000 = 3437 francs.

1000 = 1000 + premium p.m. in Paris.

Fixed number = 1959·883374.

Rule.  $F. N. \times (1000 + \text{prem. p.m.}) = \text{rate of exchange} \times \text{London price in s.}$

Given any two the third may be found from this equation.

2. Gold coins. Chain for par of exchange.

$$? = 20s.$$

$$\text{London price} = 1 \text{ oz. wt.}$$

$$\text{wt. in oz.} = 1000 \text{ coins quoted.}$$

$$1 = \text{price in Paris.}$$

Fixed numbers are easily found for various coins.

Rule.  $F. N. \times \text{price in Paris} = \text{rate of exchange} \times \text{London price.}$

3. Bar silver. Chain for par of exchange.

$$? = 240d.$$

$$\text{London price in } d. = 1 \text{ oz. st.}$$

$$1 \text{ oz.} = 31.1035 \text{ grammes st.}$$

$$40 = 37 \text{ gr. f.}$$

$$1000 = 218.89 \text{ Francs.}$$

$$100 = 100 - \text{disct. p.c.}$$

Fixed number = 1511.406.

Rule.  $F. N. \times (100 - \text{disct. p.c.}) = \text{rate of exchange} \times \text{London price.}$

4. Silver coins. Chain for par of exchange.

$$? = 240d.$$

$$\text{London price in } d. = 1 \text{ oz. wt.}$$

$$\text{wt. in oz.} = 1000 \text{ coins quoted.}$$

$$1 = \text{price in Paris.}$$

Fixed numbers are easily found for various coins.

Rule.  $F. N. \times \text{price in Paris} = \text{rate of exchange} \times \text{London price.}$

### EXAMPLES.

1. Bar gold. London, 77s. 10½d. per oz. st. Paris, 2 p.m. premium. Rate? *Ans.* = 25.217.

2. Gold coins. London, Doubloons, 77s. 6d. Paris, 83.50 Fr. Rate? *Ans.* = 25.149.

3. Bar silver. London, 55d. per oz. st. Paris, 8 p.c. discount. Rate? *Ans.* = 25.2817.

4. Silver coins. London, Sp. Dollars, 57½d. per oz. Paris, 5.30 Fr. each. Rate? *Ans.* = 25.545.

5. Bar gold. London price? Paris, 2 p.m. premium. Rate 25·20. *Ans.* = 77s. 11d.

6. Silver coins. London price of Mexican Dollars? Paris, 5 Fr. each. Rate 25·20. *Ans.* 54 86d.

### **Bills and Exchanges.**

The French laws are the same as the English in all essentials.

There are no days of grace on bills or legal claims.

A bill must be protested for non-acceptance or non-payment within 24 hours.

If the acceptor becomes a bankrupt protest may be made at once, and a claim made on the drawer and endorsers even though the bill's term is not expired.

The holder of a foreign bill payable at or after sight is obliged to present it for payment or acceptance within certain specified periods called *délais*, regulated by the distance, under penalty of losing all claim upon the endorsers or drawer.

Similarly there are also *délais* under a like penalty for suing parties to dishonoured bills.

Usance is fixed at 30 days, not including the day of date or sight.

Bills drawn, accepted or payable in France are stamped at 5 centimes per 100 fr. up to 1000 fr., then 50 centimes per thousand or part of a thousand.

Bills for endorsement are stamped 50 c. per 2000 fr. and fractions.

Bills due on a Sunday or a Holiday are payable the day before.

Cheques must be "on demand" and fully dated.

If on the same place the stamp is 10 cts.—on another 20 cts.

Negotiated foreign bills are paid for the day after.

Interest is always reckoned by taking the exact number of days and the year at 360 days.

In the case of long bills on places for which sight exchanges are quoted the seller allows the buyer the interest for the term of the bill at the bank rate where the bill is payable.

In the case of bills on places for which the exchanges are quoted at 3 mos. and which have less time to run, interest for the difference in days is charged to the buyer at  $\frac{1}{4}$  p.c.

### Paris. Course of Exchange.

London.	short.	Fr.	25·15 to 25·20	for £1.
Belgium.	"		99·40 to 99·45	for 100 Fr. (Belgium).
Switzerland.	"		99·40 to 99·45	for 100 Fr. (Swiss).
Italy.	"		89·30 to 89·35	for 100 L. (Paper).
Germany.	3 mos.		122·75 to 122·85	for 100 M.
Holland.	"		206·55 to 211·60	for 100 Fl. (Dutch).
Portugal.	"		500·00 to 510·00	for 100 Mlr.
Spain.	"		432·00 to 437·00	for 500 Pes. (100 Pesos).
Petersburg.	"		258·80 to 260·00	for 100 Ro.
Vienna.	"		208·00 to 208·25	for 100 Fl. (Austrian).

### EXAMPLES.

#### 1. Direct exchanges.

(1) London—Paris, £824. 6s. 8d., 3 mos. 25·27 $\frac{1}{2}$ , short 25·18. Discount, 2 p.c., 1 $\frac{1}{2}$  p.c.

(2) London—Paris, Fr. 7538·60, 3 mos. 25·28, 3 mos. 25·11. Discount, 2 $\frac{1}{2}$  p.c., 1 $\frac{1}{2}$  p.c.

(3) London—Paris, £1250 cheque 25·16 $\frac{1}{2}$ , 3 mos. 25·09 $\frac{1}{2}$ . Discount, 3 p.c., 2 p.c.

(4) London—Paris, Fr. 8517·20, short 25·18 $\frac{1}{2}$ .

(5) London—Paris, £382. 5s. 10d., 3 mos. 25·31. Discount, 1 $\frac{1}{2}$  p.c.

#### 2. Indirect exchange (Paris—London).

(1) Through Berlin, 123 and 20·40.

(2) Through Amsterdam, 212 and 12·15.

(3) Through Portugal, 557 and 52 $\frac{1}{2}$ d.

Direct rates, 3 mos., London on Paris, 25·35. Paris on London, 25·10. Discount, 3 p.c., 2 p.c.

How best to pay debts of £920. 16s. 8d. and 3586 Fr. ?

**3. Arbitrated pars and prices.**

(1) Bar gold. In London, 77*s.* 10½*d.* per oz. st. In Paris, 3 per mille premium. Rate?

(2) Gold coins. In London, Doubloons, 77*s.* 6*d.* per oz. wt. Rate 25·21. Price in Paris?

(3) Bar silver. In London, 43½*d.* per oz. st. In Paris, 8 p.c. discount. Rate?

(4) Silver coins. In London price? Rate 25·19. Spanish Dollars in Paris, 5·42 each.

**4. Simple circuit.**

What is profit or loss on £1000 expended in Hamburg Bills at 20·40, and sent to be sold in Paris at 125·70, my returns being sent me in bills of 3 mos. date at 25·33, the time of the transaction being three days?

**5. Compound arbitration. London—Paris.**

By bills on Vienna bought in London, at 119·40, and sold in Berlin at 179. The proceeds being sent to Paris in bills on Amsterdam bought at 169·80 and sold there at 213.

**Weights and Measures.—The Metric System.**

Unit of length.—The *mètre* = the 10-millionth part of a semi-meridian approximately.

Unit of surface.—The *àre* = the square of ten metres.

Unit of volume.—The *stère* = the cube of the metre.

Unit of capacity.—The *litre* = the cube of  $\frac{1}{10}$  of a metre.

Unit of weight.—The *gramme* = the weight of pure water at 4° C. in the cube of  $\frac{1}{100}$  of a metre.

The multiples proceed by tens and the fractions by tenths.

Greek prefixes denote multiples—deca, hecto, kilo, myria, 10 times, etc.

Latin prefixes denote fractions—deci, centi, milli, the 10th part, etc.

The words double and demi are used for the halves of the integers.

Practically the are and hectare are used for land—the sq. metre, etc. being used for small areas. Also the cubic metre is used as the name for solids, and the cubic centimetre very often for capacity.

The kilogramme is the ordinary commercial weight and is also used for bullion.

Medicine is weighed in grammes.

The Approximate Equivalents are:—

#### 1. LENGTH.

1 metre	= 3·2809 ft.	1 foot	= 3047945 metres.
1 metre	= 1·093633 yds.	1 yd.	= 91438 metres.
1 metre	= 39·37079 inches.	1 inch	= 0·0254 metres.
1 kilometre	= 0·62138 E. miles.	1 mile	= 1·609315 kilom.

#### 2. SURFACE.

1 sq. metre	= 10·7643 sq. ft.	1 sq. ft.	= 0·0929 sq. m.
1 sq. metre	= 1·196 sq. yds.	1 sq. yd.	= 8361 sq. m.
1 sq. metre	= 1550·06 sq. ins.	1 sq. inch	= 0·064514 sq. decim.
1 hectare	= 2·4712 acres.	1 acre	= 40467 hectares.

#### 3. SOLIDITY.

1 c. m. (stère)	= 35·317 cub. ft.	1 cub. ft.	= 0·283153 c. m.
1 c. m. (stère)	= 1·308 cub. yd.	1 cub. yd.	= 7645 c. m.
1 c. m. (stère)	= 61027·05 c. ins.	1 cub. inch	= 16·386 cub. cm.

#### 4. CAPACITY.

1 Litre	= 2201 imp. gall.	1 imp. gall.	= 4·543458 lit.
1 Hectol.	= 2·7512 imp. bush.	1 imp. bush.	= 36348 hectol.
1 Hectol.	= 3439 imp. qrs.	1 imp. qr.	= 2·9078 hectol.

#### 5. WEIGHT.

1 gramme	= 15·43234874 grains.	1 grain	= 6·48 grammes.
1 kilog.	= 2·205 lbs. Av.	1 lb. Av.	= 4536 kilog.
1 kilog.	= 32·151 oz. Tr.	1 oz. Tr.	= 31·1035 grammes.

The only weights and measures of the old system (le système usuel) remaining to any large extent in use are:—

Corn. The Charge de Marseilles = 1·6 hectolitre  
= 4·402 imp. bush. = 55025 imp. qrs.

Liquids. The Millerole de Marseilles = 64 litres = 14.08 gallons. This is divided into 4 escandaux for wine, and 16 quarterons for oil.

Wine and Brandy. The Bordeaux Barrique of 30 veltes = 228 litres = 50.2 imp. gallons nearly.

Medicines. The Ounce of 8 gros. of 80 grains or 4 grammes.

*Note.* The French call 1000 million a billion, and 1000 billion a trillion.

Thus a French trillion = an English billion.

## GERMANY.

### Moneys and Bullion.

1 Reichsmark = 100 pfennige.

The unit is the silver mark but gold is the standard—silver only being legal tender for small amounts.

The gold coins are 5, 10, 20-mark pieces all 900 fine.

125.55 10-marks are coined out of 1 pfund (500 grammes).

∴ 1 kilogramme fine gold = 2790 marks and 1 pfund fine gold = 1395 marks.

The silver coins are 5, 2, 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$  marks, all 900 fine.

100 marks are coined out of 1 pfund.

∴ 1 kilogramme fine silver = 222 $\frac{2}{3}$  marks and 1 pfund fine silver = 111 $\frac{1}{3}$  marks.

Nickel coins of 10, 5 pfennige and copper coins of 2, 1 pfennige are also issued.

Gold coins 5 per mille light cease to be legal tender, but are exchanged for full value.

The silver coins issued are 12 marks per head of the population.

The thaler alone of the old silver coins yet remains in circulation and is legal tender for 3 marks—until this is demonetised the German system is like the *étalon boiteux* of the Latin Union and is not purely a gold standard.

The thalers in circulation are about worth £20,000,000—the gold coins in circulation are worth £80,000,000—a proportion of 1 : 4, whereas in the Latin Union the proportion is 1 : 2½.

This silver money has no present effect upon the foreign exchanges, and they adjust themselves as if the gold standard was complete.

The notes of the Imperial Bank or of the 18 other issue-banks or of the Government paper money are not legal tender.

Assay reports are in millièmes.

The bullion weights are the kilogramme or pfund (500 grammes).

Bar gold and silver are quoted at Berlin and Hamburg in marks per kilogramme fine, and at Frankfort in marks per pfund fine.

Foreign gold coins are quoted per piece.

Napoleons (900 fine) and imperials (916 fine) are quoted per pfd. weight or per pfd. fine.

The Mint coins gold for private account at 3 marks per pfd. fine, and charges for assaying 3 marks per bar. The gold bars must be not less than 5 pfds. in weight, nor under 900 in fineness.

The Reichsbank at Berlin and branches (Dresden, Frankfort, Hamburg, Hanover) buy bar gold at M. 1392 per pfund fine when the bars are not less than 5 pfds. in weight nor under 900 in fineness. They also buy sovereigns, eagles, napoleons, imperials at prices announced from time to time (M. 1392—1393·50 per pfd. fine), taking eagles and napoleons as 899½, and sovereigns and imperials as 916½ fine. Dutch gold



coins of 10 and 20 Fl. are bought at about M. 1252·6608 per pfd. fine.

Mint Par chain.

? = 1*£*.

1869 = 480 oz. st.

12 = 11 oz. fine.

1 = 31·10349552 grammes fine.

500 = 1395 marks.

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M. 20·42945.

Coins. 20-marks = 19*s.* 6·95*d.*      10-marks = 9*s.* 9·48*d.*

Specie Points. M. 20·31.      M. 20·53.

Leaving.      Coming.

### London and Berlin. Arbitrated Pars and Prices.

1. Bar gold. Chain for Par of Exchange.

? = 20*s.*

London price = 1 oz. st.

12 oz. = 11 oz. f.

1 = 31·1035 grammes fine.

1000 = Berlin price in marks.

Fixed number = 57023083.

Rule.  $F. N. \times \text{Berlin price} = \text{London price} \times \text{rate of exchange}.$

2. Gold coins. Correct above rule for fineness by a proportion.

3. Bar silver. Chain for Par of Exchange.

? = 240*d.*

London price = 1 oz. st.

40 = 37 oz. fine.

1 = 31·1035 grammes fine.

1000 = Berlin price in marks.

Fixed number = 6·904986.

Rule.  $F. N. \times \text{Berlin price} = \text{London price} \times \text{rate of exchange}.$

4. Silver coins are not much sold.

**EXAMPLES.**

1. Bar gold. London, 77s. 9d. per oz. st. Berlin, M. 2780 per kilog. fine. Rate? *Ans.* M. 20·389.

2. Bar silver. London price? Berlin, M. 163 per kilog. fine. Rate 20·40. *Ans.* 55·171d.

**Table for rate of exchange** arbitrated from bar gold at 77s. 9d. per oz. st. in London.

Price per kilog.  
fine in Berlin.

1 = ·007334158  
2 = ·014668316  
3 = ·022002474  
4 = ·029336692  
5 = ·036670790  
6 = ·044004948  
7 = ·051339106  
8 = ·058673264  
9 = ·066007422

Ex. Berlin price 2784 per kilog. fine (1392  
per pfund fine).

14·668

5·134

·587

·029

20·418 rate of exchange.

Similar Tables may be made for other prices of gold.

**Bills and Exchanges.**

The bill stamps are:—up to 1000 M., 10 pfennige per 200 M. or fraction: beyond 1000 M., 50 pfennige per 1000 M. or fraction ( $\frac{1}{2}$  per mille).

Exempt are all bills drawn, accepted, and payable out of Germany.

If bills are in sets, only the one in actual circulation must be stamped.

Bills must bear the words "Nur zum Accept bestimmt" if for acceptance only: they need no stamp.

There are no days of grace.

Bills due on a Sunday or holiday are paid the day after.

Protest for non-payment must be made not later than the second working-day after the bill's date.

Notice must be sent to the last indorser within two days of the protest.

Bills must be accepted on presentation.

Interest is reckoned by taking the month at 30 and the year at 360 days.

Berlin quotes exchanges on Petersburg, Warsaw, and Vienna daily—others on Tuesdays, Thursdays, Saturdays.

Frankfort quotes the foreign exchanges daily.

Hamburg quotes them on Tuesdays and Fridays, but there is business every day.

Bills of less or greater currency than the quoted rates are reckoned at the quoted rates, allowance being made for interest at the bank rate, where payable.

No allowance is made for foreign bill stamps.

The brokerage at Berlin is  $\frac{1}{2}$  p.m., at Hamburg  $\frac{3}{4}$  p.m. and at Frankfort  $\frac{1}{2}$  p.m.

"Short" is a currency of 10 days or less.

No interest is given on bills shorter than 8 days.

Berlin is the centre for Russian bills, and controls the price of the paper rouble.

## Course of Exchange.

		Berlin.	Frankfort.	Hamburg.	
London.	sight.			20'41½	for £1.
"	8 days (short).	20'39½	20'40	20'39	for £1.
"	3 mos. (long).	20'35		20'34	for £1.
Petersburg.	3 weeks.	205'25	201'30		for 100 Ro.
"	3 mos.	200'15		200'10	for 100 Ro.
Warsaw.	8 days.	205'25			for 100 Ro.
Vienna.	8 days.	170'30	170'35		for 100 Fl. (A).
"	2 mos.	169'50		169'40	for 100 Fl. (A).
Paris.	8 days.	81'00	81'01		for 100 Fr.
"	3 mos.	80'90		80'50	for 100 Fr.
Amsterdam.	8 days.	168'45	168'80	167'90	for 100 Fl. (D).
"	2 mos.	169'95			for 100 Fl. (D).
Belgium.	8 days.	81'10	81'05		for 100 Fr.
"	2 mos.	80'80			for 100 Fr.
Switzerland.	8 days.	80'90	81'05		for 100 Fr.
Italy.	8 days.	77'90	78'00		for 100 L.
New York.	short.	4'18	4'25	4'20	for 1 Dollar.
"	3 mos.			4'14½	for 1 Dollar.

**EXAMPLES.**

1. Direct. London—Berlin (Frankfort, Hamburg).

- (1) £835. 12s. 6d. at 3 mos. 20·53, 3 mos. 20·35, 2 p.c.,  
 $1\frac{1}{2}$  p.c.  
 (2) 73125 marks at 3 mos. 20·52 $\frac{1}{2}$ , 3 mos. 20·32 $\frac{1}{2}$ , 3 p.c.,  
 $2\frac{1}{4}$  p.c.  
 (3) £10000 at 3 mos. 20·33 $\frac{1}{4}$ . Discount,  $2\frac{1}{2}$  p.c.  
 (4) 8345·6 marks at 3 mos. 20·53 $\frac{1}{2}$ . Discount, 3 p.c.  
 (5) £645. 7s. 6d. at 3 mos. 20·53, 8 days 20·39 $\frac{1}{2}$ . Discount,  
 2 p.c.,  $1\frac{1}{4}$  p.c.

2. Indirect. (London—Berlin.)

- (1) Through Paris, 25·21 and 82·36.  
 (2) Through Vienna, 117·85 and 176·12.  
 (3) Through S. Petersburg, 101·00 and 217·90.

Direct rates, 3 mos. 20·55 and 20·36 $\frac{1}{2}$ . Discount, 4 p.c.,  
 3 p.c.  
 How best to pay M. 1257·82 and £326. 8s. 7 $\frac{1}{2}$ d.?

3. Arbitrated p<sup>ars</sup> and prices.

- (1) Bar gold. In Berlin, 2781 M., in London, 77s. 9d.  
 Rate?  
 (2) Bar silver. In London, 43 $\frac{1}{8}$ d. Rate 20·41. Price in  
 Berlin?

4. Simple circuit.

What is profit or loss on £1000 invested in Russian bills at 23 $\frac{1}{2}$ d. and sold in Berlin at 221, my returns being in bills of 3 mos. at 20·50, time being 5 days? (Int. at 5 p.c.)

5. Compound arbitration. London—Berlin.

By bills on Russia bought in London at 100·90 and sold in Paris at 265, the proceeds being sent to Berlin in Amsterdam bills bought at 211 and sold there at 170.

**Weights and Measures.**

The metric system came into general use Jan. 1, 1872.

The meter is also called stab: the dekameter, kette: the centimeter, neuzoll: the millimeter, strich. The meile = 7 $\frac{1}{2}$  kilometers.

The measures of surface are called quadratmeter, etc.

The are is called ar—the hektare, hektar.

The liter is also called kanne (of 2 schoppen) and the hektoliter, fass (of 2 scheffel).

Firewood is measured by the cubikmeter.

The capacity of ships is also stated in cubikmeters, but the equivalent in British register tons is always added.

1 cubikmeter = 0.353 register ton.

The dekagram is also called neuloth.

The pfund is half a kilogram.

The centner is 100 pfunde = 50 kilogs.

The tonne is 1000 kilograms = 2000 pfunde.

The old weights and measures have practically disappeared.

## AUSTRIA.

### Moneys and Bullion.

100 kreuzer = 1 florin Austrian currency (Gulden oesterreichische Währung).

In 1857 Austria, Prussia and other German States concluded a convention to coin

30 N. German standard thaler	} respectively from the Mint pfund of 500 grammes fine silver, and 50 gold crowns from the Mint pfund fine gold.
52½ S. „ „ gulden	
45 Austrian „ „ gulden	

The Austrian silver coins were to be 900 fine.

The gold crowns are no longer coined and have almost disappeared, for in 1866 Austria officially seceded from the 1857 convention and with Hungary is now coining in gold 8-florin and 4-florin pieces.

77½ 8-florin pieces weigh 1 pf. 900 fine, i.e. 620 fl. weigh 1 pf. 900 fine.

The 8-florin is thus designed to be equivalent to the 20-franc in weight and fineness.

But these gold coins are commercial money—the nominal standard remains the 1857 gulden (silver), but in reality it is a paper money in which the gold coins are quoted per piece and the silver coins per Fl. 100. Gold and silver are legal tender as well as the paper currency.

In 1870 a decree made 8-Fl. pieces legal tender for Fl. 8·10, and the 4-Fl. pieces for Fl. 4·05—this makes the metallic standard of Austria a “Double Valuation” with silver to gold as 1 : 15½.

Owing to the fall in the price of silver, the premium (in paper money) on the silver Florin has now disappeared and at present they are quoted at par.

Also the Mints since 1879 have ceased to coin silver for private account—thus largely avoiding any disturbance in their currency on a further fall in silver.

There is also a silver commercial money called the “Levant” or “Maria-Theresa” dollar largely used on the East shores of Mediterranean.

Assay reports are in millièmes.

The bullion weight is the kilogramme.

Bar gold is quoted at the Mint 1395 Fl. per kilogramme fine in Austrian gold coins (ducat = 4·80 Fl. 8-Florin = Fl. 8·10).

The Mint charges ½ per mille for coinage, 2 Fl. per kilog. for refining, Fl. 0·50 per bar for assaying.

Ducats, 4-Florins, 8-Florins, foreign gold coins are quoted in paper florins per piece on the Vienna Bourse—the following weights are fixed for gold coins:—

500 German 20-marks must weigh	3975 grammes	} and for
500 Russian ½-imperials „ „	3267 „	
500 English sovereigns „ „	3987 „	
500 French napoleons „ „	3220 „	

every  $\frac{1}{2}$  gramme short the seller must allow the buyer,  $1\frac{1}{4}$  marks,  $\frac{1}{8}$  imperial,  $\frac{1}{8}$  sovereign,  $\frac{31}{100}$  napoleon.

Silver is quoted in paper Florins per 100 Fl. silver—at present "Par."

The only silver coin now coined is the Maria Theresa Thaler and only for private account. 1 M. T. T. = 28.0644 grammes  $833\frac{1}{8}$  fine.

Mint Par Chain for gold.

? = £1.

1869 = 12 oz. st.

12 = 11 oz. fine.

1 = 31.10349552 gr. fine.

900 = 1000 gr.  $\frac{9}{10}$  fine.

500 = 77 $\frac{1}{2}$  Fl.

1 = 8.10 Fl.

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Fl. 10.215.

Coins. 8-Fl. = 15s. 10.30d.      4-Fl. = 7s. 11.15d.

Specie Points. Fl. 10.11.      Fl. 10.31.

Leaving.      Coming.

Arbitrated pars for bar gold are calculated as with Germany.

### Bills and Exchanges.

Bills drawn, accepted, or payable in Austria are stamped thus:—

(a) If currency is less than 6 months—below 75 Fl., 5 Kr.: from 75 to 150 Fl., 10 Kr.: afterward 10 Kr. per 150 Fl. up to 1500 Fl.—then 1 Fl. per 1500 Fl. or fraction.

(b) If currency exceed 6 months—various charges up to 100 Fl.—then 31 Kr. additional per 100 Fl. up to 400 Fl.—afterward 1.25 Fl. per 400 Fl. or fraction.



Bills passing through Austria for indorsement pay 2 Kr. per 100 Fl.

Bills in sets are each subject to the full stamp unless one has "only for acceptance" upon it and a crossing to prevent indorsement.

The bill stamp in Hungary is  $\frac{5}{8}$  per mille.

There are no days of grace.

Bills due on a Sunday or Holiday are paid the day after.

Protest for non-payment must be made not later than the 2nd working day after the bill is due.

Notice must be sent to the drawer and indorsers within two days of the protest.

[See Appendix for new Austro-Hungarian Coinage.]

### Course of Exchange.

Quotations in Austrian paper currency.

Vienna gives to

London	3 mos.	Fl. 119·65	for £10 sterling.
Amsterdam	"	Fl. 97·50	for 100 Fl. (Neth.).
Brussels	"	46·50	for 100 Fr.
Germany	"	58·16	for 100 M.
Italy	"	42·10	for 100 L. (Paper).
Paris	"	47·50	for 100 Fr.
Petersburg	"	118·00	for 100 Ro.
Zurich	"	46·50	for 100 Fr.

### EXAMPLES OF EXCHANGES.

1. Direct. London—Vienna.

(1) £875. 16s. 3d. at 3 mos. 12·06, 3 mos. 118·90. Discount, 2 p.c.,  $2\frac{1}{2}$  p.c.

(2) £1000 at 3 mos. 118·51 $\frac{1}{2}$ . Discount, 3 p.c.

(3) Fl. 8652·38 at 3 mos. 12·08, 3 mos. 119·05. Discount,  $1\frac{1}{2}$  p.c., 2 p.c.

(4) Fl. 10000 at sight 119·70.

(5) £925. 7s. 6d. at 12·07 $\frac{1}{2}$  3 mos. Discount, 1 p.c.

## 2. Indirect. London—Vienna.

(1) Through Brussels,  $20\cdot17\frac{1}{2}$ ,  $46\cdot35$ .(2) Through Italy,  $26\cdot30$ ,  $42\cdot15$ .(3) Through S. Petersburg, 102,  $117\cdot84$ .

Direct rates 3 mos. London on Vienna,  $12\cdot07\frac{1}{2}$ , Vienna on London,  $118\cdot65$ , 3 mos. Discount, 2 p.c., 3 p.c.

How best to pay debts of £875 and Fl. 8350 ?

## 4. Compound arbitration. London—Vienna.

By bills on Hamburg, bought in London at  $20\cdot40$  and sold in Paris at 123, the proceeds being sent to Vienna in bills on Italy at  $89\cdot75$  and sold there at 43.

**Weights and Measures.**

The metric system introduced in 1876—with the German form of the denominations.

The old weights and measures still used are :—

Weight. The Vienna pfund (of 32 loth) =  $1\cdot2347$

lb. Av. 1 lb. Av. =  $\cdot80991$  V. pfund.

The stein is 20 pfunde and the centner 100 pfunde.

Dry. The Vienna metze =  $\cdot2115$  quarters.

1 imp. qr. =  $4\cdot7281$  V. metzen.

The Trieste staro =  $\cdot2841$  quarters. 1 imp. qr. =  $3\cdot51988$  T. stari.

The metze is divided into halves, quarters, eighths.

Liquid. The Vienna mass or kanne =  $\cdot311445$  galls.

1 gall. =

The Vienna eimer (of 41 mass) =  $12\cdot769$

gallons. 1 gall. =  $\cdot0783129$  V. eimer.

Wine. The Trieste orna (Barile) =  $14\cdot45$  imp.

gallons. 1 gall. =  $\cdot0692$  T. orna.

Length. The Vienna elle =  $\cdot85215$  yds. =  $\cdot77919$

metre. 1 yard =  $1\cdot1735$  V. elle.

Wool. The Trieste braccio =  $\cdot7401$  E. yds.

Silk. " " =  $\cdot7021$  "

**RUSSIA.****Moneys and Bullion.**

1 ruble (rouble) = 100 copeks.

In 1839 silver was made the standard money of account and exchange with the silver rouble as unit of 20·7315 grammes  $868\frac{1}{8}$  fine.

In 1886 the silver rouble was made of 19·99 grammes 900 fine, the exact equivalent of the former rouble.

The "Credit Notes" issued in 1843 by the State Bank for the Government were first exchangeable against silver and gold, and were at par or a small premium, but in 1844 these notes were issued beyond the amount of bullion held, and though still exchangeable between 1844 and 1854 they lost 1 or 2 p.c. in comparison with gold.

In 1854 they were made inconvertible and have in consequence fallen considerably (to about 70 p.c. of their nominal value). They form the present legal tender of Russia (excluding Finland), which is therefore an inconvertible paper currency. Gold and silver are however legal tender.

The gold coins of Russia are the new imperials and half-imperials of 1886.

These weigh 12·902 grammes and 6·451 grammes 900 fine.

The imperial is legal tender for 10 roubles.

Thus the relation of gold and silver is 1 :  $15\frac{1}{2}$  as against 1 :  $15\frac{9}{10}$  before.

The half-imperial is equal to the French 20-franc piece.

The Mint par on the 1 :  $15\frac{1}{2}$  basis is £1 = 6·305 roubles (gold).

The bullion weight is the pud = 40 funts of 96 zolotniks of 96 doli.

Bullion is reported in zolotniks, and  $\therefore$  an assay 96 fine represents pure metal.

Silver is quoted in copeks per zolotnik fine, and gold in paper roubles per 1 funt fine. Gold coins are quoted in paper roubles per piece.

The State Bank buys gold and silver at prices fixed from time to time.

Mint Par for Gold.

? = 1 Ro.

10 = 12.902 grammes fine 900.

1000 = 900 fine gr.

31.1035 = 1 oz. fine.

11 = 12 oz. st.

480 = £1869

£1 = 240*d*.

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38.06*d*.

Coins. 1 Half-Imperial = 15*s*. 10.3*d*.

1 Sovereign = 6.305 gold Roubles.

### Exchanges and Bills.

These are quoted on Tuesdays and Fridays.

Bills are for immediate or deferred delivery.

Bills are sent to the buyer the next day with interest calculated to the day of negotiation.

The rate of interest is the bank rate of the foreign place except for bills on Germany, when it is the open market rate.

Interest is calculated at 30 days to the month and 360 to the year.

The brokerage is  $\frac{1}{4}$  p.c.

There are 10 days of grace—not taken advantage of by banks and the best houses.

On sight bills there are 3 days of grace.

There is no grace on unaccepted bills.

Bills must be presented for acceptance within 24 hours of receipt.

Bills due on Sunday or a Holiday are payable the day after.

Russian merchants pay on promissory notes of 6 to 9 months' currency.

The State Bank charges  $\frac{1}{2}$  p.c. (bills below Ro. 1000) or 1 per mille (bills above Ro. 1000) for collections on provincial centres.

The Russian calendar is 12 days behind the Gregorian—a fact not to be lost sight of in bills of exchange transactions.

The bill stamps run in a very irregular manner, being about 10—15 copeks per 100 Ro., and 60—70 copeks per 500 Ro. and so on.

These stamps are in force in Poland, but not in Finland.

Inland cheques pay no stamps, but cheques drawn out of Russia do so.

Money orders up to 5 days' sight drawn in Russia are exempt.

Bills in sets must all be stamped except in the case of one marked "for acceptance only" and crossed at the back.

### Course of Exchange.

S. Petersburg gives to

London	3 mos.	Ro. 101'00	for £10.
Amsterdam	"	83'45	for Fl. 100.
Germany	"	49'50	for M. 100.
Paris	"	40'10	for Fr. 100.

**EXAMPLES.****1. Direct. London—S. Petersburg.**

(1) £9760, 3 mos.  $24\frac{5}{8}$ , 3 mos. 101·18. Discount,  $2\frac{1}{4}$  p.c.,  $1\frac{1}{2}$  p.c.

(2) Ro. 40,000, 3 mos.  $24\frac{1}{2}$ , 3 mos. 97·79. Discount, 3 p.c., 2 p.c.

(3) £712. 13s. 6d., 3 mos.  $24\frac{1}{4}$ , 3 mos. 97·85. Discount, 2 p.c.,  $1\frac{1}{4}$  p.c.

(4) 15,000 Ro., 3 mos.  $24\frac{5}{8}$ . Discount,  $2\frac{1}{2}$  p.c.

(5) £756. 8s. 4d., 3 mos. 97·90. Discount,  $2\frac{1}{4}$  p.c.

**2. Indirect. London—S. Petersburg.**

(1) Through Amsterdam, 12·3, 81·92.

(2) Through Berlin, 20·38 $\frac{1}{2}$ , 51·25.

(3) Through Paris, 25·19, 41·05.

Direct rates, 3 mos., London on S. Petersburg  $23\frac{1}{4}$ , S. Petersburg on London 97·50. Discount 3 p.c., 2 p.c.

How best to pay debts of £712. 10s. 4d., and R. 6834·72.

**3. Compound arbitrated rate. London—Petersburg.**

By bills on Paris, bought in London at 25·26 and sold in Berlin at 82, the proceeds being sent to S. Petersburg in bills on Amsterdam at 171 and sold there at 83.

**Weights and Measures.**

The metric system is contemplated.

**WEIGHT.**

96 doli = 1 zolotnik.

96 zolotniks = 1 funt.

40 funts = 1 pud.

10 puds = 1 berkowitz.

1 funt = 6319·733 grains Troy (basis).

1 funt = ·90281900 lb. avoird.

1 pud = 36·11276000 lb. avoird.

1 zolotnik = ·00940436458 lb. avoird.

1 lb. avoird. = 1·10764 R. funt.

1 cwt. = 123·993 R. funts = 3·998 R. puds.

LIQUID.

100 tscharkey	= 1 vedro.
3 vedro	= 1 anker.
6 anker	= 1 oxhoft.
2 oxhoft	= 1 pipe.
40 vedro	= 1 botchka.
1 vedro = 750·568 c. ins.	= 2·7069 imp. galls. = 12·2985 litres.
1 imp. gall.	= 3694 vedro.
100 vedro	= 271 galls. (nearly).
1000 galls.	= 369 vedro (nearly).

DRY.

8 garnez	= 1 tschetwerik.
8 tschetweriks	= 1 tschetwert.

1 garnez = 200·1548 c. ins.	= 0·72186 imp. gallon = 3·2797 litres.
1 tschetwert = 0·72185 imp. quarters	= 209·9 litres.
1 imp. qr. = 1·38533 chetwert.	
1 chetwert = 5½ imp. bushels (nearly).	
100 chetwert = 72 qrs. (nearly).	

LENGTH.

16 werschok	= 1 arschin = 2 Russian feet.
1 fachin (sajen-fathom)	= 3 arschin.
500 sajen = 1 werst	= 3500 Eng. ft. = 1·06678 kilometres.
1 arschin	= 28 inches (basis).
9 arschin	= 7 Eng. yds.
1 sajen	= 2·1386 metres = 7 Eng. ft.
1 werst	= 6628754 Eng. miles.

The English foot is used for timber.

The S. Petersburg standard (120 deals 12 feet long, by 1½ inches thick, by 11 inches wide) is the basis of the Russo-English timber trade.

Land is measured by the crown dessatine (2400 sq. sajen) = 2·7 acres = 1⅓ hectare.

The common dessatine = 1½ crown dessatine.

**SPAIN.****Moneys and Bullion.**

1 Peseta = 100 Centesimos.

Since 1871 Spain reckons in pesetas and centesimos.

These coins are the same as the franc and centime.

The peseta is not legal tender: it weighs 5 grammes 835 fine silver.

The standard of Spain is exactly that of France, and the 5-peseta silver pieces of 25 grammes 900 fine are legal tender.

Before 1871 Spain reckoned in duros, escudos, reales, maravedis.

34 maravedis = 1 real.                      1 peseta = 4 reales.

10 reales = 1 escudo.                      5 pesetas = 1 duro.

2 escudos = 1 duro (hard dollar).

The English exchange is quoted in pence per duro.

The Mint Par (London—Madrid) is 1 duro peso = 47·5785*d.* on the basis of 3100 pesetas per kilogramme gold 900 fine.

French 5-francs (silver) circulated freely at 19 reales—they are now current at 5 pesetas.

The principal old gold coin still in circulation is the doblon (quadrupel, onza de oro) = 320 reales. Mint value 64*s.* 8*d.*—taken at 64*s.* in our Colonies.

The new gold coins are 5, 10, 20, 25, 100-pesetas, by law equal to the corresponding French coins in weight and fineness.

The peso duro or peso fuerte is the Spanish piastre or dollar which circulates largely in Africa, China, etc.

The Mexican dollar or peso—the principal export of Mexico—is the legal equivalent of the old Spanish silver (carolus) dollar, which it has quite displaced. It has become a large medium of exchange,



especially in the East. Its weight and fineness are  $416\frac{1}{2}$  gr. Troy 898 fine ( $6\frac{1}{4}$  w.)  $\equiv$  24.433039 grammes fine silver and its sterling value fluctuates with the price of silver (3s. 2d. nearly, at present). Its average weight is 24.094 grammes fine silver.

Assay reports are in millièmes.

The bullion weight is the kilogramme.

The Mint coins gold for private account and pays 3444.44 pesetas per kilogramme fine in bons de monnaie (of currency 50 days usually) discountable at 5 p.c.

Silver is also quoted in pesetas per kilogramme fine.

Mint Par on the basis of 3100 pesetas per kilog. fine.

? = 1 peso duro.

1 = 5 pesetas.

3100 = 1000 grms. 900 fine.

900 = 1000 grms. 90 fine.

31.10349552 = 1 oz. Troy.

480 = £1869.

1 = 240*d*.

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47.5785*d*.

Coins. £1 = 5.0443 pesos = 25.2215 pesetas.

### Exchanges and Bills.

There are no days of grace.

Bills due on Sunday or a Holiday are payable the day before.

Protest must be levied the day after maturity.

The brokerage is 1 per mille.

Interest is reckoned by taking the exact number of days and 360 to the year.

Bills are paid in the paper money of the place where payable—this paper money is only legal tender in the place itself.

Only bills drawn "payable in gold or silver" must be paid in specie.

Bills are stamped thus:—Up to pes. 200, 10 ctos.: 250—500, 25 ctos.: 500—1000, 50 ctos.: above 1000 at the rate of 4 pes. per 10000.

Cheques with indorsements must be fully stamped.

Bills must be drawn and accepted on paper with an impressed Spanish stamp.

### Course of Exchange.

Madrid—

London.	8 days' sight: 90 days' date.	29'20	Pesetas for £1.
Paris.	"	4'20	Francs for 1 Duro.
Hamburg.	90 days' date.	3'60	Marks "
(occasionally)			
Italy.	8 days' sight.	4'90	Lire "
Lisbon.	"	9'50	Reis "

### EXAMPLES OF EXCHANGES.

1. Direct. London—Madrid.

(1) £865. 9s.  $4\frac{1}{2}d.$ , at 3 mos.  $41\frac{1}{8}$ , 3 mos. 28·90. Discount, 3 p.c., 2 p.c.

(2) £1978. 10s.  $5d.$ , at 3 mos.  $42\frac{1}{8}$ . Discount, 4 p.c.

(3) Pes. 23567, at 3 mos.  $41\frac{1}{8}$ , 3 mos. 28·70. Discount, 2 p.c.,  $1\frac{1}{4}$  p.c.

2. Indirect. London—Madrid.

(1) Through Paris,  $20\cdot17\frac{1}{2}$ , 4·30.

(2) Through Hamburg,  $20\cdot40$ , 3·70.

(3) Through Lisbon,  $40\cdot\frac{3}{8}$ , 9·80.

(4) Through Italy,  $26\cdot10$ , 5·00.

Direct rates, London on Madrid, 3 mos.  $40\cdot\frac{1}{8}$ , and Madrid on London, 3 mos. 28·10. Discount, 3 p.c., 2 p.c.

How best to pay debts of £875. 10s.  $6d.$  and Pes. 75840·65?

3. Simple circuit.

What profit or loss is there on £1000 invested in Paris bills at 25·21 and sent to be sold in Madrid at 5·05, my returns being in bills of 3 mos. at  $42\frac{1}{8}$ , the time being one week? Interest at 4 p.c.

**Weights and Measures.**

The metric system was adopted throughout Spain and its possessions in 1859.

The names are made Spanish by easily-recognised modifications.

The old wts. and measures still in large use are :—

**Weight.** The libra castellana (of 16 onzas)  
 $= 1.01443 \text{ lb. av.} = .460142 \text{ kilog.}$   
 The quintal (of 4 arrobas)  $= 100 \text{ libras.}$   
 $1 \text{ lb. av.} = .98577 \text{ lb. cast.}$   
 Most of the Provinces have libras of their own.

**Liquid.** The Spanish cantara (arroba mayor)  $= 3.5520 \text{ imp. gall.} = 16.138 \text{ litres.}$   
 The arroba menor  $= 2.7653364 \text{ imp. galls.}$   
 $= 12.564 \text{ litres.}$   
 $1 \text{ imp. gall.} = .28153 \text{ cantara} = .36198128 \text{ arroba menor.}$   
 The pipe of oil  $= 27 \text{ cantaras} = 34\frac{1}{2} \text{ arrobas menores.}$

**Dry.** The Castilian fanega  $= 1.52669 \text{ imp. bush.}$   
 $= 55.49 \text{ litres.}$   
 $1 \text{ imp. bushel} = .65502 \text{ Castilian fanegas.}$   
 The cahiz  $= 12 \text{ fanegas, } \therefore 1 \text{ Cast. cahiz} = 2.29 \text{ imp. qrs.} = 6.6588 \text{ hectol.}$   
 There are other Provincial fanegas.

**Length.** The Castilian vara (of 4 palmos or 3 pies or 36 pulgadas)  $= .91318 \text{ yds.}$   
 $1 \text{ English yard} = 1.09507 \text{ Castilian varas}$   
 $= 1.08 \text{ Cadiz varas.}$   
 The Cadiz vara  $\therefore = .926 \text{ yard.}$   
 There are other varas.

In Cuba and Porto Rico the metric and old Castilian systems are used.

1 Cuban quintal = 101·5228 lbs. av. = 4605 kilogs.

1 Cuban corn fanega = 105·71 litres = 3·094671 imp. bushels.

## PORTUGAL.

### Moneys and Bullion.

1000 reis = 1 milreis.

Since 1855 Portugal has adopted the single gold standard of value.

The gold coins are the gold crown (10 mlr.), half-crown (5 mlr.), fifth-crown (2 mlr.), tenth-crown (1 mlr.), all of 916 $\frac{2}{3}$  fineness.

The legal weight of the crown is 17·735 grammes 916 $\frac{2}{3}$  fine.

The silver coins are the 500, 200, 100, 50-reis of fineness 916 $\frac{2}{3}$ , but they are made 8% under weight and are  $\therefore$  not legal tender above 5 mlr. (5\$000).

The 100-reis is called a teston.

The copper coins are 40, 20, 10, 5 reis.

The crusado of exchange = 400 reis.

A conto of reis = 1000 milreis. A conto de conto = 1,000,000 contos.

The silver 500-reis weighs 12·5 grammes 916 $\frac{2}{3}$  fine, and is equal to 2s. nearly.

The English sovereign is legal tender at 4\$ 500 reis.

The old gold peça (14·188 grammes) is taken at 8 milreis (35s. 7d. nearly).

Mint Par Chain.

$$\begin{aligned}
 &? = 1 \text{ milreis.} \\
 &10 = 17.735 \text{ grammes.} \\
 31.10349552 &= 1 \text{ oz. Troy.} \\
 480 &= £1869. \\
 1 &= 240d. \\
 \hline
 &53.285d.
 \end{aligned}$$

Coins. 10 mlr. = £2. 4s. 5d.      5 mlr. = £1. 2s. 2½d.

Specie Points.      52½d.      53½d.  
                          Coming.      Leaving.

N.B. On basis of £1 = 4\$ 500 reis, the Mint Par is 1 mlr. = 53½d.

**Bills and Exchanges.**

Bills payable in Portugal are stamped thus:—from 5 mlr. to 20, 20 reis: from 20 to 100 mlr., 100 reis: above 100 mlr., 100 reis per 100 mlr. or frn.

Bills passing for endorsement pay half the above.

Inland bills (8 days or on demand) pay 200 reis per 1000 mlr.

Bills drawn in Portugal, payable out of Portugal, pay 21 reis per 100 mlr. or frn.

Bills in sets pay half the stamp for every copy.

Brokerage on Foreign bills is 1 per mille.

Interest is calculated by taking the exact number of days and the year at 365 days.

The law on bills of exchange is as in France.

**Course of Exchange.**

Lisbon and Oporto receive from

London.	30, 60, 90 days' date.	41½ Pence.	for 1 Milr.
Amsterdam.	3 mos.	34 Fl.	for 16 Milr.

Lisbon and Oporto give to

Paris.	8 days, 3 mos.	430 Reis.	for 3 Francs
Hamburg.	3 mos.	170 Reis.	for 1 Mark.
Italy.	3 mos.	390 Reis.	for 1 Lira.
Spain.	8 days' sight.	910 Reis.	for 5 Pesetas (1 Piastre).

Rio de Janeiro is quoted at 30 days' sight in Brazilian reis per 100 Port. reis.

**EXAMPLES.**

1. Direct. London—Lisbon.

- (1) £685. 9s. 8d., at 3 mos. 41, and 3 mos. 41½. Discount, 2 p.c., 2½ p.c.  
 (2) 7286 Mlr., at 3 mos. 41½. Discount, 3 p.c.  
 (3) £1000, at 3 mos. 40½. Discount, 2 p.c.  
 (4) 6512.64 Mlr., at 3 mos. 40½, and 3 mos. 41½. Discount, 1 p.c., 3½ p.c.  
 (5) £712. 5s. 6d., at 3 mos. 40½, and 3 mos. 41½. Discount, 1 p.c., 2½ p.c.

2. Indirect. London—Lisbon.

- (1) Through Paris, 25.22½, and 435.  
 (2) Through Hamburg, 20.41, and 176.

Direct rates, London on Lisbon, 3 mos. 40½, Lisbon on London, 3 mos. 41½. Discount, 1 p.c., 3 p.c. Debts, £756. 8000 Mlr.

**Weights and Measures.**

The metric system was adopted in 1868.

The old weights and measures still in use to some extent are:—

**WEIGHT.**

The arratel (of 16 onzas) = 1.011857142 lb. av. = .458976 kilog.

The quintal (of 4 arrobas) = 96 arratei = 129.517714285 lb. av.

1 lb. av. = .9882819 arratei. 1 cwt. = 110.69 arratei. 1 English ton = 17½ Port. quintals nearly.

LIQUID.

The Lisbon almude = 3·6845 imp. gall. = 16·74 litres.

The Oporto almude = 5·5826 imp. gall.

1 imp. gallon = ·27140 Lisbon almudes = ·179158 Oporto almudes.

The Oporto pipe = 115 gallons. The almude = 2 alqueires (liquid).

The Lisbon pipe = 117 gallons.

The pipe of oil = 80 almudes.

DRY.

The Lisbon alqueire = ·3808005 bushels = 13·841 litres.

The Oporto alqueire = ·4805058 bushels = 17·465 litres.

1 imp. bus. = 2·626047 Lisbon alqueires = 2·081140 Oporto alqueires.

The fanega = 4 alqueires (dry).

LENGTH.

The Covado de craveira (of 3 palmos, or 2 pés., or 24 polegadas) = 7186 yds.

The Covado avantajado (of 24½ polegadas) = 74165 yds.

The vara (of 5 palmos) = 1·1977 yds.

1 Eng. yd. = 1·3916. Cov. crav. = 1·34835. Cov. adv. = ·83494 vara.

Eng. yds. are used at times. 6 yds. are taken as 5 varas, or 20 yds. as equal to 27 Cov. avant.

Multiples may be constructed for conversion into English weights and measures.

SWITZERLAND.

Moneys.

1 franc = 100 centimes.

Switzerland is a party to the French Monetary Convention of 1865.

The value of the franc is that of the French one (9½d. nearly).

The coins are those of France with 5, 10, 20 centimes in nickel.

Switzerland does not coin gold pieces: the French coins circulate instead.

The Swiss silver ones (when struck) are struck at the Paris Mint.

The coining of silver is now suspended.

**Exchanges.**

There are 3 days of grace in Glaris, 6 in St Gall, one in Geneva.

The brokerage is  $\frac{1}{2}$  per mille.

Bill-stamps are charged thus: in Basle,  $\frac{1}{2}$  per mille: in Fribourg,  $\frac{1}{4}$ : in Geneva,  $\frac{1}{2}$ : in Vaud,  $\frac{1}{4}$ : in Valais, 1.

In Lucerne each bill is charged 10 centimes.

In Tessin the charges are—up to 250 fr., 10 cts.: from 250 to 500, 15 cts.: from 500 to 1000, 25 cts.: above 1000, 50 cts. per 1000 or fraction.

In Basle cheques pay 10 cts.

**The Course of Exchange** is as in Paris.

**Weights and Measures.**

The metric system fully adopted in 1877.

**BELGIUM.****Moneys.**

100 centimes = 1 franc.

Belgium belongs to the French Monetary Union, and the value of the Belgian money in francs is the same as that of France—the par of exchange on London being the same.

**Course of Exchange.**

Antwerp gives to

London.	short or 3 mos.	Fr. 25'20	for £1 sterling.
Amsterdam.	"	211'50	for Fl. 100 (Neth.).
Rotterdam.	"	211'60	for Fl. 100 (Neth.).
Germany.	"	123'00	for M. 100.
Italy.	"	89'00	for L. 100 (Paper).
Lisbon.	"	5'50	for Milr. 1.
Madrid.	"	5'10	for Peso 1.
Paris.	"	99'50	for Francs 100.
Petersburg.	"	270'00	for Ro. 100.



### Bills and Exchanges.

The short exchange on London in Antwerp and Brussels is usually  $2\frac{1}{2}$ —5 centimes higher than that in Paris.

Bills drawn or payable in Belgium are stamped thus:

Up to 200 fr. 10 centimes—from 200 to 500 fr. 25 centimes—from 500 to 1000 fr. 50 centimes, and then for every 1000 fr. or fraction 50 centimes.

Bills circulating in Belgium only by indorsement pay same 10 centimes up to 200 fr.—from 200 to 500 only 13 centimes—from 500 to 1000, 25 centimes, and then for every 1000 or frn. 25 centimes.

**Weights and Measures** are those of the Metric System with some slight difference in the names (livre or pond for kilogramme, litron or kan for litre, aune or el for metre).

In Antwerp these old weights and measures are occasionally used:—

The commercial pond = 4701734 kilogs.

The aune = 695048 metres.

The last of corn = 40 hectolitres.

### EXAMPLES.

1. Direct. London—Antwerp (Brussels).

(1) £7256, at 3 mos.  $25\cdot33\frac{1}{2}$ , 3 mos.  $25\cdot15\frac{1}{2}$ . Discount, 2 p.c., 2 p.c.

(2) Fl. 1658·75, at 3 mos.  $25\cdot29$ , 3 mos.  $25\cdot18$ . Discount, 1 p.c.,  $1\frac{1}{2}$  p.c.

(3) £7256, short  $25\cdot19$ .

(4) Fl. 7569, at 3 mos.  $25\cdot16\frac{1}{4}$ . Discount,  $1\frac{1}{4}$  p.c.

2. Indirect. London—Brussels (Antwerp).

(1) Through Paris, short  $25\cdot23$ , 3 mos.  $98\cdot75$ .

(2) Through Berlin, 8 days  $20\cdot40$ , short  $123\cdot50$ .

(3) Through Rotterdam, 3 mos.  $12\cdot2\frac{1}{2}$ , 3 mos.  $209\cdot70$ .

Direct rates, 3 mos.  $25\cdot34$ , short  $25\cdot19$ . Discount, 2 p.c.,  $1\frac{1}{2}$  p.c.

Minimum payments for £1250. Fr. 8596 ?

**THE NETHERLANDS AND LUXEMBOURG.****Moneys and Bullion.**

1 florin or guilder = 100 cents.

1 fl. is also 20 stivers, still used in the London rate of exchange but not in the Amsterdam.

The ryksdaaler (still in circulation) = 50 stivers =  $2\frac{1}{2}$  Fl.

From 1847 to 1872 the standard was silver, with the silver florin weighing 10 grammes 945 fine as unit—thus the kilogramme fine silver = fl. 105·82.

In 1872 silver fell rapidly and the coinage of silver was suspended, and at last gold was coined conjointly with the legal currency of silver coins owing to the rapid fall in the foreign exchanges which were then favourable.

The new standard coin is the 10 florin-piece weighing 6·720 grammes (wigtjes) 900 fine, containing 6·048 grammes fine gold.

Thus the Mint Par is £1 = Fl. 12·1071.

Also the ratio of gold to silver is 1 : 15·625 (or 60·35*d.* per oz. in London).

Thus the currency of Holland is a gold standard with the former silver coins as legal tender—that is, the étalon boiteux is in force but no inconvenience has been felt owing to the foreign exchanges being in favour of Holland, and  $\therefore$  adjusting themselves as on a purely gold standard.

In Luxembourg the moneys are 1 franc = 100 centimes, as in France or Belgium.

Assay reports are in millièmes.

The bullion weight is the kilogramme.

Bar gold and silver are quoted in florins per kilog. fine.

Foreign gold coins are quoted per piece or per kilog. fine.

The Netherlands Bank buys bar gold (not under 900 fine or over 7 kilog. in weight) at a price fixed from time to time (now fl. 1647 per kilog. fine).

It also accepts 20-francs, 10-francs, 20-marks, 10-marks, 20-kronas, 10-kronas, and eagles at 899, and sovereigns and imperials at 916 fine.

It sells gold bars at fl. 1653, gold coins at fl. 1658 per kilog. fine.

The Utrecht Mint coins 10-florins for private account at a charge 5 fl. per kilog. 900 fine.

It coins 1653·44 fl. per kilog. fine, and ∴ pays fl. 1648·44 per kilog. fine—one month after receipt.

Mint Par Chain.

? = £1.

1869 = 480 oz. st.

12 = 11 oz. fine.

1 = 31·10349552 grms. fine.

1000 = 1653·439 fl.

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Fl. 12·1071.

Coins. 20-Fl. = £1. 13s. 0·46d.      10-Fl. = 16s. 6·23d.

Specie Points.      12·02.      12·17.  
Leaving.      Coming.

Arbitrated pars and prices as in Germany, using florins for marks.

### **Bills and Exchanges.**

Bills payable in Holland require these stamps—up to fl. 300, 21 cts.: from 300 to 500, 34½ cts.: from 500 to 1000, 34½ cts. per 500 or frn.: from 1000 the stamp is 69 cts. per 1000 or frn.

Bills “on demand” require the stamp legally.

Bills for endorsement require up to 600, 21 cts.: from 600 to 1000,  $34\frac{1}{2}$ : above 1000,  $34\frac{1}{2}$  cts. per thousand or frn.

Copies require the stamp legally, but only the "acceptance" copy is usually stamped.

Brokerage is  $\frac{3}{4}$  per mille on bills on Germany or England,  $\frac{1}{2}$ —1 per mille on other foreign bills.

Interest is reckoned by taking the exact number of days, and the year at 360 days.

There are no days of grace.

Bills due on Sunday or a Holiday are paid the day after.

Protest for non-payment is made out the first working-day after maturity.

In Luxembourg the bill stamp is:—up to 100 fr., 5 cts.: from 100 to 300, 20 cts.: from 300 to 600, 40 cts.: from 600 to 1200, 80 cts.: above 1200, 80 cts. per 1200 or frn.

### Course of Exchange.

Amsterdam gives to

		Florins.	
London.	short, 2 mos.	12'08	for £1.
Paris.	"	47'60	for 100 Fr.
Germany.	short, 3 mos.	59'00	for 100 M.
S. Petersburg.	3 mos.	126'00	for 100 Ro.
Vienna.	"	100'00	for 100 Fl. V.
Belgium.	short, 3 mos.	47'50	for 100 Fr.
Switzerland.	"	47'50	for 100 Fr.
Italy.	"	43'00	for 100 L.

### EXAMPLES.

1. Direct. London—Amsterdam.

(1) £1856. 10s. 10d., at 3 mos.  $12'3\frac{1}{2}$ , 2 mos. 12'07. Discount, 3 p.c., 2 p.c.

(2) Fl. 5000, at 3 mos.  $12'3\frac{1}{2}$ , 2 mos. 12'06 $\frac{1}{2}$ . Discount, 1 p.c.,  $1\frac{1}{2}$  p.c.

## 2. Indirect. London—Amsterdam.

- (1) Through Paris, 25·19, 46·90.  
 (2) Through S. Petersburg, 101·50, 126·80.  
 (3) Through Berlin, 20·41, 60·30.

Direct rates, 3 mos. London on Antwerp, 12·3½, Amsterdam on London, 2 mos. 11·98. Discount, 3 p.c., 2½ p.c.

Minimum cost of debts, £865. 7s. 6d. Fl. 4756·92.

## 3. Compound arbitration. London—Amsterdam.

By bills on Berlin, bought in London at 20·36, and sold in Paris at 124, the proceeds being sent to Amsterdam in bills on Hamburg, bought in Paris at 125·30 and sold in Amsterdam at 59½.

## Weights and Measures.

The Metric System is used with different (the old) names.

## WEIGHT.

Wigtjes	= grammes.
Looden	= decagramme.
Onsen	= hectogramme.
Pond	= kilogramme.
Korrel or Greinen	= decigramme.

The Old Medicine Pond = 375 grammes.

20 grs. = 1 scruple. ∴ 1 medicine pond = 5760 grains.

8 scr. = 1 dram. and 1 medicine grain

8 drams = 1 oz. = 0·06510416 grammes.

12 ozs. = 1 pond. = 1·0047 gr. Tr.

## CORN.

Kop	= litre.
Mudde or Zak	= hectolitre.
Schepel	= decalitre.

The Half-mudde = 50 Koppen.

The Last = 30 Zakken or Mudden = 10·3172 imp. qrs.

These are used in the wholesale corn trade.

## LIQUID.

Kan	= litre.
Maatje	= decilitre.
Vingerhoeden	= centilitre.
Vat	= hectolitre = 100 Kannen.

Whale and seal oil are sold by the Kwardeel of 12 Steekkannen = 51½ imp. galls.

**LENGTH.**

El	= metre.
Palm	= decimetre.
Durin	= centimetre.
Streep	= millimetre.
Roede	= decametre.
Mije	= kilometre.

**SURFACE.**

Bunder	= hectare = 2·471169 Eng. acres.
Vierkant-Roede	= are = 119·6046 sq. yds.
Vierkant-El	= centiare = 10·764414 sq. ft.

**SOLIDITY.**

Kubrek-El	= cubic metre = 35·81714 c. ft.
Kubrek-Palm	= cubic decimetre = 61·028 c. ins.

The former Weights and Measures are still used in some of the Dutch Colonies and Settlements.

1 Amsterdam lb.	=	1·0893 lb. Av.
1 Mudde of corn	=	·3826 imp. qrs.
1 Stoop of liquid	=	·5338 imp. gallons.
1 Amst. Ell	=	·7592 yards.
1 Leggar (240 stoops)	=	128·11 gallons.
1 lb. Av.	=	·91803 Amst. lb.
1 imp. qr.	=	2·613 Mudden.
1 imp. gall.	=	1·873 Stoops.
1 yd.	=	1·317 Amst. Ell.

**ITALY.****Moneys.**

1 lira Italiana = 100 centesimi.

The lira is of the same value as the franc.

Italy joined the 1865 French Monetary Convention.

The monetary system is the same as that of France.

**Exchanges.**

Stamps are thus required:—On cheques, 10 cti.—no endorsements are allowed.

On bills—up to 100 lire, 5 cti.: from 100 to 200, 10 cti.: from 200 to 300, 18 cti.: from 300 to 600,

36 cti. : from 600 to 1000, 60 cti. : above 1000 at 60 cti. per 1000 lire or fraction.

Bills for a currency greater than 6 months—double the above stamp.

Copies under lire 2000 pay as above—over lire 2000 they pay lira 1·20 whatever be the value.

If a second is sent for acceptance the first must be stamped in full.

Interest is calculated by taking the exact number of days, and 360 to the year.

The laws on bills of exchange are those of France.

### Course of Exchange.

Italy gives to

London.	short and 3 mos.	L. 26·10	for £1.
Paris.	"	103·60	for Fr. 100.
Germany.	"	127·10	for M. 100.
Austria.	"	247	for Fl. 100.

### Weights and Measures.

Those of the metric system with Italianised names easy to distinguish. The chilogramma is also called *libbra metrica*.

In Milan and Lombardy the denominations were different though the metrical system has been in force since 1803—these names still survive, but the ordinary names are being adopted.

decilitro = coppo.	chilogramma = libbra (metrica).
litro = pinta.	hectogramma = oncia "
decalitro = mina.	decagramma = grosso "
hectolitro = soma.	gramma = denaro
	decigramma = grano "
decimetro = palmi.	are = tavola.
centimetro = diti.	hectare = tornatara.
millimetro = atomi.	
chilometro = miglio.	

**EXAMPLES.****1. Direct. London—Italy.**

- (1) £2600. 18s. 9d., at 3 mos. 26·27½, 3 mos. 26. Discount, 2½ p.c., 2 p.c.  
 (2) £375. 9s. 10d., at 3 mos. 26·04. Discount, 2 p.c.  
 (3) £8000, at 3 mos. 26·25½. Discount, 1 p.c.  
 (4) L. 7325·25, at 3 mos. 26·27½, 3 mos. 26·05. Discount, 2 p.c., 3 p.c.

**2. Indirect. London—Turin.**

- (1) Through Paris, 25·20, 112.  
 (2) Through Berlin, 20·38½, 129.  
 (3) Through Vienna, 12·07½, 250.

Direct rates, 3 mos. 26·26, 26·10. Discount, 1 p.c., 2½ p.c.  
 Minimum payments for debts, £651. 8s. 9d., L. 12000?

**GREECE.****Moneys.**

1 drachme = 100 lepta.

Since 1867 the monetary system was to be assimilated to that of France—the drachme being equal to the franc. This reform is not complete.

The old drachme was ⅓ of the Spanish silver peso.

100 new drachmes are taken for 112 old drachmes.

Since 1877 the two issue Banks, the National and the Ionian, have greatly increased their note-issues by law. These notes are legal tender and pass at the exchange 100 new drachmes for 112 old drachmes, but they have fallen to a discount of 10 p.c. as regards metallic money.

Only the silver coins of the Latin Union are legal tender in addition.

20 new lepta are taken for 25 old lepta.

12 p.c. duty is charged on the export of Greek silver coins below 5 drachmes.



**Exchanges and Bills.**

Bills are stamped thus:—up to dr. 1500, 50 lepta per 750 dr.: from 1500 to 6000, 1 dr. per 1500: from 6000 to 15000, 2 dr. per 3000: above, 4 dr. per bovo or fraction.

**Course of Exchange.**

Athens gives to

London.	3 mos.	Dr. 28'50	paper for £1.
Paris.	31 days.	110'50	paper for Fr. 100.
Trieste.	3 mos.	236'20	paper for Fl. 100.
Amsterdam.	3 mos.	236'00	paper for Fl. 100.
Hamburg.	3 mos.	136'00	paper for M. 100.

In some Grecian Islands (Lyra, etc.) the monetary unit is the current drachme, then the quotation for Athens is about 125 (in current drachmes per 100 dr.).

The Course of Exchange at Corfu is as at Athens.

**Weights and Measures.**

The metric system, with different names in some cases.

The gramme is called the drachme: the decigramme, the obolos: the centigramme, the gram.

The common commercial weight is the mina = 1500 drachmes =  $1\frac{1}{2}$  kilogrammes.

The decilitre is called the kotylos: the centilitre, the mystra: the millilitre, the kubos.

The royal kilo is the same as the hectolitre (for corn).

The metre is called piki.

The royal stadeon = 1000 pikis.

The Greek mile = 10000 pikis = 10 kilometres.

1 royal piki = 1.5423 Constantinople piki = 1.4948 large piki.

The weights and measures of the Ionian Islands are those of Great Britain with Italian denominations.

**SCANDINAVIA. DENMARK.****Moneys and Bullion.**

1 krone = 100 öre.

Denmark is a party to the Scandinavian Mint Convention of 1872—at first with Sweden alone—Norway joining in it afterwards.

This convention established the gold standard.

The unit is the silver krone—legal tender up to 20 kroner only.

The gold coins are 20-kroner, and 10-kroner pieces.

The legal weight is given by this rate, 124 20-kroner pieces or 248 10-kroner pieces 900 fine must be coined from 1 kilogramme fine gold.

Thus the 10-kroner weighs 4·480286 grammes (4·032258 grammes fine).

The Danish National Bank buys bars and German gold coins at 2480 kr. per kilog. fine less  $\frac{1}{4}$  p.c., sovereigns and napoleons at the same price less 3 per mille. The German rules apply with corrections for the charges deducted.

The bullion weight at the Mint is the gramme (formerly the Danish-Cologne mark).

1 krone is taken as equal to  $\frac{1}{4}$  the old regsdaler of 96 skillinger.

Mint Par Chain.

? = £1.

£1869 = 480 oz. st.

oz. st. 12 = 11 oz. fine.

oz. f. 1 = 31·10349552 grammes fine.

gr. f. 1000 = 2480 kronas.

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Kr. 18·15952.

Coins. 20-Kronas = £1. 2s. 0·328d. 10-Kronas = 11s. 0·164d.

Specie Points. Kr. 18·06. Kr. 18·26.

Leaving. Coming.

**Exchanges and Bills.**

The Scandinavian law of 1880 on bills of exchange abolished the 8 days of grace before existing in Denmark.

Protest for non-payment must be made the second working day after maturity.

The legal commission is  $\frac{1}{3}$  p.c. for each indorsement, but the total commission must not exceed 2 p.c.

Sight bills must be presented within 6 mos. when drawn in Europe, within one year when drawn out of Europe.

Bills due on Sunday or a Holiday are payable the day after.

Bills are stamped thus:—on bills not exceeding 8 days' sight or 14 days' date, 20 ore: on all others up to kr. 1000, 20 vere: 1000—2000, 35: 2000—4000, 70: 4000—6000, 1 kr.: above, every 6000 additional to the above, 1 kr. (e.g. 14000 = 235).

**Course of Exchange.**

Copenhagen gives to

London.	short, 3 mos.	Kr. 18·20	for £1.
Paris.	"	72·00	for Fr. 100.
Hamburg.	"	88·70	for M. 100.
Amsterdam.	"	150·00	for Fl. 100.

**EXAMPLES OF EXCHANGES.**

1. Direct. London—Copenhagen.

(1) £584. 9s. 7½d., at 3 mos. 18·34, 3 mos. 18·12. Discount, 2 p.c., 3½ p.c.

(2) £312. 5s. 6½d., at 3 mos. 18·19½. Discount, 2 p.c.

(3) Kr. 9564, at 3 mos. 18·35. Discount, 1½ p.c.

(4) Kr. 10,000, at 3 mos. 18·33, 3 mos. 18·14. Discount, 2 p.c., 2½ p.c.

## 2. Indirect. London—Copenhagen.

- (1) Through Paris,  $20\cdot22\frac{1}{2}$ ,  $72\cdot10$ .  
 (2) Through Hamburg,  $20\cdot37$ ,  $89\cdot20$ .  
 (3) Through Amsterdam,  $11\cdot97\frac{1}{2}$ ,  $151\cdot35$ .

Direct rates, 3 mos.  $18\cdot35$ , 3 mos.  $18\cdot15$ . Discount, 2 p.c.,

3 p.c.

Which is best rate for paying £1000 or 10,000 Kr. ?

## Weights and Measures.

The Metric System is contemplated.

## WEIGHT.

2 Lod	= 1 Unser.
16 Unser	= 1 Pund (Danish).
10 Ort	= 1 Quintin.
100 Quintins	= 1 Pund.
100 Punds	= 1 Centner.
12 D. Punds	= 1 Bismar Pund (steelyard).
8 Bismar Punds	= 1 Vog.
4000 D. Punds	= 1 Last.
1 Skibslast	= $16\frac{1}{2}$ Skippund = 5200 D. Punds.
1 Pund	= 500 grammes = $\frac{1}{2}$ kilog. (Pfund).
1 Pund	= 1·10231 lb. Avoir.
1 lb. Avoir.	= ·907185 D. Pund.
1 cwt.	= 101·605 D. Punds.
1 skippund	= 352·7393 lb. Av.
8 skippund	= 25 cwt. nearly.

## LIQUID.

4 Paegle	= 1 Potte.
8 Potter	= 1 Viertel.
38 $\frac{1}{2}$ Potter	= 1 Anker.
6 Ankers	= 1 Oxhoved.
2 Oxhoveder	= 1 D. Pipe.
32 Potter	= 4 Viertel = 1 D. cub. ft.
1 Oxhoved	= 49·4393 imp. gall. = 224·623 litres.
1 Imp. Gall.	= 4·7027 Potter = ·5879 Viertel.
1 Dan. c. ft.	= 1886·71 Eng. cub. ft.

## DRY.

4 Fjerdingkar	= 1 Skjepper.
8 Skjepper	= 1 Tønde (barrel).
22 Tønder	= 1 D. Corn Last.
12 Tønder	= 1 D. Lune (Salt) Last.
18 Tønder	= 1 D. Coal Last.
144 Potter (Liq.)	= 1 Tønde = $4\frac{1}{2}$ cub. ft. D.
1 Tønde	= 3·827566 imp. bushels.
1 Tønde	= 478445 imp. qrs.
1 imp. qr.	= 2·0901 Tønder.
1 Dan. Corn Last	= 10·5258 imp. qrs. = 306·066 hectol.
1 Corn Shipping Last	= 80 c. ft. Danish = 2 tons (Eng. Reg).

## LENGTH.

12 Tommer	= 1 Fod.
1 Fod	= 2 Alen.
12000 Alen	= 1 Mål (Mile D.).
1 D. Ell (Alen)	= 68647 Eng. yds.
1 Eng. yard	= 1·45673 D. Ell.
1 D. Mål	= 8237·82 E. yds. = 4·68058 E. mi. = 7·5325 kilom.

Norwegian Deals are reckoned by the Norwegian Standard Hundred = 120 pieces, each 12 feet long, 3 inches thick, 9 inches broad = 270 English cubic feet.

**NORWAY.****Moneys.**

1 krone = 100 öre.

Norway has joined the Scandinavian Mint Convention.

Her monetary system is the same as Denmark.

4 kroner are taken as equal to the old species daler of 120 skilling (= 2 Danish rigsdaler).

**Exchanges.**

The Scandinavian law on bills of exchange abolishes the 10 days of grace hitherto existing in Norway.

**The Course of Exchange** is the same as at Copenhagen.

**Weights and Measures.**

The metric system since 1875.

The former weights and measures were those of Denmark.

Deals shipped at Christiania, Drammen, Fredericks-hall and Gothenburg (in Sweden) are measured by the Norwegian standard hundred of 120 pieces 12 ft. long, 3 inch. thick, 9 inch. broad Eng. measure (= 270 c. ft.).

**SWEDEN.****Moneys.**

1 krona = 100 öre.

By the Scandinavian Mint Convention of 1872 (Sweden, Denmark—afterwards Norway) the gold standard has been introduced.

The money is the same as in Denmark.

Legally the new krona is taken as equal to the old rigsdaler riksmünt.

This gives silver to gold as 1 : 15·81.

Also 8 daler banco are taken as equal to 3 daler species or 12 rigsdaler riksmünt, but these are rapidly disappearing.

**Course of Exchange.**

Stockholm gives to

London.	Short, 90 days' date.	Kr. 18·10	for £1 stg.
Paris.	" "	71·20	for Fr. 100.
Hamburg.	" "	88·00	for M. 100.
Amsterdam.	" "	150·00	for Fl. 100.
S. Petersburg.	" "	190·00	for Ro. 100.
Copenhagen.	" "	99·80	for Kr. 100.

**Bills and Exchanges.**

The Scandinavian law on bills of exchange is in force.

Bills pay 1 krona stamp without regard to the amount.

Seconds or copies are exempt.

Letters of credit pay 50 öre (as stamp).

**Weights and Measures.**

Since 1889 the French metrical system exclusively. It has been in use since 1875 with the old system. These former weights and measures were also decimal, but the integers were various and very confusing, e.g. there were six weights and four dry measures.

**EXAMPLES OF EXCHANGES.****1. Direct.**

(1) £625. 7s. 8½d., at 3 mos. 18'37, 3 mos. 18'12. Discount, 3 p.c., 2 p.c.

(2) Kr. 18561·74, at 3 mos. 18'34½, 3 mos. 18'11. Discount, 1 p.c., 2 p.c.

**2. Indirect. London—Stockholm.**

(1) Through Paris, 25'17½, 71'32.

(2) Through Copenhagen, 18'25, 98'95.

(3) Through S. Petersburg, 102'31, 195'00.

Direct rates, 3 mos. 18'40, 3 mos. 18'09. Discount, 3 p.c., 2 p.c.

Find minimum payment for debts of £8000. 20,000 Kr. ?

**TURKEY IN EUROPE.****Money.**

100 aspres = 1 piastre = 40 paras.

100 piastres = 1 gold medjidié or Turkish lira.

This coinage was issued in 1844 and is called medjidié, after the Sultan Abdul-Medjid.

The gold coins (not now coined) are :

The lira turca (medjidié) = 100 piastres, weight 111.359 gr. Tr. = 7.216 grammes fine.

The ellilik =  $\frac{1}{2}$  lira turca = 50 piastres.

The tzeirek =  $\frac{1}{4}$  „ = 25 „

The standard is  $\therefore$  nominally gold, but practically silver, as gold is no longer coined.

The silver coins are 1-piastre, 2 (ikilik), 5 (bechlik or  $\frac{1}{2}$  silver medjidié), 10 (onlik or  $\frac{1}{4}$  medjidié), 20 (gumneh or silver medjidié) piastres, 830 fine.

There are many base silver coins, and every European nation has given coins to Turkey.

£1 is taken at 125 piastres, 1 napoleon at 100.

Five-franc pieces and smaller Russian and Austrian coins form the chief part of the foreign silver currency.

Gold and silver coins are at a premium against caïmés—a paper currency issued during the Russo-Turkish War—now almost out of circulation.

Mint Par for gold £1 = 110.700 T. piastres. £1 T. = 18/0.8. This is useless owing to discontinuance of coinage.

**Exchanges.**

The bill stamp is  $\frac{1}{2}$  per mille.

Of bills in sets only one need be stamped.



### Course of Exchanges.

Constantinople gives to

London. 3 mos' date : 31 days' sight. 109·50 P. for £1.

Constantinople receives from

Paris.	3 mos.	23 Fr. for £1 Turkish.
Vienna.	3 mos.	12 Fl.    "    "

Salonica gives to

Paris. 3 mos. 4½ P. for 1 Franc.

### EXAMPLES.

1. Direct. London—Constantinople.

(1) £512. 18s. 9d., at 3 mos. 110·56, 3 mos. 108·75. Discount, 1½ p.c., 2 p.c.

(2) £T.4000, at 3 mos. 111·96. Discount, 1 p.c.

(3) £684. 17s. 6d., at 3 mos. 109·71. Discount, 1½ p.c.

(4) £T.300·74, at 3 mos. 111·40, 3 mos. 108·25. Discount, 2 p.c., 1 p.c.

### Weights and Measures.

The metric system introduced in 1874.

In 1882 by decree of the Sultan the archine was declared to be one metre in length and adopted as the unit.

The oka was divided into 100 drachmas subdivided in tenths.

The old weight (oka) remains in force only for gold and silver.

Former weights and measures still in use to some extent :—

4 grains	= 1 carat.
16 carats	= 1 dram.
400 drams	= 1 oka.
4 chekys	= 1 oka.

- 1 rottolo (of 180 drams) = 1·27 lb. av.  
 1 oka (of 400 drams) = 2·818565 lb. av.  
 1 cheky ( $\frac{1}{4}$  oka) = 4942·3503 grams Tr. = 320·259 grammes.  
 1 alma or almud (liquid measure) = 1·14555 imp. gall.  
 1 killow (of corn) = 0·97 imp. bushel.  
 1 pik or droá = 26 $\frac{1}{2}$  Eng. inches = 27 inches (usually).  
 1 halebi or arschin (for land) = 27·9 Eng. inches.

**Turkey in Asia** has the same moneys, but the Smyrna piastre is at a discount, the exchange on Constantinople being at 110 S. piastre per lira.

It gives to London 3 mos' date S. piastres (130) per £1.

The metric system is in use since 1874, but there is a bewildering variety of weights and measures. The real unit of weight is however the dirhem.

## EGYPT.

### Moneys.

10 ochr el guerches = 1 piastre.—Introduced by the law of November, 1885.

100 piastres = 1 Egyptian pound.

40 paras = 1 piastre.

The Egyptian pound weighs 8·500 grammes 875 fine.

20, 10, 5, 2, 1 piastre pieces are issued in silver—legal tender to £2.

The Government receives English sovereigns at 97·50, napoleons at 77·15, Turkish pounds at 87·75, imperials at 79·45 piastres, with fractions for the halves.

There is also an inland money called "corrente"—a depreciated currency of the tariff piastre at a considerable discount.

**Course of Exchange.**

Alexandria gives to

London.	3 days' sight, 3 mos.	98 piastres for £1.
Paris.	3 days' sight, 3 mos.	382 „ 100 Fr.
Switzerland.	3 mos.	381 „ „
Italy.	3 mos.	380 „ 100 L.

**EXAMPLES.**

1. Direct. London—Cairo.

(1) £584. 8s. 10½d., at 3 mos. 99½, 3 mos. 95. Discount, 2 p.c., 3 p.c.

(2) £E.8340, at 3 mos. 100, 3 mos. 96. Discount, 2½ p.c., 3¼ p.c.

**Weights and Measures.****WEIGHT.**

144 dirhems = 1 rottolo.

100 rottoli = 1 cantar.

36 oke = 1 cantar.

Unit = 1 dirhem = 47·66131 grains = 3·0884 grammes.

1 Government cantar = 98·0461 lbs.

1 common cantar = 99 lbs.

40 oke = 108·94 lbs. (exactly) = 110 (commercially).

**MEASURE.**

Legal unit = 1 ardeb.

24 Cairo rubbre = 1 ardeb.

100 ardebs = 63 imp. qrs. (commercially).

100 imp. qrs. = 159 ardebs „

**LENGTH.**

Unit = the Turkish pik of 27 inches.

The introduction of the metric system is decreed.

**Abyssinia** reckons in sequins (9s. 6d.) and borjookes (glass corals). 1 sequin = 6210 borjookes.

Large accounts are paid in gold (bars or dust) weighed in wakeas.

The Abyssinian rottolo of 12 wakeas (ounces) of 10 derimes = 4800 grains Troy.

The Gondar ardeb = 0·121055 imp. bushels, the Massowah ardeb = 0·290532 imp. bushels,  $\therefore$  10 Massowah ardebs = 24 Gondar ardebs.

The pik (27 inches) is the unit of length.

## DOMINION OF CANADA.

### Moneys.

100 cents = 1 dollar.

The standard is gold based upon the legal rate  $\text{£}1 = 4\cdot80\$$ .

This gives  $\$108 = \text{£}22\cdot5$ . 1 dollar = 4s. 2d.

The Dominion has no coinage of its own except pieces of 20, 10, 5 cents in silver and some copper tokens.

English and American coins circulate freely.

**Weights and Measures** as in Great Britain.

### Exchanges.

The bill stamp is—up to 25\$, 1 cent; from 25 to 50\$, 2 cents; from 50 to 100, 3 cents; above 100\$, 3 cents per 100 or fraction.

There are 3 days of grace.

### Course.

London is quoted 60 days' sight (demand) at a premium on the fixed exchange,  $\$100 = \text{£}22\frac{1}{2}$  or  $\$40 = \text{£}9$ .

This gives \$1 = 4s. 6d., and so the "par of exchange" is 8 inasmuch as the legal value of \$1 is 4s. 2d.

New York is quoted at a premium or discount per 100\$.

Chain for sterling value of 1000 dollars at par (i.e. 8 p.c. premium) remitted from Montreal.

$$\begin{array}{r} ? = 1000\$ \\ 40 = £9. \\ 108 = 100. \\ \hline 12 \mid \frac{2500}{100} \mid 208\frac{3}{4} \\ \hline \frac{100}{4} \\ \hline £208. 6s. 8d. \end{array}$$

At any other premium multiply  $\frac{100 + \text{premium}}{108}$  by £208 $\frac{3}{4}$ .

e.g. at 3 p.c.

$$\begin{array}{r} 208\cdot3333 \\ 301 \\ \hline 208\ 3333 \\ 6\ 2500 \\ \hline 1,08 \mid 21458\cdot33 \mid 198\cdot689 \\ \hline 106\ 5 \\ \hline 9\ 38 \\ \hline 743 \\ \hline 95 \\ \hline 9 \end{array}$$

13s. 9 $\frac{1}{2}$ d.

Table for value of Silver Dollar at any Premium.

1 =	£·00192901234567
2 =	·00385802469135
3 =	·00578703708703
4 =	·00771604938271
5 =	·00964506172839
6 =	·01157407407407
7 =	·01350308641975
8 =	·01548209876543
9 =	·01736111111111

*Example.* Premium 3. Find 103.

$$\begin{array}{r}
 .192901 \\
 .005787 \\
 \hline
 1 = £198688 \\
 1000 = £198\cdot688
 \end{array}$$

### EXAMPLES.

1. Find value in dollars of £875. 16s. 1d., at 6 p.c. premium.
2. Find value in sterling of \$25186·72, at 5 p.c. premium.
3. Find value in dollars of £1000, at 10 p.c. premium.
4. Find value in sterling of \$7125, at  $7\frac{1}{2}$ .

## THE UNITED STATES.

### Moneys.

1 dollar = 100 cents.

Gold and silver are both standards.

Gold is naturally at a considerable premium for inland exchange.

By the "Sherman Bill" the Government and it alone must coin silver, and to the value of not less than  $4\frac{1}{2}$  million dollars per month.

The gold coins are the eagle (10\$), and half-eagle (5\$).

The eagle weighs 258 grs. Troy 900 fine (0·5375 oz. 900 fine).

Hence Mint price of gold is \$20·67183 per oz. fine or \$18·60465 per oz. 900 fine.

The chief silver coin is the silver dollar—it weighs 412½ grs. Tr. 900 fine.

Its value varies with the price of silver.

Assay reports are in millièmes and halves.

Gold is quoted on the basis of 43 oz. 900 fine for 800\$—generally par.

Silver is quoted in gold dollars and cents per oz. silver 999 fine.

The bullion weight is the Troy lb. and its divisions.

The U.S. Mint receives bars or coins only at their actual weight and fineness on the above bases for gold and silver.

The Mint charge for bars is 1 per mille for gold 900 fine, 4 per mille for fine gold. Payment is made a few days after receipt.

The charge for melting coins and bars is very small.

Mint Par for gold.

$$? = 1\text{£}.$$

$$1869 = 480 \text{ oz. st.}$$

$$12 = 11 \text{ oz. fine.}$$

$$9 = 10 \text{ oz. } \frac{9}{10} \text{ fine.}$$

$$1 = 18\cdot60465\$.$$

---


$$\text{£}1 = \$4\cdot866564.$$

$$\text{Coins } 10\$ = \text{£}2. 1s. 1\cdot16d. \quad 5\$ = \text{£}1. 0s. 6\cdot58d.$$

$$\therefore 1 \text{ Gold dollar} = 49\cdot316d.$$

Specie points

\$4\cdot83  
Leaving.

\$4\cdot90.  
Coining.

### Arbitrated Pars and Prices.

1. Bar gold. Chain for Par.

$$? = 20s.$$

London price = 1 oz. st.

$$12 = 11 \text{ oz. fine.}$$

$$9 = 10 \text{ oz. } \frac{9}{10} \text{ fine.}$$

$$43 = \$800 \pm \text{p.c. prem. or disct.}$$

Fixed Number = .2545118.

Rule.  $F.N. \times 800 \pm \text{prem. or disct p.c.} = \text{London price} \times \text{rate of exchange.}$

Example. In London 77s. 9d. per oz. st. } Rate?  
In U.S. \$800 per 48 oz. 900 fine } Ans. \$4.8744.

2. Silver dollars. Chain for price.

? = 1 dollar.

1 =  $412\frac{1}{2}$  gr. 900 fine.

10 = 9 gr. fine.

480 = 1 oz. fine.

37 = 40 oz. st.

1 = price of bar silver per oz. st. in London.

Fixed Number = .8861486.

Rule.  $F.N. \times \text{London price} = \text{par of 1 dollar.}$

### EXAMPLES.

Find value of silver dollar, when bar silver is in London per oz. st.

(1) $40\frac{5}{16}$ .	(4) $37\frac{1}{16}$ .	(7) $39\frac{1}{16}$ .	(9) $41\frac{1}{2}$ .
(2) $41\frac{1}{8}$ .	(5) $39\frac{5}{8}$ .	(8) $40\frac{1}{8}$ .	(10) $40\frac{5}{8}$ .
(3) 43.	(6) $38\frac{1}{16}$ .		

Table for Rate of Exchange, arbitrated from bar gold at 77s. 9d. per oz. st. in London.

1 = .006093	Ex. If premium is $1\frac{1}{2}$ p.c.
2 = .012186	Find $800 + 12 = 812$ .
3 = .018279	
4 = .024372	4.8744
5 = .030465	.0609
6 = .036558	.0122
7 = .042651	4.9475 = Rate.
8 = .048744	
9 = .054837	

Similar Tables may be made for 77/11, 77/10 $\frac{3}{4}$  etc.



Table for Price of Silver Dollar, given price of bar silver in London per oz. st.

Bar silver per oz. st.		Ex. Price = $41\frac{1}{8}$ .
1d. = .8361486		
2 = 1.6722972		
3 = 2.5084459		33.446
4 = 3.3445945		.836
5 = 4.1807432		.261
6 = 5.0168918		$34.543d. = 1\$.$
7 = 5.8530405		
8 = 6.0891891		
9 = 6.9253378		

### 3. Inland exchange.

Gold and silver are both legal tender, so there are two problems of inland exchange.

(1) To find value in currency (cents of the silver dollar) of a gold dollar—given market price of silver currency.

Divide 100 by market price.

*Example.* Market price of currency = 72 cts.

$\therefore$  Value of gold dollar =  $\frac{100}{72} = 138\frac{1}{3}\%$ .

(2) To find market price of currency (silver cents)—given value of a gold dollar.

Divide 100 by value of dollar.

*Example.* Gold dollar = 140 cents.

$\therefore$  Price of currency =  $\frac{100}{140} = 71\frac{2}{7}\%$ .

### Bills and Exchanges.

The bill stamp is 2 cents on all cheques and bills drawn on the U. S.

There are three days of grace, but demand bills have none.

Bills due on a Sunday or Holiday are paid the day before.

**Course of Exchange.**

New York gives to

London.	Cable transfers. Sight and 60 days' sight.	\$4·86½ for £1.
Hamburg.	" "	\$95 for M. 400.
Amsterdam.	" "	\$42 for Fl. 100.

New York receives from

Paris	" "	Fr. 5·20 for \$1.
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S. Louis, Orleans, etc. are quoted in premium or discount p.c. on 100\$.

**EXAMPLES.**

1. Direct. London—New York.

(1) £851. 7s. 6d., at 60 days' sight  $49\frac{3}{8}$  and  $4·85\frac{1}{2}$ . Discount,  $2\frac{1}{4}$  p.c.,  $2\frac{1}{4}$  p.c.

(2) \$1270·36, at 60 days' sight  $49\frac{1}{2}$ . Discount,  $1\frac{1}{2}$  p.c.

(3) £912. 18s. 6d., at 60 days' sight 4·83. Discount, 2 p.c.

(4) \$3000, at 60 days' sight  $49\frac{5}{8}$  and  $4·87\frac{1}{2}$ . Discount, 2 p.c.,  $2\frac{1}{2}$  p.c.

2. Indirect. London—New York.

(1) Through Paris, short 25·19, 60 days 5·21.

(2) Through Hamburg, 3 mos. 20·53, 60 days 96·25.

(3) Through Amsterdam, 3 mos. 12·09, 60 days 41·75.

Direct rates, 2 mos. (60 days)  $49\frac{1}{4}$  and  $4·85\frac{1}{4}$ . Discount, 1 p.c.,  $2\frac{3}{4}$  p.c.

Minimum payments of \$8000 and £950?

3. Simple circuit.

What is gain or loss on £1000 invested in Amsterdam bills at 12·07, and sold in New York at 41·85, my returns being bills of 60 days' sight at  $4·85\frac{1}{2}$ , the time being 16 days? Bank interest (deposit)  $2\frac{3}{4}$  p.c.

**Weights and Measures.**

The same as Great Britain, with the old wine gallon and the Winchester bushel.

## Tables for Conversion.

O. W. Galls. to Imp. Galls.

1 =	·833109
2	1·666218
3	2·499327
4	3·332436
5	4·165545
6	4·998654
7	5·831763
8	6·664872
9	7·497981

Imp. Galls. to O. W. Galls.

1 =	1·20032
2	2·40064
3	3·60096
4	4·80128
5	6·00160
6	7·20192
7	8·40224
8	9·60256
9	10·80288

Winch. Bush. to Imp. Bush.

1 =	·9694472
2	1·9388944
3	2·9083416
4	3·8777888
5	4·8472360
6	5·8166832
7	6·7861304
8	7·7555776
9	8·7250248

Imp. Bush. to Winch. Bushel.

1 =	1·031557
2	2·063114
3	3·094671
4	4·126228
5	5·157785
6	6·189342
7	7·220899
8	8·252456
9	9·284013

**THE EAST INDIES.****Calcutta. Madras. Bombay.****Moneys and Bullion.**

3 pie	= 1 pice.
4 pice	= 1 anna.
16 annas	= 1 rupee.

In 1862 the Government rupee took the place of the Company rupee of the same value (weight and fineness).

The coins are—silver, 1 rupee of 180 gr. Tr. silver  $\frac{11}{12}$  fine, and  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$  rupee of same fineness and weight (called 8, 4, 2-anna pieces): copper,  $\frac{1}{2}$ -anna, pice

( $\frac{1}{4}$ -anna), half-pice and pie, which weigh at rate of 400 grains Troy per anna.

N.B. The anna does not exist actually.

Gold is also coined for private account but treated as a commodity.

The coins are the gold mohur of same weight and fineness as the rupee (nominal ratio 15 : 1), the double mohur, a ten-rupee, a five-rupee.

In the bazaars, the cowrie is used for small payments—its value rising and falling with demand and supply.

Generally 320 cowries make 1 anna.

At £3. 17s. 10 $\frac{1}{2}$ d. per oz. stand. gold (British Mint)  
1 gold mohur = £1.46015625.

1 lac = 100,000 rupees, 1 crore = 100 lacs, 1 mas = 100 crore, 1 crore = 10 million rupees.

The Indian standard was a silver one until the closing of the Mints to the free coinage of silver, and steps are now being taken to introduce the gold standard.

The fineness of gold and silver is reported in millièmes.

Gold is quoted in rupees per 1 tola fine.

Silver is quoted in rupees per 100 tolas sycee (980 fine).

The Mint price for 100 tola fine is 109 $\frac{1}{10}$  rupees or 106 $\frac{3}{4}$ —106 $\frac{1}{2}$ , deducting charges. This is calculated from the rupee's weight and fineness (1 tola  $\frac{11}{12}$  fine). Any demand for silver in the bazaar raises the price to 107—108.

The Calcutta Mint receives silver for coinage at 1 rupee per 1 tola  $\frac{11}{12}$  fine less 2 $\frac{1}{10}$  p.c. seignorage.

The standard is silver and  $\therefore$  there is no Mint par.

**Arbitrated Par for Silver** (rupee).

- $\frac{1}{2}$  = 1 rupee.  
 1 =  $\frac{3}{8}$  oz. st. India ( $\frac{1}{4}$ ).  
 12 = 11 oz. fine.  
 37 = 40 oz. st. England.  
 1 = price in d. of bar silver per oz. st. in London.

Fixed Number = 37162.

Rule. F. N.  $\times$  price in London = rate of exchange.

The charges are not included and must be added to the rate as a final answer or before working (when given).

They amount to  $3\frac{1}{2}$  p.c. (including  $2\frac{1}{10}$  seignorage).

The connection between London price of bar silver and London—Calcutta rate of exchange is thus obvious.

**Table of Exchanges** (London—Calcutta, sight).

Bar silver in London. Pence per oz. st.	Calcutta sight in London. Exc. charges.	Calcutta sight in London. Inc. $3\frac{1}{2}$ p.c. charges.
1 =	37162	38462837
2 =	74324	76925675
3 =	111486	115388513
4 =	148648	153851351
5 =	185810	192334189
6 =	222972	230797027
7 =	260135	269259864
8 =	297297	307722702
9 =	334459	346185540

*Example 1.* Price  $39\frac{1}{2}$ .      115389  
    34619  
    3365  
 1 rupee = 153373d.

*Example 2.* Price  $40\frac{1}{2}$ .      153851  
    0481  
 1 rupee = 154332d.

**EXAMPLES.**

Find exchanges (Calcutta sight on London), when price of bar silver is (1)  $39\frac{1}{2}$ , (2)  $38\frac{1}{2}$ , (3)  $40\frac{1}{2}$ , (4)  $39\frac{1}{2}$ , (5)  $41\frac{1}{2}$ .

**Bills and Exchanges.**

"Documentary" bills upon India are discounted at the current rate of Government acceptances (if not above 5 p.c.) when taken up before maturity.

The bill-stamps are—"on demand" above 20 R., 1 anna.

Bills "not on demand":—up to Rs. 2500, 2 annas per 200 (very nearly): then from 2500 to 10000, 1 R. 8 annas per 2500 or fraction: from 10000 to 30000, 3 R. per 5000 or part: above 30000 Rs., 6 R. per 10000 or part.

Bills in sets of two and three pay the nearest whole number of annas above the  $\frac{1}{2}$  and the  $\frac{1}{3}$  of the above prices.

Telegraphic Transfers are largely employed.

**Course of Exchange.**

Calcutta receives from

London.	sight, 4 & 6 mos' sight.	1s. 4d.	for 1 Rupee.
Paris.	sight, 3 & 6 "	Fr. 2'04	for 1 "
Australia.	60 days' sight.	1s. 4d.	for 1 "
Réunion.	"	Fr. 2'05	for 1 "

Calcutta gives to

Hong-Kong & Canton.	60, 90 days' sight.	R. 220	for 100 Ds.
Shanghai.	"	R. 310	for 100 Tls.
Colombo.	sight, 30 days' sight.	5 Annas	premium on 100 Rupees.

Calcutta quotes

Mauritius.	60 days' sight discount p.c.
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Bombay receives from

London.	sight, 4 & 6 mos' sight.	1s. 4d.	for 1 Rupee.
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**EXAMPLES.****1. Direct. London—Calcutta.**

(1) 15000 Rs. at 4 mos. 1s.  $3\frac{1}{8}d.$ , and 6 mos. 1s.  $4\frac{1}{8}d.$  Discount, 3 p.c.,  $3\frac{1}{2}$  p.c.

(2) £875. 9s. 10d., at 4 mos. 1s.  $3\frac{1}{8}d.$ , and 6 mos. 1s.  $5\frac{1}{8}d.$  Discount, 3 p.c., 4 p.c.

**2. Indirect. London—Calcutta.**

(1) Through Paris, short 25·19, 3 mos. 2·15.

(2) Through Hong-Kong, 4 mos. 2s.  $11\frac{1}{8}d.$ , 3 mos. 225.

Direct rates, 4 mos. 1s.  $3\frac{1}{8}d.$ , 6 mos. 1s. 6d. Discount, 3 p.c.,  $3\frac{1}{2}$  p.c.

Minimum cost of bills for 10,000 Rs. and £825 ?

**Weights and Measures.**

The basis of Indian weights is the tola of 180 grs. Tr.

The tola itself is used for weighing bullion.

Its subdivisions are used by jewellers to weigh pearls and precious stones.

Its multiples are the general commercial weights in use.

**Bengal.**

5 tolas = 1 chittack.

16 chittacks = 1 seer.

40 seers = 1 maund (Bazaar).

**Madras.**

3 tolas = 1 pollam.

8 pollams = 1 seer.

40 seers = 1 maund.

20 maunds = 1 candy.

**Bombay.**

40 seers = 1 maund.

20 maunds = 1 candy.

The Bazaar maund = 100 lb. Troy =  $82\frac{2}{3}$  lbs. Av.

The Factory „ =  $74\frac{1}{8}$  lbs. Av.

The Madras „ = 24·685703125 lbs. Av.

The Bombay „ = 28 lb. Av.

Bazaar M. : Factory M. = 100 : 91.

The Candy = 493·7143 lb.

The Candy = 560 lb. Av.

Liquids are sold by weight or by the imp. gall. or O. E. gallon.

The Calcutta kahoon of corn weighs 40 Factory maunds = 2986 $\frac{2}{3}$  lbs. Av.

The Madras garse (corn) of 80 paratis of 5 marcals = 16·906 imp. qrs.

The Bombay candy (dry) is taken at 24·5 imp. bush.

The English yard is increasingly used.

The guz (original Indian linear unit) varies in the different provinces.

1 hanth is taken at 18 inches. 1 guz at 27 inches.

1 ady (Malabar foot) = 10·46 English inches.

The “**Indian Weights and Measures Act**” of 1870 enacted as follows:—

Standard of weight = the ser (the French kilogramme) of 80 tolas.

Standard of length = the mètre (3·280899 ft.).

Standard of capacity = the ser (1·760773 pints = ·220096 gallons).

1 tola = 0·01164 ser.

1 seer = 0·9331 ser.

1 mun = 37·3242 ser = 100 lb. Troy.

1 gramme = 0·08573526 tola.



**HONG-KONG and the Treaty Ports  
CANTON, FOOCHOW, AMOY.**

**Moneys and Bullion.**

At these places accounts are kept in dollars and cents.

At Hong-Kong, Canton, Foochow chopped dollars circulate. These are examined and weighed at 717 taels Canton weight per 1000\$.

At Foochow any sort of chopped dollars pass, but only the good ones are taken at par in Hong-Kong and Canton.

At Amoy accounts are kept in currency dollars, at 720 taels Canton weight per 1000\$.

Clean or unchopped Mexicans command a small premium.

These dollars circulate—Japanese yens (416 grs. Tr.), Japanese trade dollars (420 grains Tr.), Hong-Kong dollars (416 grains), Mexican dollars (416½ grains 898 fine), and Spanish Carolus dollars (414 grains 894 fine).

The value of these in sterling varies with the price of bar silver in London and fluctuates between 2s. and 3s.

Sycee and bar silver are taken and quoted at a premium on the fixed par of 717 taels Canton weight per 1000\$.

These ports also quote gold leaf 100 touch in dollars per tael, gold bars 98 touch in dollars per tael, and sovereigns and U.S. eagles in dollars per piece.

The standard is silver and ∴ there is no Mint par.

**Arbitrated Par for Silver Dollar (Hong-Kong).**

$$? = 1.$$

100 + prem. p.c. = 71.7 taels Canton weight.

$$1 = 580 \text{ grains.}$$

$$480 = 1 \text{ oz. of given fineness.}$$

$$240 = (222 \pm \text{betterness or worseness}) \text{ oz. fine.}$$

$$37 = 40 \text{ oz. st.}$$

$$1 = \text{price per oz. st. bar silver in London.}$$

Fixed Number = 39025900.

Rule.  $F. N. \times (222 \pm B. \text{ or } W.) \times \text{price in London in } d = \text{Rate of Exchange.}$

This gives the par of exchange without charges—these are  $1\frac{1}{2}$  p.c. and must be added to the rate before working or to get a final answer.

**EXAMPLES.**

1. Bar silver, 17 B. in London,  $40\frac{1}{8}d.$  per oz. st., in Hong-Kong, 3 p.c. Rate?
2. Bar silver, 16 B. in London,  $41\frac{1}{2}d.$  per oz. st., in Hong-Kong, 4 p.c. Rate?
3. Bar silver, 14 B. in London,  $40\frac{1}{8}d.$  per oz. st., in Hong-Kong,  $3\frac{1}{4}$  p.c. Rate?

**Bills and Exchanges.**

The bill stamps are—"On demand" 2 cents.

"Not on Demand":—up to 100\$ 30 cents singly, 15 cents each in sets: from 100 to 3000\$ 1 dollar singly, 50 cents in sets: above 3000\$ 1 dollar 50 cents singly, 75 cents. in sets.

**Course of Exchange.**

The Four Ports receive from

London.	Bank 4 mos.' sight drafts.	2s. 10 $\frac{1}{2}$ d.	per 1\$.
Paris.	"	4'18 Fr.	per 1\$.
Bombay. }	Bank demand drafts.	219 Rs.	per 100\$.
Calcutta. }			
Shanghai.	"	74 Ts.	per 100\$.

Japan is quoted at a premium in \$ for Bank demand drafts.

**EXAMPLES.**

1. Direct. London—Hong-Kong.

- (1) £875. 16s. 7 $\frac{1}{2}$ d., at 4 mos. 2s. 8 $\frac{1}{2}$ d., and 4 mos. 2s. 11 $\frac{1}{2}$ d.  
Discount, 2 p.c., 3 $\frac{1}{2}$  p.c.  
(2) \$1000, at 4 mos. 2s. 9 $\frac{3}{8}$ d. Discount, 2 p.c.  
(3) £1000, at 4 mos. 3s. 1 $\frac{1}{4}$ d. Discount, 3 p.c.  
(4) \$8514.75, at 4 mos. 2s. 9d., and 4 mos. 3s. 0 $\frac{1}{2}$ d. Discount, 3 p.c., 3 $\frac{1}{4}$  p.c.

2. Indirect. London—Canton.

- (1) Through Paris, short 25.17 $\frac{1}{2}$ , 4 mos. 4.21.  
(2) Through Shanghai, 4 mos. 3s. 11 $\frac{3}{8}$ d. Demand, 75.  
Direct rates, 4 mos. 2s. 10 $\frac{1}{8}$ d., 4 mos. 3s. 1 $\frac{1}{8}$ d. Discount, 2 p.c., 3 p.c.  
Minimum cost of bills for £1825, \$6530?

**CHINA (SHANGHAI, ETC.).****Moneys and Bullion.**

- 10 cash or li = 1 candareen.  
10 candareen = 1 mace.  
10 mace = 1 tael.

This is the money of account, only the cash or li are actual coins. The others are fixed weights of silver.

The Canton tael weight = 580.036434 grains Troy  
= 580 roughly,  $\therefore$  taels = oz. Tr.  $\times \frac{3}{4}$ .

$\frac{3}{4}$  = 1.208 $\bar{3}$  (decimals) or 1—64 (aliquots).

The li is not a silver coin, but an alloy of copper, iron, and tin with a hole in centre. It is cast in moulds and nominally  $\frac{1}{1000}$  of a tael, but owing to depreciation 1 tael = 1600—1800 cash now.

The unit is the tael.

At Shanghai the money tael used is the quantity of silver weighing a tael 898 fine (as the Mexican dollar),  $\therefore$  1 money tael = 1.3937 dollars. 1 dollar = .717517 money tael, and the quotations are given in taels.

\$100 = 717 taels or 1 tael = 1.395 dollar.

There are local tael weights at all the Treaty Ports in addition to the chief ones which are:—

The Canton tael = 580.036434 grs. Troy = 37.573 grammes.

The Hai-Kwan tael = 590.35 grs. Troy = 38.246 grammes (the Government tael).

The Shanghai tael = 564.20 grs. Troy = 36.56 grammes (for gold).

A uniform currency is sadly needed.

In large transactions ingots of silver called shoes are used of variable weight.

The Shanghai currency consists of such shoes of about 50 taels' weight each.

These ingots are assayed, stamped to show their touch or purity by the Hong Koo, and so rendered current.

The Shanghai sycee (pure silver) is 6 p.c. premium on the Hong Koo's standard.

The Haikwan sycee (made by melting and refining at the Customs Bank, Mexican, Spanish, and other foreign dollars paid as duties) has a premium of 3—10 p.c. over the dollar currency.

The Chinese report the fineness of metals by a percentage called toques or touch.

Gold bars are quoted in taels per 10 taels Shanghai weight 98 touch fine (365·6 grains).

Silver bars are quoted in taels per 100 taels Canton weight.

Mexican and Carolus (Spanish) dollars are quoted in taels per 100\$.

The standard is silver, so there is no Mint par.

### Arbitrated Par for Silver (Tael).

? = 1 tael (money).

Quoted no. of taels = 100 taels C. W.

1 = 580 gr. Troy.

480 = 1 oz. of given fineness.

240 = (222 ± betterness or worseness) oz. fine.

37 = 40 oz. st.

1 = London price of bar silver in d.  
per oz. st.

Fixed Number = 54429429.

Rule.  $F. N. \times (222 \pm B. \text{ or } W.) \times \text{London price} = \text{Rate of Exchange}.$

This does not include charges—which are generally  $1\frac{1}{2}$  p.c. This must be added to the rate of exchange before working (when given).

### EXAMPLES.

1. Bar silver, 17 B. in London,  $39\frac{5}{16}d.$  per oz. st., in Shanghai,  
110. Rate? *Ans.*  $5s. 11d.$

2. Bar silver, 15 B. in London,  $40\frac{1}{8}d.$  per oz. st., in Shanghai,  
111 $\frac{1}{2}$ . Rate?

3. Bar silver, 16 B. in London,  $41\frac{3}{16}d.$  per oz. st., in Shanghai,  
109 $\frac{1}{4}$ . Rate?

**Course of Exchange.**

Shanghai receives from

London.	sight and 4 mos' sight.	3s. 10½d. for 1 Tael.
Paris.	" "	5.18 Fr. for 1 Tael.
Bombay & Calcutta. }	on demand.	315 Rs. for 100 Ts.

quotes Hong-Kong. sight & 15 days' sight. 27 p.c. disc't. on  
the par of 73 Taels per 100\$.

**EXAMPLES.**

1. Direct. London—Shanghai.

(1) 3520 Taels, at 4 mos. 3s. 9½d. Discount, 3 p.c.

(2) £780. 19s. 5d., at 4 mos. 3s. 10½d., and 4 mos. at 4s. 1½d. Discount, 1 p.c., 2½ p.c.

2. Indirect. London—Shanghai.

(1) Through Paris, 3 mos. 25.53, 4 mos. 5.35.

(2) Through Calcutta, T.T. 1s. 4½d. Demand, 313.

Direct Rates, 4 mos. 3s. 9d. and 4 mos. 4s. 1½d. Discount,  
3 p.c., 2 p.c.

Minimum cost of £310. 15s. 8d. and 13750 Taels?

**Weights and Measures.****WEIGHT.**

16 Taels = 1 Catty (ie).  
100 Catties = 1 Picul (Pecul).

1 Tael = 583½ gr. Tr.  
1 Tael = 1½ oz. Av.  
1 Cattie = 1½ lb. Av.  
1 Picul = 133½ lb. Av.  
1 lb. Av. = ¼ Cattie.  
1 cwt. Av. = 84 Catties.

In wholesale tea trade and in freights, Avoirdupois weight is used but it is at once changed to Piculs and Catties as above.

Most articles (liquids, wood, silk, cloth, grain, live stock) are sold by weight. Grain is retailed by measure.

## CAPACITY.

Decimal scale.  
 10 Tsho = 1 Ko.  
 10 Ko = 1 Shung.  
 10 Shung = 1 Tow.  
 10 Tow = 1 Shih.

1 Shih weighs 120 Catties.  
 Arrack is sold by the English gallon.  
 Bottled wines by the number of bottles.

## LENGTH.

Decimal scale.  
 10 fan = 1 tsun (punt).  
 10 tsun = 1 chih (coord, cobre).  
 5 chih = 1 pu.  
 10 chih = 1 chang.  
 180 chang = 1 li.

1 chih = 14 $\frac{1}{2}$  E. in. (common)  
 1 chih = 14.1 E. inches (tariff).  
 12 chang = 47 English yds.  
 1 E. yd. = 2.4 chih (Canton).  
 = 2.5 chih (Shanghai).

Piece goods are measured by the chang.

Silk goods are sold by weight.

In transactions between Europeans and Chinese the English yard (ma) is used.

There are a great many chih.

The Siamese cattie (the cally) = 2 Chinese catties = 2 $\frac{1}{2}$  lbs. Av.,  
 but the Siamese picul = 50 callies = 1 Chinese picul.

## JAPAN.

## Moneys.

1 yen = 100 sen.  
 1 sen = 10 rin.

Since 1871 the legal money has been the yen of 100 sen.

The yen is silver 900 fine weighing 416 grains Troy = 26.9563 grammes and is equivalent to the Mexican dollar which it has now virtually supplanted since 1879, when the new yen was declared legal tender and on a par with the Mexican dollar.

The standard was however double, as the Japanese coined gold yens at a proportion of 1 : 16.173818. In consequence of the fall in silver these gold yens have

been largely sent to Europe and the actual standard was silver till recently.

In Oct. 1897 Japan adopted the gold standard.

### Course of Exchange.

Yokohama receives from

London.	sight, 4 m. & 6 mos' sight.	3s. 9½d. for 1 Yen or Dol.
Paris.	sight, 4 mos' sight.	5 Fr.       "       "
New York.	sight, 30 days' sight.	90\$       for 100 Yen.
Shanghai.	sight, 10 days' sight.	74 Taels   "       "

Hong-Kong is quoted sight and 10 days' sight at disc't. or prem. p.c. Yens for Dollars.

Gold Yens are generally quoted per 400 Dollars, e.g. 390 (i.e. gold Yens) for 400 Yen-Dollars.

### Weights and Measures.

#### WEIGHT.

10 momme	= 1 jumomme (tael).
16 jumomme	= 1 king or kin (catty).
100 kins	= 1 tan (picul).

These weights are exactly the Chinese.

1 jumomme	= 1½ oz. Av.
1 kin	= 1½ lb. Av.
1 tan	= 133½ lb. Av.

#### CAPACITY.

10 sai	= 1 shujakn.
10 shujakn	= 1 ngoō.
10 ngoō	= 1 shoō.
10 shoō	= 1 to.
10 to	= 1 koku.

The sai or choh = 1·107277458 cub. inch.

The koku = 6·4078557 cub. ft.

= 4·99918017 imp. bushels.

#### LENGTH.

The keng (inch) = 74·9 Eng. inches (Kelly) = 6·2416 ft. = 2·0805 yds.

For cloth and textile fabrics, the tsune sasi is used = 1½ Eng. ft.



**EXAMPLES.****1. Direct.**

(1) £768. 10s. 9d. at 4 mos. 3s. 8 $\frac{5}{16}$ d. Discount, 3 p.c.

(2) 1056 yens at 6 mos. 3s. 7 $\frac{7}{16}$ d. Discount, 2 p.c.

**2. Indirect. London—Yokohama.**

(1) Through New York, 4·85, 92.

(2) Through Hong-Kong, 3s. 4 $\frac{1}{2}$ d., 5 p.c. premium.

Direct rates, 4 mos. 3s. 6d., 3s. 11d. Discount, 2 $\frac{1}{2}$  p.c.,

3 p.c.

Debts, £250. 8s. 10d. and 7128 yens.

**SOUTH AMERICA.****The Argentine Republic.****Moneys.**

1 peso = 100 centesimos or 8 reales.

The standard is nominally gold, but actually an inconvertible paper currency much depreciated and fluctuating is alone in use.

The premium on gold is about 220 p.c.

The peso is silver and weighs 27·11 grammes 900 fine.

There are 5, 10, 20-peso gold pieces.

**Exchanges.**

Buenos Ayres receives from London pence per (gold) peso, from Paris francs per (gold) peso, from Hamburg marks per (gold) peso at 90 days' sight, but this is quite useless at present.

Mint par for gold.

£1 = 4·882 pesos.      1 peso = 49·164d.

**Weights and Measures.**

Metric and Old English (American and English merchants).

**Brazil.****Moneys.**

1 milreis = 1000 reis.

The standard is nominally gold—silver is not legal tender—but really the medium of exchange is an inconvertible paper at a heavy discount.

The 20, 10, 5-milreis pieces are gold, the 2, 1,  $\frac{1}{2}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$ -milreis pieces are silver.

Mint par for gold.

1 mlr. = 26·93*d*.      £1 = 8·912 milreis.

This is of course totally useless at present.  
Sovereigns are taken at 8890 reis gold.

**Exchanges.**

The bill stamp is 1 per mille additional for every 200 mlr. up to 1000, and then for every 1000 mlr.

Rio de Janeiro receives from London 90 and 60 days' sight 12½*d*. (Bank paper) per milreis—it gives to Paris, Hamburg and Lisbon at 90 days' sight 230, 270, 1230 reis (paper) per franc, mark, and milreis respectively.

**Weights and Measures.**

The Metric (since 1874) and Portuguese.

**Chili.****Moneys.**

1 peso = 100 centavos.

The peso is the equivalent of the 5-franc.

The 10, 5, 2-peso pieces are gold, the 1,  $\frac{1}{2}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$ ,  $\frac{1}{20}$ -peso pieces are silver.

In Chili there is a premium of 6 p.c. on gold over silver, but practically the standard is silver.

For exchanges the medium is a depreciated paper for which silver is also at a premium.

**Exchanges.**

Valparaiso receives from London, Paris, and Hamburg 90 days' sight 18 $\frac{1}{2}$ d., 2·70 Fr., 1·48 M. per peso (paper) respectively.

**Weights and Measures.**

Metric and Spanish.

**Venezuela, Columbia States, and Ecuador.****Moneys.**

1 peso = 100 centavos.

The peso = the 5-franc silver piece.

The standard is a double valuation at 1 : 15 $\frac{1}{2}$ .

Mint par for gold £1 = 5·045 pesos.

**Exchanges.**

Bogota gives to London 90 days' sight 5 pesos for £1, and receives from Paris, Hamburg, at 90 days' sight 4·80 Fr., 4 M. per peso.

**Weights and Measures.** Spanish.

**Paraguay.****Moneys.**

1 peso = 100 centavos or 8 reales.

A depreciated paper currency exists for which gold is at a premium.

**Exchanges** depend on Buenos Ayres.

**Weights and Measures.** Spanish.

**Peru and Bolivia.****Moneys.**

1 sol = 100 centesimos.

The sol = the silver 5-franc piece.

The standard is silver, and gold coins are commercial money.

The 20, 10, 5, 2, 1-sol pieces are gold, the 1,  $\frac{1}{2}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$ ,  $\frac{1}{20}$ -sol pieces are silver.

The exchange medium is inconvertible paper.

**Exchanges.**

London and Paris are quoted in pence or francs per paper sol at 90 days' sight, and New York at 90 days' sight in premium p.c. on 100 dollars.

**Weights and Measures.** Spanish.

**Uruguay.****Moneys.**

1 peso = 100 centesimos.

The standard is nominally gold, the peso being silver.

**Exchanges.** As in Buenos Ayres.

**Weights and Measures.** Spanish.

**THE BRITISH COLONIES.****Gibraltar.**

The peso duro is legal tender since 1838 in all British Colonies at 50*d.* stg., and the doubloon at 64*s.*

In Gibraltar and Spain the doubloon is always taken at 16 dollars or pesos, i.e. £3. 6*s.* 8*d.*, and is the chief money of payment.

The weights and measures are those of Great Britain and the Old Spanish.

Wine is sold by the Cadiz pipe = 18·949121 imp. gallons.

Gibraltar receives from London 90 days pence per peso, and from Italy lire per peso.

Between Gibraltar and Spain exchanges are quoted at a premium or discount p.c.

**Malta.**

The Sicilian dollar (Maltese pezza) is legal tender at the rate of 5 per £. It is divided into 2½ scudi of 12 tari of 20 grani.

Commercially the pezza is reckoned at 50*d.*, and the scudo at 20*d.*

The Mint value of the pezza is 49½*d.*

£ *s d f* are also used.

The rottolo ( $\frac{1}{100}$  of a cantaro) = 1¼ lb. Av. ∴ 1 cwt. = 64 rottoli.

The barile of wine = 9·35 imp. gallons.

The caffiso of oil = 4·375 imp. gallons.

The English yard = 3·5036 palmi (3½).

Malta receives from London 90 days' pence per pezza, but gives to Italy and Marseilles grani (100 ±) per lira or franc, and to Austria tari (15 ±) per florin.

The exchange on London is quoted at a premium p.c. on the pezza at 50*d.*, thus the sovereign is in reality a silver coin = 4½ Sic. dollars.

**The West Indies.**

Accounts are kept in dollars and cents or *£ s d f*.

The standard is silver. 1 dollar (Mexican) = 4*s.* 2*d.*  
1 doubloon = 64*s.* The *£* (a silver coin) = 4½ Mex. doll.

Weights and measures are those of Great Britain with the old measures of capacity. 1 old wine gallon = .833109 imp. gallons.

1 Winchester bushel = .9694472 imp. bushels.

Exchanges on London are quoted in pence per dollar or in a premium p.c. on sterling (2 p.c. premium means £100 nominal = £102 stg.).

New York is quoted in premium p.c. at the fixed par of 100 W. I. \$ = 100 U.S. \$.

**British Guiana.**

Demerara reckons in dollars (gurds) and cents, in *£ s d f*, or in guilders (of 20 stivers of 16 pfennige) for inland trade.

Great Britain sends the silver and copper coins (including guilders and stivers) required.

The exchange value of the dollar fluctuates with silver.

The London exchange is quoted 90 days' sight in Spanish dollars per £100.

The weights and measures for inland trade are the old Dutch; for wholesale, the English.

**Ceylon.**

The coins are the rupee of 100 cents—the standard is silver.

The rupee is the Indian Government rupee.

Colombo receives from London 4 and 6 mos' sight pence per rupee, but gives to India and Mauritius

rupees (100  $\pm$ ) per 100 rupees, and to China rupees (220  $\pm$ ) per 100 taels.

The British weights and measures are used.

The Candy or bahar = 496 lbs. (500 lbs. commercially).

Arrack is sold in leggers of 75 or 80 veltes = 150—160 old wine gallons.

### **Burmah.**

The rupee etc. of India from the currency.

100 kyats = 3.65 lb. Avoir. = 1 paktha.

The ten of rice weighs 58.4 lbs. ( $\frac{1}{2}$  cwt. nearly).

The royal ell = 19.1 inches.

### **Penang.**

Accounts are kept in Spanish piastres of 100 cents.

Exchanges on London pence per dollar, and on Calcutta rupees (220  $\pm$ ) per 100 dollars.

The Chinese weights are used.

The Penang bahar is sometimes reckoned in Malay catties, which is heavier than the Chinese. 1 bahar (Malay) = 428 lbs.

The Malay picul is hence often called the large picul.

Rice and salt are measured by gantongs of 520 parras. 1 gantong = 0.97973113 imp. gall.

Piece goods are measured by the hasta of 18 inches or by the English yard.

### **Singapore.**

Accounts are kept in dollars and cents (as in Hong-Kong).

The standard is silver.

The bill stamp is  $\frac{1}{2}$  per mille.

The exchange on London is 1—2 p.c. above the Hong-Kong rate, and on India it is the same as the Hong-Kong rate.

The Chinese weights and measures are chiefly in use.

Siam salt, sago, and rice are sold by the koyang.

Bengal rice and wheat by the sack of 2 mahuds = 164½ lbs.

### **Cape of Good Hope and Natal.**

The moneys are sterling.

In Cape Colony the bill stamps are 6d. up to £50, 1s. from £50 to £100, above £100 at 1s. per £100 or fraction.

Bills in sets pay  $\frac{1}{3}$  of the stamp each.

There is no bill stamp in Natal.

London exchange is quoted 30 days', 4 or 6 mos'. sight in premium or discount p.c.

Cape Town and Natal give to Amsterdam and Paris 30 days' sight pence per florin or franc, and to Bombay and Calcutta pence per rupee (as in London).

The English and Old Dutch weights and measures are used.

### **Mauritius.**

The coins are the I. rupee divided into 100 cents.

The standard is silver. The dollar (4s. 2d. per value) is taken at 2 rupees.

The London exchange is quoted in pence per rupee or in premium p.c. on the fixed par of 1 rupee = 2s.

Calcutta, Madras, Ceylon are quoted in premium discount p.c.

The metric system was introduced in 1875, but English and Old French weights and measures are still used.



**Guinea (West Africa).**

The moneys are the Spanish dollar of 100 cents.

Business is chiefly by barter. In the gum trade by strips of cloth called "guineas."

The Natives use the moco (Danish rigsdaler) and macute of 2000 cowries = 4·91 pence.

The British Mint supplies silver macute (10 cents) at  $4\frac{3}{4}d.$  exchange.

The weight is the beudo of 8 usanos (ounces) = 989·43 grains Troy.

The jacktan = 12 English feet. The pik = 22·756438 English inches.

**Australia.**

Moneys as in Great Britain.

The gold coins of the Melbourne and Sydney Mints are legal tender in England.

The bill stamp for Tasmania is 6*d.* per £50 or fraction, for the rest of Australia 1*s.* per £50 or fraction.

Exchanges on London are quoted in premium p.c. on £100 (i.e. £100 in London = £101 to £105 in Australia), on East Indies in pence per rupee, on China, etc. in pence per dollar, on United States in dollars per £.

The weights and measures as in Great Britain.

The section = 80 acres.

The Victorian flour ton = 2000 lbs. Av.

## APPENDIX.

**AUSTRIA-HUNGARY.****1 Krone = 100 heller. (A.)****1 Corona = 100 filler. (H.)**

Austria-Hungary has adopted the Gold Standard since 1892. The unit is the gold Krone or Corona divided into 100 heller or filler. 1 Krone =  $\frac{1}{3}$  Gulden. 3280 Kronen contain 1000 grams of fine gold and are  $\frac{9}{10}$  fine. Austria issues in Vienna and Hungary in Kremnitz 20-Kronen and 10-Kronen pieces. Also for private account similar pieces are struck at the following charges: Assaying 2 Kr. per bar, Minting 4 Kr. per 1000 gr. fine gold for the Austro-Hungarian Bank and 6 Kr. per 1000 gr. fine gold for private persons; partings 4 Kr. per 1000 grams gross weight if bars are under 898 fine gold or the silver is demanded back.

The Austro-Hungarian Bank also buys gold and gives 3276 Kronen for every 1000 grams fine gold.

**Mint Par Chain for Gold.****? = £1.****1869 = 440 oz. fine gold.****1 = 31.1035 g. „****1000 = 3280 Kronen.**

---

**£1 = 24.02.**

The earlier silver florins (2 Kronen) remain in use and form the currency.

The State coins 1- and 5-Kronen pieces of silver solely for its own account.

The 1-Krone piece weighs 5 grams and is 0·835 fine—the 5-Kronen piece weighs 24 grams and is 0·900 fine. The State Treasuries are bound to accept these to any amount but in private business only 50 pieces need be taken.

Nickel pieces of 20 and 10 heller, and bronze pieces of 2 and 1 heller are also struck.

In Vienna also the Levant or Maria-Theresa silver dollar is struck for use in Asia Minor and Abyssinia—23·387 g.  $\frac{5}{8}$  fine. Also ducats of weight 23·387 g. and  $\frac{7}{8}$  fine originally coined in 1867 are still in use—but the 8 and 4-florin pieces originally designed to rank with 20 and 10-franc pieces are no longer struck.

The reckoning in Kronen was made obligatory for both parts of the Monarchy from Jan. 1, 1900.

### Bills and Exchanges.

Bills drawn, accepted, or payable in Austria-Hungary are stamped thus:

I. Up to 6 mos. Currency.	II. Above 6 mos. Currency.
To 150 Kr. ... 10 H.	To 40 Kr. ... 14 H.
„ 300 Kr. ... 20 H.	„ 80 Kr. ... 26 H.
„ 600 Kr. ... 40 H.	„ 120 Kr. ... 38 H.
„ 900 Kr. ... 60 H.	„ 200 Kr. ... 64 H.
„ 1200 Kr. ... 80 H.	„ 400 Kr. ... 1·23 Kr.
	„ 600 Kr. ... 1·88 Kr.
	„ 800 Kr. ... 2·50 Kr.
	„ 1600 Kr. ... 5·00 Kr.

Bills issued in Austria must bear the Hungarian Stamp in Hungary and *vice versa*.

Bills passing through Austria or Hungary for indorsement pay 4 Heller per 200 Kr.

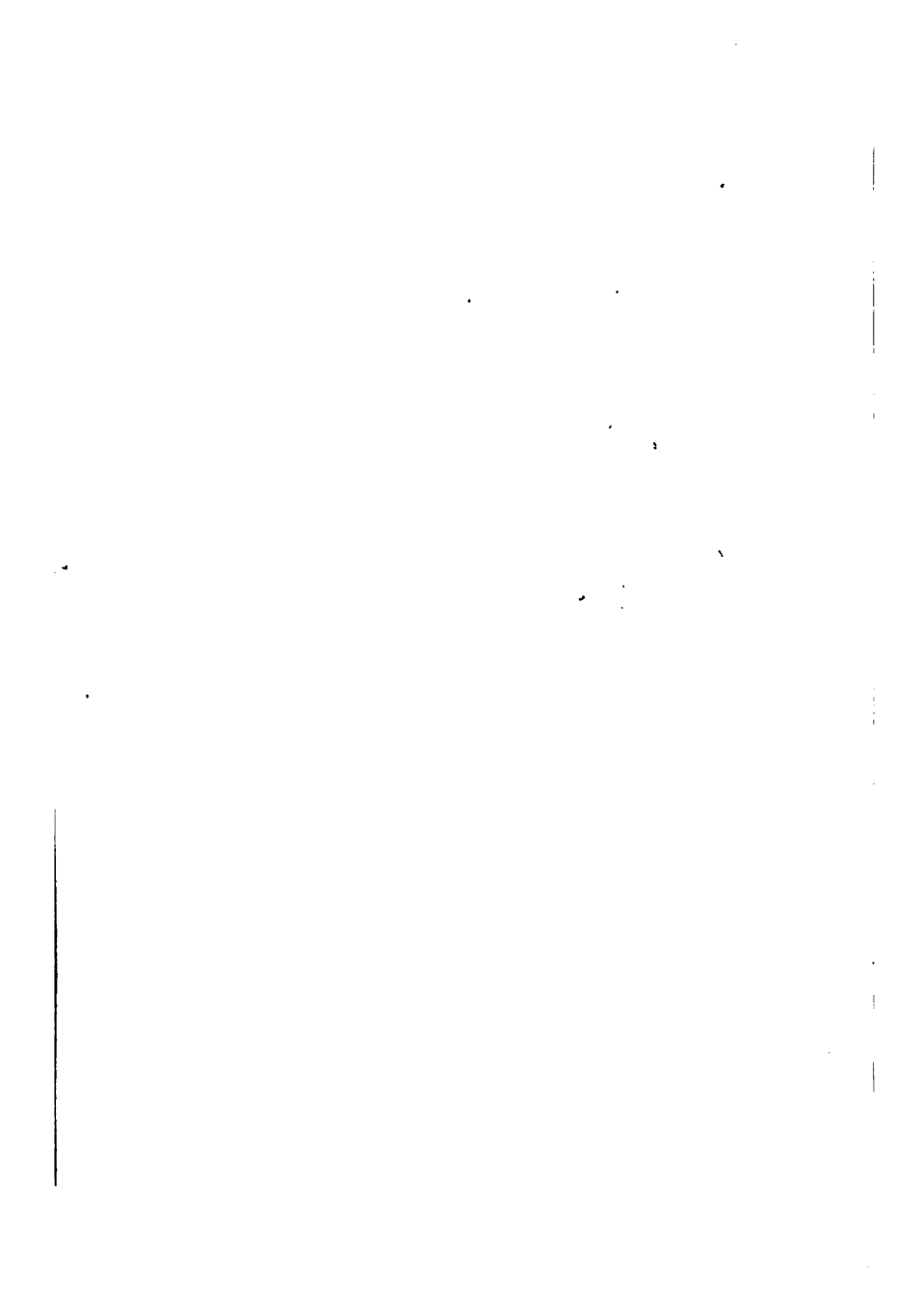
**Course of Exchange.**

Quotations in Kronen from Jan. 1900.

Vienna gives to

London	sight	Kr. 241·80	for £10 sterling.
Amsterdam	"	200·20	for 100 Fl. (Neth.).
Brussels	"	93·00	for 100 Fr.
Germany	"	118·10	for 100 M.
Italy	"	89·95	for 100 L. (Paper).
Paris	"	95·60	for 100 Fr.
Petersburg	"	236·00	for 100 Ro.
Zürich	"	95·30	for 100 Fr.
New York	"	503·75	for 100 Doll.

**ANSWERS**  
**TO**  
**COMMERCIAL ARITHMETIC**



ANSWERS  
TO  
COMMERCIAL ARITHMETIC.  
SECTION I.

PAGE 2.

- |  |  |  |
|--|--|--|
| 1. 86896797372,<br>86818558959,<br>8614918587. | 2. 462278125,<br>2311390625,<br>184911250. | 3. 774267·616,<br>3871338·08,<br>30970·7046. |
| 4. 9642556,<br>106068116,<br>1166749276.       | 5. 1383·896,<br>691·948,<br>345·974.       |  |
| 1. 4772191697,<br>2941762005,<br>5491289076.   | 2. 711590760,<br>786935664,<br>159061464.  | 3. 236434064,<br>196189968,<br>130793312.    |
| 4. 75016104,<br>115649827,<br>221922641.       | 5. 350529290,<br>507236502,<br>1253657696. |  |

PAGE 3.

1. 398161, 64112049, 81234169, 225360144, 719104.
2. 546121, 793881, 558009, 99900025, 579121.
3. 59319, 68921000, 822656953, 912673, 1157625.
4. 857375, 54872, 387420489, 68417929, 14142076.
5. 5720, 35000, 2160, 12879, 4319.

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- |                 |                  |                 |
|-----------------|------------------|-----------------|
| 1. 25921781472. | 2. 139555312.    | 3. 73052309356. |
| 4. 1641748992.  | 5. 377207225812. | 6. 19604208.    |
| 7. 77241167934. | 8. 349391773471. | 9. 24682304284. |
| 10. 1055104500. |                  |                 |

## PAGE 7.

1. 233100.	2. 197892.	3. 208094.	4. 2339695.
5. 13066997.	6. 4095322.	7. 103780.	8. 1426674.
9. 2027188.	10. 7605·7.	11. £1·217.	12. £8·971.
13. £1·712.	14. £7·797.	15. £11·186.	16. £0·491.
17. £0·236.	18. £1·144.	19. £9·255.	20. £8·069.
21. 138450·04.	22. 1178...	23. 1·4...	24. 1289·3...
25. ·00011.	26. 26·2...	27. 23423.	28. ·011...
29. 5·16.	30. 2·4...	31. 2.	32. 2.
33. 3.	34. 5.	35. 6.	36. 225.
37. 2.	38. 2.	39. 1.	40. 4.
41. £1·033.	42. £2·684.	43. £10·375.	44. £2·663.
45. £0·119.	46. £46·216.	47. £0·810.	48. £631·682.
49. £1·284.	50. £28·818.		

## PAGES 15, 16.

1. (1) 18·4802083.	(2) 17·265625.	(3) 931·871875.	
(4) 864·403125.	(5) 91·67083̄.	(6) 72·334375.	
(7) 3306·8739583̄.	(8) 123·727083̄.	(9) 904·8302083̄.	
(10) 75·6677083̄.	(11) 324·04375.	(12) 66·790625.	
(13) 8107·5510416̄.	(14) 926·9552083̄.	(15) 841075·1635416̄.	
(16) 73·0510416̄.	(17) 96·377083̄.	(18) 875·94375.	
(19) 4375·5875.	(20) 8·610416̄.		
2. (1) £81. 13s. 5d.	(2) £793. 11s. 3d.	(3) £184. 9s. 10½d.	
(4) £71. 6s. 8½d.	(5) £361. 2s. 1¼d.	(6) £137. 0s. 0¾d.	
(7) £87. 6s. 3d.	(8) £931. 5s. 5½d.	(9) £710. 6s. 2½d.	
(10) £876. 19s. 0d.			
3. (1) 3601878.	(2) 84269.	(3) 873670.	(4) 46409.
(5) 49096.	(6) 782581.	(7) 59207.	(8) 73438.
(9) 144836.	(10) 68392.		



4. (1) 73·39427083̄. (2) 18·317447916̄. (3) 0·3641927083̄.  
 (4) 0·022135416̄. (5) 0·09114583̄. (6) 0·16953125.  
 (7) 3·5188802083̄. (8) 0·01614583̄. (9) 0·056510416̄.  
 (10) 0·158984375.
5. (1) £19. 6s. 3½d. (2) £3. 2s. 6½—¾d. (3) 7s. 1½—¾d.  
 (4) 7s. 6½—¾d. (5) £7. 12s. 11d. (6) £13. 6s. 3d.  
 (7) £27. 5s. 11½—⅞—¾d. (8) £139. 15s. 3½d.  
 (9) £78. 16s. 8½d. (10) £1. 18s. 9d.

## PAGES 21, 22.

- |                   |                  |                   |
|-------------------|------------------|-------------------|
| 1. 33930·878.     | 2. 13·221771.    | 3. 2310·11643.    |
| 4. 681733·552.    | 5. ·58022.       | 6. 5731·261.      |
| 7. ·117753.       | 8. ·0044289.     | 9. ·000000021.    |
| 10. 5480400·55.   | 11. £2972·011.   | 12. £28569·928.   |
| 13. £415125·000.  | 14. £2459·300.   | 15. £176094·841.  |
| 16. £775131·321.  | 17. £150567·188. | 18. £84293·378.   |
| 19. £5448·469.    | 20. £10605·468.  | 21. £55179·132.   |
| 22. £8045·951.    | 23. £603875·078. | 24. £346850·586.  |
| 25. £23673·710.   | 26. £169762·586. | 27. £128226·058.  |
| 28. £2069139·067. | 29. £41012·674.  | 30. £1062156·104. |

## PAGES 25, 26.

- |                   |                   |                     |
|-------------------|-------------------|---------------------|
| 1. 2225·12109.    | 2. ·00900966.     | 3. 11·4078          |
| 4. ·0000488.      | 5. ·00012.        | 6. 14·6892          |
| 7. 6·495.         | 8. ·108567.       | 9. ·00188.          |
| 10. ·00000265.    | 11. £93·616.      | 12. £0·461.         |
| 13. £25·527.      | 14. £2483·607.    | 15. £0·978.         |
| 16. 2s. 1d.       | 17. £35·807.      | 18. 1s. 9¼d.        |
| 19. £2. 4s. 9d.   | 20. £1. 8s. 1d.   | 21. £2. 2s. 2d.     |
| 22. £7. 5s. 2d.   | 23. £31. 16s. 6d. | 24. £4. 9s. 8d.     |
| 25. £3. 8s. 4d.   | 26. £2. 9s. 1d.   | 27. £117. 19s. 10d. |
| 28. £171. 9s. 8d. | 29. £74. 8s. 4d.  | 30. 19s. 1¾d.       |

## JACKSON'S LIMITS OF ERROR.

The exact degree of correctness in De Morgan's method may be verified in the following way for the four operations of Addition, Subtraction, Multiplication, and Division.

Obtain the result approximately correct to any number of places desired by De Morgan's method, with this modification, that care must be taken that the full effect of all omitted parts must be considered.

To each approximation which is too small add + and to each approximation which is too great add -.

Then from the nature of the operation a + can represent no more than 5 too little nor less than 0 too little; and a - can represent no more than 5 too much nor less than 0 too much.

Hence

1. To obtain a *superior limit* neglect the -s and count all +s as 5s; the result will be a quantity which the expression cannot exceed.

2. To obtain an *inferior limit* neglect the +s and count all -s as -5s; the result will be a quantity which the expression will certainly exceed.

By comparing these two limits it is at once seen how much of the approximation is certainly correct.

## 1. Addition.

p. 17.

$$\begin{array}{r} 83\cdot1714 - \\ 206\cdot3413 - \\ 1308\cdot1216 + \\ \hline 1597\cdot6343 \end{array}$$

Sup. limit 1597·63435

Inf. limit 1597·63420.

∴ certainly correct to 1597·634.

## 2. Subtraction.

$$\begin{array}{r} 78\cdot8341 + \\ 27\cdot2983 - \\ \hline 51\cdot5358 \end{array}$$

Sup. limit 51·53590

Inf. limit 51·53580.

∴ correct to 51·535.

*Note.* The subtraction changes a - in the lower line into a +, and a + in the lower line into a -.

3. *Multiplication.*

p. 20.

$$\begin{array}{r}
 373 \cdot 86150 \\
 59172 \\
 \hline
 747 \ 72300 \\
 261 \ 70305 \\
 3 \ 73862 - \\
 3 \ 36475 + \\
 18693 + \\
 \hline
 10167 \cdot 1635 \\
 \text{Sup. limit } 10167 \cdot 16360 \\
 \text{Inf. limit } 10167 \cdot 16345. \\
 \therefore \text{ correct to } 10167 \cdot 163
 \end{array}$$

4. *Division.*

p. 23.

$$\begin{array}{r}
 8564 \overline{) 3 \ 731 \cdot 530} \quad | \quad 8 \cdot 541 \\
 \underline{685 \ 14 +} \\
 46 \ 39 \\
 \underline{42 \ 82 +} \\
 3 \ 57 \\
 \underline{3 \ 43 -} \\
 14 \\
 \underline{9 -} \\
 5
 \end{array}$$

Changing signs owing to the subtraction,

Sup. limit of remainder = 60, 85 will not go,

Inf. " " " = 40, always positive.

$\therefore$  correct to 8·541.

## SECTION II.

PAGES 29, 30.

1. (a) 2s. 9d., 17s. 6d., 3s. 7½d., £5. 15s. 9d.  
 (b) £3. 18s. 10d., £2. 15s. 5d., £7. 10s. 0d., 15s. 3d.  
 (c) £6. 6s. 0d., £9. 9s. 0d.
2. £6. 17s. 6d., £13. 15s. 0d., 19s. 6d., £16. 17s. 6d., £144. 7s. 6d.
3. 5s. 10d., £3. 2s. 6d., £8. 6s. 3d., £36. 17s. 10d., £66.
4. £23. 5s. 0d., £27. 6s. 0d., £103. 10s. 0d., £367. 4s. 0d., £198. 15s. 0d.
5. £40. 10s. 0d., £67. 13s. 4d., £48. 15s. 0d., £37. 2s. 6d., £41. 1s. 8d.
6. 13s. 6½d., £11. 5s. 10d., £156. 5s. 0d., 17s. 8½d., £23. 10s. 0d.

7. £92. 8s. 4d., £45. 4s. 8d., £10. 19s. 4d., £18. 17s. 4d.,  
£13. 7s. 9d., £30. 11s. 0d., £18. 15s. 4d.

8. 19s. 10d., 7s. 7d., £165. 13s. 4d., £33. 15s. 4d., £3. 7s. 6d.

9. 5s. 10d., 4s. 3d., 4s. 7½d., £1. 3s. 5½d., 2s. 2½d.

10. 3s. 6d., 9s. 7d., £3. 18s. 0d., 10s. 3d., 19s. 7d., £7. 19s. 0d.,  
13s. 6d., £1. 19s. 7d., £15. 6s. 0d., £2. 19s. 0d., £5. 11s. 8d.,  
£54. 0s. 0d.

## PAGES 34, 35.

1. (1) 612·998. (2) 481·692. (3) 1362·484.

2. (1) 53·902. (2) 83·420. (3) 980·643.

3. (1) 823·826. (2) 28·821. (3) 141·5514. (4) 71·632.  
(5) 84·6667.

4. (1) 19·93125. (2) 27·8626. (3) 15·67633. (4) 18·953125.

(5) 1803·12.

5. (1) £30. 6s. 8d., £21. 11s. 8d., £148. 3s. 4d.

(2) £373. 1s. 8d., £509. 4s. 2d., £199. 2s. 11d.

(3) £10s. 10s. 0d., £16. 6s. 8d., £25. 1s. 8d.

(4) £1. 8s. 0d., £8. 2s. 8d., £5. 1s. 4d.

(5) £3. 19s. 1d., £6. 7s. 0d., £16. 1s. 0d.

6. (a) £6·167, £1·812, £1·677, £3·874, £8·017, £9·867, £19·271,  
£14·857, £11·967, £1·599, £3·700, £10·792.

(b) £53·877, £20·820, £39·206, £481·623, £552·155, £1601·563,  
£192·188, £348·244, £673·105, £561·508.

(c) £191·552, £677·680, £792·741, £1058·106, £3315·275,  
£5177·083, £11348·837, £19104·755.

## PAGES 37, 38.

1. £6. 2s. 11d., £15. 14s. 7d., £3. 19s. 2d., £11. 15s. 5d., £3. 3s. 6½d.

2. £19. 0s. 4d., £5. 9s. 8d., £9. 2s. 7d., £28. 7s. 0d., £5. 0s. 4d.

3. £8. 2s. 6d., £6. 17s. 6d., £33. 15s. 0d., £25. 12s. 6d., £28. 15s. 0d.

4. £1015, £740. 19s. 0d., £629. 6s. 0d., £406, £2476. 12s. 0d.

5. £14. 17s. 1d., £17. 19s. 4½d., £30. 15s. 8½d., £55. 2s. 1d.,  
£36. 1s. 1½d.

6. £155. 4s. 2d., £97. 18s. 4d., £492. 14s. 2d., £73. 19s. 2d.,  
£132. 5s. 10d.

7. £53. 8s. 9d., £104. 19s. 10d., £33. 14s. 6d.
8. £113. 10s. 10d., £44. 12s. 8d., £156. 11s. 3d.
9. £348. 5s. 4d., £275. 8s. 0d., £214. 8s. 0d., £141. 6s. 8d., £248. 10s. 8d.
10. £116. 14s. 4d., £321. 3s. 4d., £154. 6s. 4d., £573. 8s. 0d., £69. 14s. 4d.
11. £3. 5s. 4d., £3. 12s. 4d., £3. 19s. 4d., £2. 14s. 10d., £4. 5s. 2d.
12. £10. 10s. 0d., £16. 6s. 8d., £33. 16s. 8d., £29. 3s. 4d., £22. 3s. 4d.
13. 13s. 1½d., £1. 13s. 1½d., £1. 6s. 3d., £2. 8s. 1½d., £1. 11s. 10½d.

## PAGE 39.

1. 1 - (12) 16, 3 - 568, 13 - 64242, 0 - 422(16), 1 - 228(15).
2. 0 - 2634, 0 - 4534, 0 - 22(24), 0 - 2(15), 4 - (16) 2.
3. 0 - (10) 52, 0 - 5(10) 2, 0 - 525, 0 - 51.
4. 0 - (20) 5151, 0 - (20) 255, 0 - (20)(10) 225, 0 - (20) 5(10).
7. £53. 8s. 9d., £104. 19s. 10d., £33. 14s. 6d.

## PAGES 40, 41.

1. (1) 1 - 2223(10) 22. (2) 0 - 24 + 5(12) 84. (3) 3 - 2246.  
(4) 17 - 22(15). (5) 0 - 222(120). (6) 1 - 42(10) 4. (7) 0 - 2223(40).  
(8) 2 - 22453. (9) 1 - 24 + 56(16) 4. (10) 3 - 4(15) 8.
2. (1) 0 - (80)(12) 22. (2) 0 - (60) 842. (3) 0 - (40)(12) 22.  
(4) 0 - (20) 464. (5) 0 - (120) 822. (6) 0 - (40) 2624.
3. (1) £2239. 18s. 7d., £3681. 2s. 4d., £11571. 14s. 2d.  
(2) £145. 12s. 1d., £168. 3s. 11d., £23. 16s. 11d., £23306. 14s. 2d.  
(3) £25. 19s. 4d., £33. 13s. 2d., £2034. 15s. 8d.  
(4) £259. 4s. 3d., £30. 6s. 9d., £35. 15s. 11d., £45. 6s. 6d.  
(5) £447. 18s. 4d., £63. 3s. 0d., £44. 2s. 9d., £49. 4s. 4d.  
(6) £634. 2s. 3½d., £91. 9s. 5d., £131. 16s. 8d., £200. 16s. 11d.  
(7) £60. 18s. 9d., £13. 13s. 5d., £42. 19s. 4d., £21. 1s. 10d., £181. 5s. 0d.  
(8) £146. 3s. 8d., £125. 3s. 1d., £34. 14s. 2d., £43. 15s. 10d.  
(9) £20. 18s. 7d., £29. 2s. 7d., £22. 15s. 3d., £61. 13s. 9d., £265. 7s. 1d.  
(10) £61. 9s. 2d., £13. 5s. 7d., £81. 15s. 5d., £835. 8s. 4d., £954. 8s. 4d.

## PAGE 41.

1. £36. 11s. 4d.      2. £13. 11s. 1d.      3. £953. 16s. 6d.
4. £549. 14s. 9d.      5. £45. 9s. 0d.

## PAGE 43.

1. £76·949, 110·976, 132·548. 2. £354·375, 602·438, 104·344.
3. £621·875, 864·063, 481·250. 4. £191·363, 136·688, 208·913.
5. £393·759, 23·688, 157·500. 6. £330·115, 398·601, 277·888.
7. £83·758, 267·950, 162·725. 8. £665·508, 684·491, 391·288.
9. £210·156, 384·375, 110·938. 10. £259·217, 265·225, 154·500.
11. £100·583, 141·576, 194·010. 12. £112·111, 226·252, 87·254.
13. £442·897, 306·316, 184·651. 14. £253·500, 282·750, 302·250.
15. £112·416, 119·146, 159·916.

## PAGE 45.

1. £4. 1s. 3d. £14. 5s. 0d., £23. 17s. 0d.
2. (1) 54·150, 142·500, 192·85, 82·65, 131·1.  
 (2) 55·734, 65·344, 74·953, 96·094, 197·953.  
 (3) 111·879, 142·301, 193·245, 203·416, 783·151, 834·005.  
 (4) 146·531, 166·069, 84·663, 100·944, 74·894, 260·500.  
 (5) 43·333, 38·667, 42·000, 46·000, 49·333, 58·000.  
 (6) 29·288, 33·725, 24·850, 39·938, 55·025, 59·463.  
 (7) 38·438, 28·125, 143·438, 193·125, 226·875, 224·063.  
 (8) 64·167, 71·667, 97·500, 155·000, 164·167.  
 (9) 51·350, 75·050, 71·100, 101·384, 56·017.
3. £3. 15s. 0d., £2. 4s. 0d., £4. 13s. 0d., £8. 0s. 5d., £114.
4. £6. 3s. 8d., £276. 14s. 4d., £1300. 15s. 0d., £80. 5s. 6d.,  
 £118. 9s. 9d.

## PAGE 46.

1. £23948. 14s. 5d. 2. £1404. 14s. 4d. 3. £12503. 13s. 7d.
4. £35676. 11s. 3d. 5. £7934. 7s. 6d. 6. £2018. 13s. 11d.
7. £13865. 8s. 4d. 8. £15870. 4s. 9d. 9. £16624. 19s. 9d.
10. £1690. 10s. 0d.

## PAGE 48.

1. £225. 15s. 4d. 2. £130. 19s. 1d. 3. £302. 16s. 1d.
4. £40. 9s. 10d. 5. £747. 5s. 0d. 6. £217. 2s. 8d.
7. £793. 7s. 2d. 8. £3840. 13s. 11d. 9. £1199. 6s. 2d.
10. £298. 10s. 2d.

## PAGE 49.

1. £1449. 5s. 3d. 2. £108. 6s. 10d. 3. £3191. 3s. 10d.
4. £705. 13s. 8d. 5. £1611. 18s. 5d. 6. £348. 3s. 6d.
7. £545. 7s. 10d. 8. £4628. 16s. 1d. 9. £14199. 4s. 7d.
10. £20875. 16s. 2d.

## PAGE 50.

- |                  |                 |             |                |
|------------------|-----------------|-------------|----------------|
| 1. 16s. 11½d.    | 2. 2s. 8½d.     | 3. 10s. 4d. | 4. 99s. 8d.    |
| 5. £8. 0s. 5d.   | 6. 5d.          | 7. 7s.      | 8. £4. 6s. 5d. |
| 9. £1. 15s. 11d. | 10. £1. 6s. 5d. |             |                |

## PAGE 51.

- |        |        |             |        |        |
|--------|--------|-------------|--------|--------|
| 1. 3d. | 2. ½d. | 3. 1s. 1¾d. | 4. 1s. | 5. 3d. |
|--------|--------|-------------|--------|--------|

## PAGE 53.

1. £90. 13s. 0d., £64. 2s. 2d., £28. 7s. 7d., £94. 17s. 3d., £35. 280.
2. £37. 8s. 6d., £21. 17s. 8d., £50. 2s. 3d., £62. 118, £154. 686.
3. £40. 8s. 0d., £59. 14s. 0d., £12. 2s. 0d., £25. 4s. 0d., £12. 4s. 0d.
4. £228, £148. 10s. 0d., £221. 5s. 0d., £483, £975.
5. £1717. 12s., £8015. 62s., £4681. 12s., £5942. 250, £744. 5625.

## SECTION III.

## PAGE 59.

1. £230. 10s. 0d., £92, £60, £87, £42.
2. £159646. 6s. 11d., £159390. 1s. 10d.
3. £1910. 13s. 7d., £1286. 5s. 8d.
4. £858. 8s. 6d.

## PAGE 62.

1. £3. 5s. 0d., £5. 12s. 6d., £7. 10s. 0d., £5. 3s. 9d., £13. 5s. 0d.
2. £2. 12s. 6d., £4. 1s. 8d., £7. 5s. 10d., £6. 5s. 5d., £8. 15s. 0d.
3. £16. 6s. 8d., £19. 16s. 6d., £25. 13s. 4d., £31. 10s. 0d., £32. 13s. 4d.
4. £1. 19s. 8d., £3. 10s. 0d., £5. 9s. 8d., £4. 5s. 3d., £2. 13s. 8d.

## PAGES 62, 63.

- |                     |                    |                   |
|---------------------|--------------------|-------------------|
| 1. 350000.          | 2. 8120.           | 3. 72576.         |
| 4. 7468.            | 5. £106. 3s. 4d.   | 6. £61. 19s. 6d.  |
| 7. £17. 5s. 10d.    | 8. £246. 11s. 10d. | 9. £165. 13s. 0d. |
| 10. £7163. 12s. 9d. |                    |                   |

## PAGE 64.

- |                      |                  |                      |
|----------------------|------------------|----------------------|
| 1. Lisbon Pipe 115.  | Sherry Butt 108. | Malaga Pipe 105.     |
| £ s. d.              | £ s. d.          | £ s. d.              |
| s. 5 15 0            | s. 5 8 0         | s. 5 5 0             |
| d. 9 7               | d. 9 0           | d. 8 9               |
| f. 2 4 $\frac{3}{4}$ | f. 2 3           | f. 2 2 $\frac{1}{4}$ |
2. £9. 4s. 6d., £8. 13s. 5d., £8. 8s. 5d., £43. 7s. 3 $\frac{1}{2}$ d.,  
 £40. 14s. 6d., £39. 11s. 10 $\frac{1}{2}$ d., £31. 12s. 6d., £29. 14s. 0d.,  
 £28. 17s. 6d., £78. 6s. 3d., £68. 17s. 0d., £66. 18s. 9d.,  
 £79. 10s. 10d., £74. 14s. 0d., £72. 12s. 6d.
3. £46. 4s. 0d. 4. £14. 3s. 6d., £11. 16s. 3d., £18. 7s. 6d.  
 5. 15s., 15s. 7d.

## PAGE 65.

- |                 |                 |                |
|-----------------|-----------------|----------------|
| 1. £1. 10s. 5d. | 2. £1. 12s. 0d. | 3. £1. 3s. 9d. |
| 4. £2. 3s. 6d.  | 5. £1. 3s. 8d.  |                |

## PAGE 70.

- |               |           |           |
|---------------|-----------|-----------|
| 1. 152090400, | 549120,   | 12540000. |
| 2. 2962080,   | 44061600, | 37382400. |

## PAGE 74.

- |   |                    |               |
|---|--------------------|---------------|
| 1. 210 9' 3".   | 2. 2100 c. ft.     | 3. 240 7' 6". |
| 4. SP. 12 feet $\times$ 11 in. $\times$ 1 $\frac{1}{2}$ in. Area = 25 $\frac{1}{4}$ sq. ft. |                    |               |
| N. 12 feet $\times$ 9 in. $\times$ 3 in. Area = 24 $\frac{3}{4}$ sq. ft.                    |                    |               |
| S. 14 feet $\times$ 10 in. $\times$ 3 in. Area = 30 $\frac{3}{4}$ sq. ft.                   |                    |               |
| 5. 2270 6' 4" 6'''.   | 6. £501. 13s. 2d.  | 7. £123355.   |
| 8. £2598. 15s. 0d.  | 9. £4307. 11s. 2d. | 10. Page 80.  |

## PAGE 78.

- |                   |                  |                                |
|-------------------|------------------|--------------------------------|
| 1. £17. 13s. 6d.  | 2. £17. 16s. 3d. | 3. £26. 1s. 1 $\frac{1}{2}$ d. |
| 4. £18. 6s. 8d.   | 5. £40. 17s. 6d. | 6. £117. 19s. 10d.             |
| 7. £6. 3s. 10d.   | 8. £32. 0s. 10d. | 9. £45. 6s. 6d.                |
| 10. £302. 6s. 3d. |                  |                                |

## PAGE 80.

- |                  |                 |                 |
|------------------|-----------------|-----------------|
| 1. £5. 12s. 10d. | 2. £3. 6s. 4d.  | 3. £2. 11s. 7d. |
| 4. £11. 19s. 2d. | 5. £2. 15s. 1d. |                 |

## PAGE 87.

- |  |  |  |
|--|--|--|
| 1. Tables.   |  |  |
| 2. £30. 0s. 8d., £34. 4s. 4d., £28. 0s. 5d., £40. 16s. 8d.,<br>£86. 13s. 9d.                       |  |  |
| 3. £40. 14s. 8d., £46. 3s. 0d., £66. 6s. 0d., £83. 4s. 0d., £91.,<br>£135. 4s. 0d., £163. 16s. 0d. |  |  |



## SECTION IV.

## PAGE 97.

- |                  |               |              |
|------------------|---------------|--------------|
| 1. (1) 2836·370. | (2) 774783·4. | (3) 553·116. |
| (4) 7240·990.    | (5) 3138·098. | (6) 715·665. |
| (7) 16855·747.   | (8) 71·583.   | (9) 681·882. |
| (10) 14·583.     |               |              |
2. 793·977, 63·973.      3. (1) 2·7512, (2) 0·4047.

## PAGES 101, 102.

- |                   |                |                |
|-------------------|----------------|----------------|
| 1. (1) 168850·73. | (2) 1673·780.  | (3) 8372·85.   |
| (4) 430·95.       | (5) 22502·133. | (6) 150·710.   |
| 2. (1) 7916·71.   | (2) 1208·595.  | (3) 22061·22.  |
| (4) 825·548.      | (5) 22400·22.  | (6) 144222·24. |

## PAGE 103.

- |                       |                       |                |
|-----------------------|-----------------------|----------------|
| 1. $94\frac{1}{2}$ c. | 2. $11\frac{1}{2}$ d. | 3. 56 cere.    |
| 4. $9\frac{1}{4}$ d.  | 5. 13 co.             | 6. £3. 6s. 1d. |

## PAGE 105.

It is to be noticed that 1 piastre = 5 pesetas.

1.  $8\frac{1}{2}$  d.      2. 6·873 milr.      3. 25·810d.      4. 3d. nearly.

## PAGES 107, 108.

- |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|
| 1. (1) $11\frac{1}{4}$ d. | (2) 1s. $0\frac{1}{4}$ d. | (3) 2s. $0\frac{1}{4}$ d. |
| (4) 1s. 10d.              | (5) 1s. $1\frac{1}{2}$ d. | (6) 10d.                  |
2. 1·64, 1·44,  $1\cdot17\frac{1}{2}$ , 259 reis, 0·51.
3. 6·92, 10s.  $1\frac{1}{2}$  d., 7s.  $10\frac{1}{2}$  d., 1·33, 0·18, 1s.  $8\frac{1}{2}$  d., £5. 16s.  $4\frac{1}{2}$  d.
- |                                    |   |                           |
|------------------------------------|---|---------------------------|
| 4. (1) $12\frac{3}{4}$ d.          | (2) 1s. 11d.                              | (3) 2s. $5\frac{1}{4}$ d. |
| (4) 3s. 7d.                        | (5) 1s. $11\frac{1}{2}$ d.                | (6) $9\frac{3}{4}$ d.     |
| (7) 1s. $3\frac{3}{4}$ d.—1s. 4d.  | (8) 1s. $6\frac{1}{4}$ — $\frac{1}{2}$ d. | (9) 2s. $2\frac{3}{4}$ d. |
| (10) 1s. $0\frac{3}{4}$ d.—1s. 1d. |   |                           |

## PAGE 113.

1. (1) 61s. 3 $\frac{1}{4}$ d. (2) £13. 9s. 5d. (3) 148s. 3 $\frac{1}{2}$ d.  
(4) £6. 12s. 8 $\frac{3}{4}$ d. (5) £9. 18s. 2 $\frac{1}{2}$ d. (6) £6. 7s. 1d.
2. (1) 3.41. (2) 2.12. (3) 4.32.  
(4) 5.64. (5) 1.173. (6) 4.115.
3. (1) 1.62. (2) 2.99. (3) 1.80 pesetas. (4) 220 reis.
4. (1) £1. 9s. 3 $\frac{1}{4}$ d. (2) £3. 4s. 7d. (3) £1. 11s. 7 $\frac{1}{2}$ d.  
(4) 17s. 1d.
5. (1) 17.62. (2) 1.556. (3) 0.72. (4) 1.00.
6. (1) 9s. 3 $\frac{1}{4}$ d. (2) 9s. 0 $\frac{1}{2}$ d. (3) 7 $\frac{1}{4}$ d. (4) 1s. 8 $\frac{1}{2}$ — $\frac{3}{4}$ d.

## SECTION V.

## PAGES 124, 125.

1. (1)  $\frac{7}{24}$ ,  $\frac{1}{47}$ . (2)  $\frac{9}{47}$ ,  $\frac{17}{174}$ . (3)  $\frac{1}{2}$ ,  $\frac{7}{8}$ . (4)  $\frac{39}{88}$ ,  $\frac{3}{88}$ .  
(5)  $\frac{39}{88}$ ,  $\frac{3}{88}$ .
2. (1)  $\frac{1}{13}$ ,  $\frac{1}{117}$ . (2)  $\frac{13}{117}$ ,  $\frac{1}{101}$ . (3)  $\frac{27}{88}$ ,  $\frac{2}{88}$ .
3. 138s., 161s. 5d., 145s., 159s., 134s. 4d.
4. (1) 12 $\frac{3}{4}$ d. (2) 3s. 2d. (3) 10s. 3 $\frac{1}{2}$ d.
5. (1) 19 : 24 : 30 : 30 : 3. (2) 42 : 20 : 10 : 4 : 10.  
(3) 2 : 6 : 6 : 2 : 10 : 11 $\frac{1}{2}$ . (4) 4 : 4 : 7 : 33 : 9.  
(5) 5 : 9 : 9.
6. (1) 20 : 10 : (20) : 50. (2) 33 : 60 + 50 : 88 : 37.  
(3) 3 : 8 : 11 : 6.  $\frac{3}{8} \times 348$  etc.  
(4) 1 : 3 : 7 : 5.  $\frac{1}{8} \times 100$  etc.  
(5) 37 : 37 : 37 : 28 + 70.

7. (1) 4 u.p. (2) p. nearly.  
 8. (1) 16. (2) 6 $\frac{1}{2}$ . (3) 2. (4) 2 $\frac{1}{2}$ .  
 9. (1) 12 : 8. (2) 86 : 64 nearly. (3) 160 : 23.  
 (4) 50 : 50. (5) 27 : 44 : 44.  
 10. (1) 21 $\frac{1}{2}$  ozs. (2) 750, 925.  
 (3) 950, 40, 10.

PAGE 128.

*Note.* The first formula is to be used after moving the point two places to the left : the second formula requires no such change.

1. 3-44 or 0-(40)452. 2. 4-48 or 0-(40)2+5(16).  
 2-4222 or 0-(50)8222. 0-842.  
 4-8 or 0-(40)252. 1-24 or 0-(100)24.  
 3-223 or 0-(30)(10)2. 3-82 or 0-(40)4(10).  
 1-242 or 0-(80)4+10. 7-8 or 0-(20)518.  
 3. 1-224 or 0-(80)4+5. 4. 3-2 or 0-(50)22.  
 2-2(16) or 0-(40)(80). 4-22 or 0-(25)82.  
 3-2822 or 0-(40)422(10). 1-24 or 0-(100)24.  
 5-222 or 0-(20)8(20). 3-42 or 0-(40)4+10.  
 8-225 or 0-(20)22(25). 5-24 or 0-(20)5.  
 5. 3-(10) or 0-(50)2(10).  
 5-522 or 0-(20)(20)51.  
 8-3+8 or 0-(15)4(20)2.  
 5-422 or 0-(20)(16)51.  
 9-(16) or 0-(20)228.

1. £18·925. £14·105. £23·568. £21·901. £9·641.  
 2. £36·883. £1·481. £13·999. £27·460. £61·382.  
 3. £18·125. £25·313. £36·094. £58·750. £88·000.  
 4. £24·943. £33·852. £11·581. £24·053. £40·088.  
 5. £29·865. £51·541. £81·486. £52·386. £87·307.  
 6. £175, £13. 7s. 4d., £23. 11s. 2d.  
 7. £662·5, £238·375, £29·8125.  
 8. £32. 16s. 3d., £5. 9s. 4d.  
 9. 9s. 7d., £27. 15s. 3d.  
 10. £22. 7s. 11d., £26. 17s. 6d., £29. 2s. 3d., £41. 15s. 10d.

## PAGES 129, 130.

1. £371, £494.13s.6d., £391.4s.7d., £521.12s.9d., £435.3s.8d., £580.4s.10d.
2. £652.16s.0d., £333.4s.0d., £170.
3. £451.2s.5d., £505.15s.8d.
4. £82.16s.8½d., £97.2s.8½d., £132.15s.5d. 6s.5½—7d.
5. £350, £379.13s.9d., £210.12s.0d., £488.13s.3d., £327.8s.9d.
6. £542.4s.5d., £650.13s.4d., £759.2s.3d.  
£517.17s.8d., £621.9s.2d., £725.0s.8d.  
£555.11s.1½d., £666.13s.4d., £777.15s.6¾d.
7.  $B = £60.3s.3d.$ ,  $C = £38.16s.8d.$ ,  $D = £36.1s.2d.$
8. £3769.17s.8d., £6283.2s.10d., £8796.7s.11d.
9. 5·640 p.c. £676·840, £733·243, £1410·083.
10. £464·040, £407·534, £349·315.

## PAGE 131.

- |                |                 |               |
|----------------|-----------------|---------------|
| 1. £4.18s.5d.  | 2. £7.13s.10d.  | 3. £2.8s.5d.  |
| 4. £8.6s.8d.   | 5. £45.12s.0d.  | 6. £63.0s.0d. |
| 7. £38.1s.5d.  | 8. £63.17s.10d. | 9. 12 years.  |
| 10. 13½ years. |                 |               |

## PAGE 133.

- |   |                 |                   |
|---|-----------------|-------------------|
| 1. 5s.1½d.  | 2. 2s.4d.       | 3. £6.4s.7d.      |
| 4. 6s.3d., 10s., 30s., 53s.4d.                              |                 |                   |
| 5. £79.3s.7d., £21.14s.4d., £160.8s.2d., £204.4s.1d., £225. |                 |                   |
| 1. £1·068.  | 2. £18·750.     | 3. £3.10s.0d.     |
| 4. £15443.3s.4d.  | 5. £1071.8s.0d. | 6. £2528.19s.10d. |

## PAGES 134, 135.

- |                        |                            |                       |
|------------------------|----------------------------|-----------------------|
| 1. 10s.                | 2. 24s.4d.                 | 3. £3.17s.5d.         |
| 4. 16s.7d.             | 5. £2.10s.5d.              | 6. £1203·762. £3·762. |
| 7. £1852·549. £2·549.  | 8. £7011·686. £11·686.     |                       |
| 9. £6485·671. £35·671. | 10. £21052·631. £1052·631. |                       |

## COMMERCIAL PROFIT AND LOSS.

Questions in Profit and Loss are reckoned by most merchants on the *Selling Price*. This is done on the ground that to reckon on the Cost Price would be to charge no interest on the Capital invested in the goods.

The Rules are here contrasted:—

## THEORETICAL.

## PRACTICAL.

1. To find gain or loss p.c., given cost price and selling price.

$$\left( \text{s. p.} - \text{c. p.} \right) \times \frac{100}{\text{c. p.}} \quad \left| \quad \left( \text{s. p.} - \text{c. p.} \right) \times \frac{100}{\text{s. p.}} \right.$$

*Ex.* Gain p.c. by selling for  $11\frac{1}{4}d.$  what cost  $8\frac{3}{4}d.$

$$\left( 11\frac{1}{4} - 8\frac{3}{4} \right) \times \frac{100}{8\frac{3}{4}} \quad \left| \quad \left( 11\frac{1}{4} - 8\frac{3}{4} \right) \times \frac{100}{11\frac{1}{4}} \right.$$

$$= 28\frac{1}{2} \text{ p.c.} \quad \left| \quad = 22\frac{2}{3} \text{ p.c.} \right.$$

2. To find selling price to gain a given p.c.

$$\text{c. p.} \times \frac{100 + \text{given p.c.}}{100} \quad \left| \quad \text{c. p.} \times \frac{100}{100 - \text{given p.c.}} \right.$$

*Ex.* Find s.p. to gain 12 p.c. on article costing 3s.  $2\frac{1}{2}d.$

$$38\cdot5 \times \frac{112}{100} \quad \left| \quad 38\cdot5 \times \frac{100}{88} \right.$$

$$= 43\cdot120d. \quad \left| \quad = 43\cdot75d. \right.$$

3. To mark goods so as to ensure a given p.c. profit after a given discount.

$$\text{c. p.} \times \frac{100 + \text{given p.c.}}{100 - \text{given disc.}} \quad \left| \quad \text{c. p.} \times \frac{100}{100 - \text{given disc.}} \right.$$

$$\quad \quad \quad \times \frac{100}{100 - \text{given p.c.}} \left. \right.$$

*Ex.* Mark article at 2s.  $9\frac{1}{4}d.$  to gain 20 p.c. after deducting 10 p.c.

$$33\cdot25 \times \frac{110}{90} \quad \left| \quad 33\cdot25 \times \frac{100}{90} \times \frac{100}{90} \right.$$

$$= 44\cdot3d. \quad \left| \quad = 46\cdot18d. \right.$$

4. To mark goods bought by the dozen, score, gross, hundred, thousand, to gain a given p. c.

$$\text{c. p.} \times \frac{100 + \text{given p. c.}}{x \times 100}, \quad \left| \quad \text{c. p.} \times \frac{100}{x(100 - \text{given p. c.})}, \right.$$

where  $x = 12, 20, 144, 100, 1000$ , as the case may be.

5. To mark goods bought in any quantity to gain a given p. c.

$$\text{c. p.} \times \frac{100 + \text{given p. c.}}{x \times 100}, \quad \left| \quad \text{c. p.} \times \frac{100}{x(100 - \text{given p. c.})}, \right.$$

where  $x$  is the quantity in the desired unit of quotation.

*Ex.* Price goods per lb. so as to gain 10 p. c. if bought at £1. 5s. 9d. per cwt.

$$\begin{array}{l|l} 1.2375 \times \frac{110}{x \times 100} & 1.2375 \times \frac{100}{x \times 90} \\ \text{where } x = 112 & \\ = 3\frac{1}{4}d. \text{ per lb. nearly.} & = 3\frac{3}{4}d. \text{ per lb. nearly.} \end{array}$$

The following is a comparison of the multipliers to be applied to the c. p. for different rates :

THEOR.	P.C.	COM.
$1\frac{1}{100}$	1	$1\frac{1}{100}$
$1\frac{1}{50}$	$2\frac{1}{2}$	$1\frac{1}{50}$
$1\frac{1}{25}$	5	$1\frac{1}{25}$
$1\frac{3}{100}$	$7\frac{1}{2}$	$1\frac{3}{100}$
$1\frac{1}{10}$	10	$1\frac{1}{10}$
$1\frac{1}{5}$	$12\frac{1}{2}$	$1\frac{1}{5}$
$1\frac{3}{50}$	15	$1\frac{3}{100}$
$1\frac{1}{3}$	20	$1\frac{1}{3}$
$1\frac{1}{2}$	25	$1\frac{1}{2}$
$1\frac{3}{10}$	30	$1\frac{3}{10}$
$1\frac{2}{3}$	$33\frac{1}{3}$	$1\frac{2}{3}$
$1\frac{1}{2}$	40	$1\frac{2}{5}$
$1\frac{1}{2}$	50	2

## PAGE 136. (Theoretically.)

1.  $38\frac{1}{2}$ ,  $22\frac{1}{2}$ ,  $11\frac{1}{2}$ , 20, 27,  $27\frac{1}{2}$  (nearly).
2.  $11\frac{1}{2}d.$ ,  $1s. 6\frac{1}{2}d.$ ,  $3s. 11\frac{1}{2}d.$ ,  $1s. 8\frac{1}{2}d.$ ,  $7s. 2d.$ ,  $\pounds 4. 1s. 7d.$ ,  $\pounds 16. 2s. 7\frac{1}{2}d.$  (approx.).
3. (1)  $9\frac{1}{2}d.$  (2)  $2s. 0\frac{3}{4}d.$  (3)  $5s. 6d.$  (4)  $6s. 10d.$
- (5)  $27s. 1d.$  (6)  $23s. 1d.$  (7)  $\pounds 6. 2s. 9d.$  (8)  $\pounds 16. 16s. 0d.$
- (9)  $\pounds 29. 18s. 7d.$  (10)  $\pounds 43. 3s. 7d.$

## PAGE 136. (Practically.)

1.  $27\frac{1}{2}$ ,  $29\frac{1}{2}$ ,  $10\frac{1}{2}$ ,  $16\frac{1}{2}$ ,  $21\frac{1}{2}$ ,  $21\frac{1}{2}$ .
2.  $11\frac{1}{2}d.$ ,  $1s. 7d.$ ,  $4s.$ ,  $1s. 9d.$ ,  $7s. 5\frac{1}{2}d.$ ,  $\pounds 4. 6s. 7d.$ ,  $\pounds 19. 4s. 2d.$
3. (1)  $9\frac{1}{2}d.$  (2)  $2s. 1d.$  (3)  $6s. 1d.$  (4)  $7s. 1\frac{1}{2}d.$
- (5)  $27s. 8d.$  (6)  $24s. 6d.$  (7)  $\pounds 7. 0s. 7\frac{1}{2}d.$  (8)  $\pounds 26. 5s. 0d.$
- (9)  $\pounds 30. 12s. 5d.$  (10)  $\pounds 48. 7s. 2d.$

## PAGE 139. (Theoretically.)

1.  $2s.$ ,  $3s. 3\frac{1}{2}d.$ ,  $2s. 5\frac{1}{2}d.$ ,  $1s. 11\frac{1}{2}d.$ ,  $4s. 11d.$
2.  $3s. 7d.$ ,  $5s. 2\frac{1}{2}d.$ ,  $6s. 10d.$ ,  $4s. 11\frac{1}{2}d.$ ,  $5s. 2\frac{1}{2}d.$
3.  $1s. 11d.$ ,  $2s. 6\frac{1}{2}d.$ ,  $3s. 6\frac{1}{2}d.$ ,  $16s. 6d.$ ,  $8s. 6\frac{1}{2}d.$
4.  $\pounds 1. 181.$ ,  $\pounds 2. 884.$ ,  $\pounds 1. 121.$
5.  $6s. 11\frac{1}{2}d.$ ,  $14s. 7\frac{1}{2}d.$ ,  $21s. 1d.$
6.  $6\frac{1}{2}d.$ ,  $3s. 9\frac{1}{2}d.$ ,  $1s. 2\frac{1}{2}d.$ ,  $3s. 3d.$ ,  $1s. 10d.$ ,  $7s. 3d.$ ,  $2s. 7d.$ ,  $2s. 5\frac{1}{2}d.$ ,  $3s. 1d.$
7.  $11\frac{1}{2}d.$ ,  $10\frac{1}{2}d.$ ,  $1s. 1\frac{1}{2}d.$ ,  $2s. 10\frac{1}{2}d.$ ,  $1s. 7\frac{1}{2}d.$
8.  $2s. 11d.$ ,  $3s. 11\frac{1}{2}d.$ ,  $5s. 5\frac{1}{2}d.$
9.  $2s. 1d.$ ,  $2s. 9\frac{1}{2}d.$ ,  $7s. 5d.$  10.  $14s. 2d.$ ,  $4s. 8\frac{1}{2}d.$

## PAGE 139. (Practically.)

1.  $2s.$ ,  $3s. 3\frac{1}{2}d.$ ,  $2s. 6\frac{1}{2}d.$ ,  $2s. 1d.$ ,  $5s. 3d.$
2.  $3s. 8d.$ ,  $5s. 5\frac{1}{2}d.$ ,  $7s. 1d.$ ,  $5s. 5d.$ ,  $5s. 11d.$
3.  $1s. 11\frac{1}{2}d.$ ,  $2s. 7\frac{1}{2}d.$ ,  $3s. 8\frac{1}{2}d.$ ,  $17s. 6d.$ ,  $18s. 4\frac{1}{2}d.$
4.  $\pounds 1. 3s. 8\frac{1}{2}d.$ ,  $\pounds 2. 18s. 7\frac{1}{2}d.$ ,  $\pounds 1. 2s. 11\frac{1}{2}d.$
5.  $7s.$ ,  $14s. 11d.$ ,  $22s. 5d.$
6.  $7d.$ ,  $1s. 3d.$ ,  $1s. 10\frac{1}{2}d.$ ,  $2s. 9d.$ ,  $3s. 5d.$ ,  $4s.$ ,  $3s. 5\frac{1}{2}d.$ ,  $7s. 9\frac{1}{2}d.$ ,  $2s. 7d.$
7.  $11\frac{1}{2}d.$ ,  $11d.$ ,  $1s. 2d.$ ,  $3s. 5d.$ ,  $2s. 1\frac{1}{2}d.$
8.  $3s.$ ,  $4s.$ ,  $5s. 7d.$
9.  $2s. 1d.$ ,  $2s. 10d.$ ,  $8s. 3d.$  10.  $14s. 4d.$ ,  $4s. 10d.$

## PAGE 140. (Theoretically.)

1.  $\frac{1}{2}d.$  2.  $3d.$  3.  $1s.$  4.  $4d.$  5.  $1s. 1\frac{1}{2}d.$

## PAGE 140. (Practically.)

1.  $\frac{1}{8}d.$  2.  $3\frac{1}{2}d.$  3.  $1s. 1d.$  4.  $4s. 4\frac{1}{2}d.$  5.  $1s. 2d.$

## SECTION VII.

## PAGES 151, 152.

- |  |                                  |
|--|----------------------------------|
| 1. £25. 12s. 0d., £6. 8s. 0d., £17. 1s. 4d.  |                                  |
| 2. £32. 10s. 0d., £1. 12s. 6d., £6. 10s. 0d. |                                  |
| 3. £9. 9s. 8d., £35. 0s. 5d.                 | 4. £6. 0s. 8d., £11. 6s. 3d.     |
| 5. £3. 2s. 6d., £8. 6s. 8d.                  | 6. £25. 9s. 6d., £44. 11s. 7d.   |
| 7. £5. 6s. 5d., £18. 4s. 10d.                | 8. £86. 7s. 0d., £323. 16s. 6d.  |
| 9. £22. 10s. 8d., £29. 14s. 4d.              | 10. £17. 17s. 3d., £30. 12s. 6d. |
| 11. £62. 10s. 0d., £2000.                    | 12. £17. 10s. 0d., £800.         |

## PAGE 152.

- |                   |                  |                  |
|-------------------|------------------|------------------|
| 1. £136. 5s. 11d. | 2. £37. 17s. 9d. | 3. £74. 18s. 4d. |
| 4. £167. 3s. 5d.  | 5. £22. 13s. 8d. |                  |

## PAGE 154.

1. £2. 8s. 8d., £1. 17s. 10d., £23. 6s. 11d.
2. 13s. 7d., £1. 8s. 0d., £1. 2s. 7d.
3. £19. 1s. 10d., £3. 0s. 10d., £3. 4s. 1d.
4. £15. 8s. 2d., £8. 8s. 5d., £6. 7s. 4d.
5. £6. 10s. 5d., £35. 13s. 11d., £20. 10s. 3d.
6. £13. 6s. 8d., £16. 5s. 9d., £26. 13s. 7d.
7. £20. 12s. 3d., £9. 9s. 2d., £8. 18s. 5d.
8. £48. 7s. 7d., £11. 8s. 4d., £5. 13s. 2d.
9. £1. 10s. 8d., £2. 19s. 8d., £3. 5s. 4d.
10. £16. 9s. 2d., £8. 15s. 1d., £38. 10s. 3d.
- 11—15. Aliquot Formulae.

## PAGE 156.

- |                                |                                |
|--------------------------------|--------------------------------|
| 1. £5. 11s. 0d., £3. 16s. 6d.  | 2. £12. 17s. 1d., £5. 3s. 3d.  |
| 3. £8. 12s. 7d., £8. 10s. 4d.  | 4. £5. 7s. 10d., £5. 1s. 8d.   |
| 5. £15. 9s. 6d., £14. 1s. 11d. | 6. £12. 8s. 1d., £39. 17s. 9d. |
| 7. £4. 4s. 1d., £6. 16s. 2d.   | 8. £3. 1s. 8d., £2. 8s. 0d.    |
| 9. £41. 4s. 2d., £4. 4s. 5d.   | 10. £6. 13s. 2d., £1. 11s. 0d. |



## PAGE 157.

- |                                |                               |
|--------------------------------|-------------------------------|
| 1. £1. 15s. 8d., £4. 0s. 7d.   | 2. £7. 2s. 7d., £35. 15s. 7d. |
| 3. £6. 16s. 3d., £2. 0s. 8d.   | 4. £2. 13s. 6d., £1. 5s. 7d.  |
| 5. £2. 19s. 2d., £14. 18s. 4d. |                               |

## PAGE 159.

- |                                  |                                |
|----------------------------------|--------------------------------|
| 1. £7. 7s. 2d., £3. 18s. 0d.     | 2. £2. 8s. 3d., £1. 19s. 3d.   |
| 3. £5. 2s. 1d., £1. 12s. 7d.     | 4. £20. 3s. 3d., £8. 15s. 2d.  |
| 5. £9. 7s. 9d., £5. 9s. 1d.      | 6. £89. 6s. 8d., £15. 15s. 1d. |
| 7. £3. 0s. 3d., £1. 19s. 11d.    | 8. £0. 11s. 0d., £1. 11s. 9d.  |
| 9. £1. 12s. 10d., £2. 12s. 9d.   | 10. £1. 13s. 8d., £2. 14s. 5d. |
| 11. £5. 4s. 0d., £9. 1s. 0d.     | 12. £7. 3s. 1d., £17. 15s. 1d. |
| 13. £6. 19s. 8d., £8. 7s. 0d.    | 14. £3. 4s. 2d., £11. 19s. 1d. |
| 15. £0. 15s. 7d., £2. 19s. 4d.   | 16. £0. 16s. 3d., £1. 4s. 5d.  |
| 17. £1. 9s. 1d., £3. 0s. 11d.    | 18. £3. 14s. 7d., £4. 15s. 4d. |
| 19. £33. 10s. 9d., £17. 11s. 9d. | 20. £9. 9s. 3d., £4. 2s. 11d.  |

## PAGES 163, 164.

- |                        |                          |
|------------------------|--------------------------|
| 1. £848·052, £780·971. | 2. £1266·692, £1282·609. |
| 3. £762·243, £684·796. | 4. £112·851, £112·945.   |
| 5. £348·841, £334·320. | 6. £727·033, £767·755.   |
| 7. £57·589, £228·425.  | 8. £154·946, £73·558.    |
| 9. £62·051, £123·194.  | 10. £155·859, £53·424.   |

## PAGE 169.

Tables should be constructed for £1 to six or seven places.

## PAGES 173, 174.

- |   |                                |
|---|--------------------------------|
| 1. £609·428.  | 2. £712·908 (Discount £2·583). |
| 3. £994·709.  | 4. £3·559 (Discount).          |
| 5. £3·319 (Discount).                                 | 6. £1·438 (Discount).          |
| 7. £839. 18s. 11d.                                    | 8. £3·787 (Discount).          |
| 9. £3·589 (Discount).                                 | 10. £5·888 (Discount).         |
| 11. £54. 16s. 6d., £63. 15s. 6d., £77. 0s. 2d.        |                                |
| 12. £1. 3s. 11d., 19s. 1½d., £1. 5s. 6d., £1. 0s. 5d. |                                |
| 13. £1. 15s. 7d., 15s. 4d., £6. 15s. 7d.              |                                |
| 14. £40. 15s. 1d., £125. 0s. 5d., £163. 18s. 4d.      |                                |
| 15. £4. 3s. 7d. nearly.                               |                                |

## PAGES 185, 186.

- |              |              |                        |
|--------------|--------------|------------------------|
| 1. £636·760. | 2. £390·325. | 3. £350·438<br>± 2·787 |
|--------------|--------------|------------------------|

- |                          |   |
|--------------------------|---|
| 4. £6466·163<br>± 10·633 | 5. Sale = £14218·650<br>Less Com. etc. 75·093<br><hr/> £14143·557<br><br>Purchase = £14053·000<br>Plus Com. etc. 74·190<br><hr/> £14127·190 |
|--------------------------|---|

Bal. due from Broker £16. 7s. 4d.

- |   |
|---|
| 6. Sale less Com. etc. £885·550<br>Purchase plus Com. etc. 868·685<br><hr/> £16·865 |
|---|

- |                                   |              |
|-----------------------------------|--------------|
| 7. Bal. due from Brokers £10·377. | 8. £166·125. |
|-----------------------------------|--------------|

9. Dr.		Cr.
	£	£
	270·000	287·500
	13·500	795·000
	780·000	
	3·900	<hr/> 950·000
	985·000	2100·000
	4·925	
	2050·000	<hr/> 4132·500
	10·250	6·188
	<hr/> £4117·575	<hr/> £4126·312
contr.	·250	
	<hr/> £4117·825	

Balance = £8·487.

10. The 114 Greek bonds at 79.

## COMPUTATION TABLE.

The pages 187—189 may be conveniently replaced by the following small Table:

Months.			Days.	
13	1	January	0	365
14	2	February	31	396
15	3	March	59	424
16	4	April	90	455
17	5	May	120	485
18	6	June	151	516
19	7	July	181	546
20	8	August	212	577
21	9	September	243	608
22	10	October	273	638
23	11	November	304	669
24	12	December	334	699

*Example 1.* No. of days between Feb. 17 and Aug. 13

$$\begin{array}{rcl}
 13 + 212 & = & 225 \quad \text{or} \quad 212 \\
 17 + 31 & = & 48 \quad \quad \quad 31 \\
 \hline
 & & 177 \quad \quad \quad 181 - 4 = 177
 \end{array}$$

*Example 2.* No. of days between Nov. 6, 1888 and March 10, 1889

$$\begin{array}{rcl}
 10 + 424 & = & 434 \quad \text{or} \quad 424 \\
 6 + 304 & = & 310 \quad \quad \quad 304 \\
 \hline
 & & 124 \quad \quad \quad 120 + 4 = 124
 \end{array}$$

*Example 3.* 5 mos. from Apr. 9.

$$5 + 4 = 9, \therefore \text{Sept. 9.}$$

*Example 4.* 90 days from May 5.

$$120 + 5 + 90 = 215 \text{ (Aug. = 212)}$$

$\therefore$  Aug. 3 or Aug. 6 with grace.

*Example 5.* 6 mos. bill dated Aug. 9, discounted Nov. 18.

$$6 \text{ mos. from Aug. 9, } 8 + 6 = 14, \therefore \text{Feb. 9.}$$

$$\begin{aligned}
 \text{No. of days between Nov. 18 and Feb. 12} &= 396 + 12 - (304 + 18) \\
 &= 408 - 322 = 86 \text{ days.}
 \end{aligned}$$

## SECTION VIII.

## PAGES 191, 192.

1. (1) £41. 4s. 5½.      (2) £232. 1s. 10½d.      (3) £4. 2s. 9¾d.  
     (4) £31. 9s. 11¾d.
2. (1) £242.      (2) £259. 9s. 10d.      (3) £10535. 1s. 10d.  
     (4) £889. 18s. 8d.

## PAGES 206, 207, 208.

1. (1) Oct. 30.      (2) Aug. 10.      (3) Oct. 28.      (4) Sept. 13.  
     (5) July 16.
2. (1) Jan. 1, 1890.    5 p.c. p.a.    £577·683.  
     (2) July 26, 1891.    5 p.c. p.a.    £764·621.  
     (3) June 13, 1894.    5 p.c. p.a.    £380·888.  
     (4) Dec. 25, 1897.    £26. 12s. 5d.    Ap. 22, 1894.    £387·289.
3. (1) £4426·294.      (2) £4470·349.
4. Sept. 8, 1889.

## PAGES 212, 213.

1. £231·562.      2. £1832·726.      3. £4773. 11s. 8d. credit.
4. £967·693.      5. £820·561.      6. £394·494.      7. £2688·852.

## SECTION IX.

## PAGES 231, 232.

Telegraphic Transfers are in principle "sight" or "demand" bills remitted by telegraph. If a debtor of mine lives in India (say) it would take "weeks" before even a "sight bill" sent by him reaches me. I may want the money at once, and so I get him to pay his debt to the branch of a foreign bank in his place; this branch advises its London Office, which pays me immediately. Similarly, if I owe money in distant countries and my creditor requires it at once, I pay the amount to the London Office of a foreign bank which advises (by wire) its branch to pay over at once to my creditor.

## PAGES 233, 234.

Nos. 1 and 2 of the Methods of Payment are both very frequent. Bills are bought "on Change" in London, at the "Bourses" abroad.

## PAGES 236, 237.

In the Example p. 236, the Interest should be omitted, as the London Quotation for Vienna is for 3 mos.

In the Example p. 237, a draft on Paris is intended and the endorsement (25. 18) is in francs. So far as the English creditor is concerned the cost in francs is of no importance.

- |                            |                          |              |
|----------------------------|--------------------------|--------------|
| 1. £595-835.               | 2. £385-591.             | 3. £717-565. |
| 4. £1564-734.              | 5. £378-985.             |              |
| 1. £707-844, Fr. 17961-92. | 2. £319-073, M. 6535-82. |              |
| 3. £989-764, Ro. 9760.     | 4. £412-964, \$2025-16.  |              |
| 5. £1808-543, Ts. 9503-63. |                          |              |

## PAGE 238.

These depend on the Multiples to be constructed.

PAGE 248.

## PRELIMINARY DETERMINATION OF DRAFTS.

The importance of the preliminary determination is so great that a general statement of the method is inserted here for the three kinds of joint rates.

1. *A* has a Foreign Rate (London).

*B* has a Sterling Rate (Paris).

£*x* offered in *B* at 25·15, 3 mos.

Int. at 4 p.c. in *A*. Brok. 1 p.m. Com.  $\frac{1}{8}$  p.c. Stamp.  $\frac{1}{2}$  p.m.

$$x \times 25 \cdot 15 \left(1 + \frac{1}{100}\right) \left(1 - \frac{2\frac{3}{4}}{1000}\right) = y \text{ Fr.}$$

*y* Fr. offered in *A* at 25·50, 3 mos.

Int. at 4 p.c. in *B*. Brok. 1 p.m. Com.  $\frac{1}{8}$  p.c. Stamp  $\frac{1}{2}$  p.m.

$$\frac{y}{25 \cdot 50} \left(1 + \frac{1}{100}\right) \left(1 - \frac{2\frac{3}{4}}{1000}\right) = \text{£}x.$$

2. *A* has a Sterling Rate (London).

*B* has a Foreign Rate (Lisbon).

£*x* offered in *B* at 43, 3 mos.

Int. at 4 p.c. in *A*. Brok. etc. as before.

$$\frac{x \times 240}{43} \left(1 + \frac{1}{100}\right) \left(1 - \frac{2\frac{3}{4}}{1000}\right) = y \text{ Milr.}$$

*y* Milr. offered in *A* at 41 $\frac{3}{8}$ , 3 mos.

Int. at 4 p.c. in *B*. Brok. etc. as before.

$$y \times \frac{41\frac{3}{8}}{240} \left(1 + \frac{1}{100}\right) \left(1 - \frac{2\frac{3}{4}}{1000}\right) = \text{£}x.$$

3. *A* has a Sterling Rate (London).

*B* has a Sterling Rate (Russia).

£*x* offered in *B* at 9·79, 3 mos.

Int. at 4 p.c. in *A*. Brok. etc. as before.

$$x \times 9 \cdot 79 (1 + \frac{1}{100}) \left(1 - \frac{2\frac{1}{2}}{1000}\right) = y \text{ Ro.}$$

*y* Ro. offered in *A* at  $24\frac{1}{8}$ , 3 mos.

Int. at 4 p.c. in *B*. Brok. etc. as before.

$$\frac{y \times 24\frac{1}{8}}{240} (1 + \frac{1}{100}) \left(1 - \frac{2\frac{1}{2}}{1000}\right) = £x.$$

Thus to find what must be offered to produce *y* we determine *x* given *y*, and to find what must be offered to produce *x* we determine *y* given *x*.

The working on p. 243 is a case of 3 and stands thus:—

$$x = \frac{20000}{9 \cdot 79 (1 \cdot 01) (\cdot 99725)} = £2028 \cdot 2519.$$

£2028·2519 offered in S. P. at 9·79 3 mos. will produce 19856·586 Ro.

Interest 1 p.c. 198·566

---

20055·152

Brok. 1 p.m. 20·055

Stamp  $\frac{1}{2}$  p.m. 10·028

Com.  $\frac{1}{4}$  p.c. 25·069

55·152

---

20000·00 Ro.

PAGE 244.

1. Paris draw: £989·160.

2. Amsterdam remit: Fl. 10529·15 (cost).

3. £410·439.

4. Fr. 14283·45.

5. £957·031.

6. London remit: £5808·122 (cost).

7. London draw: 7691·74 Fl.

8. London draw: 69481·94 Ro.

9. Genoa draw: £474·381.

10. \$4535·14.

## PAGE 252.

1. Discount rate, 3 p.c. Profit £14·892.
2. Discount rate 1 p.c. Loss 12 Francs.
3. Discount rate 2 p.c. Loss.

## PAGE 267.

1. (a) Paris. (b) Spain. (c) Lisbon.
2. (a) Vienna. (b) Copenhagen. (c) Spain.
3. (a) (b) (c) (d) Yes.

The Drawing Rates here should be interchanged—an instructive error. The student is recommended to reverse them and then see if the answer is still yes. e.g. (a) draw upon Hamburg 20·43 at 20·41½.

4. (a) 25·43½. (b) 51·107. (c) 20·87½. (d) 20·49.

## PAGE 275.

	Fine wt.	Standard wt.	Value in sterling.
1. (1)	181·87,	198·41,	32s. 2·3d.
(2)	372·03,	405·85,	65s. 10·2d.
(3)	175·8,	190·0,	1s. 11·8d.
(4)	370·1,	400·1,	4s. 2·0d.
(5)	374·017,	404·342,	4s. 2½d.
(6)	111·359,	120·389,	1s. 3d.
2. (1)	£2600·458.	(2) £2647·185.	
(3)	£9834·953.	(Standard = 912 nearly.)	
3. (1)	£155·622.	(2) £27·917.	(3) £175·318.
4. (1)	B 1 0½.	(2) W 21·6.	(3) 971. (4) 82·9.
(5)	96½.	(6) 958.	



## SECTION X.

## PAGE 286 (Great Britain).

- (1) London draw: Fl. 4264.02.      (2) Paris draw: £397.999.  
(3) Russia draw: £1967.120.      (4) London draw: \$ 4725.27.  
(5) London remit: £482.277.

## PAGES 294, 295 (France).

1. (1) London draw.      (2) Paris draw.      (3) London draw.  
(4) £337.712.      (5) Fr. 9732.71.  
2. £, through Berlin. Fr., through Amsterdam.  
3. (1) 25.24.      (2) 84.69.      (3) 82.10.      (4) 59d.  
(3) is very unlikely.  
4. Loss of about £3.10s. 0d.  
5. 26.66 (less brokerages etc.).

## PAGE 303 (Germany).

1. (1) Berlin remit: cost, M. 17115.62.      (2) Berlin draw.  
(3) 204902.68 (bought).      (4) £408.845 (sold).  
(5) Berlin remit: cost, M. 18188.02.  
2. M., Berlin draw. £, through S. Petersburg.  
3. (1) 20.896.      (2) 12.80.  
4. £76.283, if rate of discount be 4 p.c.  
5. 21.54 (less brokerages etc.).

## PAGES 307, 308 (Austria).

1. (1) Vienna remit: cost, 10481·08 Fl.  
 (2) Fl. 11921·48 (sold).  
 (3) London remit: cost, £720·917.  
 (4) £836·674 (bought). (5) Fl. 11185·03 (offered).
2. £, through Brussels. Fl., through S. Petersburg.
3. 12·03 (less brokerages etc.).

## PAGE 312 (Russia).

1. (1) London draw. (2) London remit: cost, £4046·998.  
 (3) S. Petersburg remit: 7018·90 Ro.  
 (4) £1546·359 (sold). (5) Ro. 7467·42 (sold).
2. £, through Amsterdam. R, through Paris.
3. 19½d. (plus brokerages etc.).

## PAGE 316 (Spain).

1. (1) London draw. (2) 11400·95 (offered).  
 (3) Madrid draw.
2. £, through Paris. Pes., through Lisbon.
3. Loss of over £130: discount at 4 p.c.

## PAGE 320 (Portugal).

1. (1) London draw. (2) £1275·051 (bought).  
 (3) Milr. 5901·19 (sold). (4) Lisbon draw.  
 (5) Lisbon remit: cost, Milr. 4173·38.
2. £, through Hamburg. Milr., London remit.

## PAGE 323 (Belgium).

1. (1) London draw. (2) London remit.  
 (3) 182982·70 (bought). (4) Rate should be larger.
2. £, through Paris. Fr., through Rotterdam.

## PAGES 326, 327 (Holland).

1. (1) London draw.  
(2) London remit.
2. £, through Paris. Fl., through S. Petersburg.
3. 11·98 (less brokerages etc.).

## PAGE 330 (Italy).

1. (1) London draw. (2) L. 9842·41 (bought).  
(3) L. 209850·11 (bought). (4) Italy draw.
2. £, London draw. L., through Vienna.

## PAGES 333, 334 (Denmark).

1. (1) Copenhagen remit. (2) Kr. 5718·85 (bought).  
(3) £523·937 (offered). (4) Copenhagen draw.
2. £, through Paris. Kr., Copenhagen draw.

## PAGE 337 (Sweden).

1. (1) London draw. (2) London remit.
2. £, through Paris. Kr., through S. Petersburg.

## PAGE 339 (Turkey).

1. (1) Constantinople remit. (2) £3581·63 (bought).  
(3) £T.747·809 (sold). (4) London remit.

## PAGE 341 (Egypt).

1. (1) Cairo remit. (2) London remit.

## PAGE 344 (Canada).

The Multipliers are given.

- |               |               |
|---------------|---------------|
| 1. ·20447530. | 2. ·20254629. |
| 3. ·21219135. | 4. ·20736883. |

## PAGE 346 (United States).

These depend on the Table given on p. 347.

## PAGE 348 (United States).

1. (1) London draw. (2) £261·067 (bought).  
(3) \$4417·51 (offered). (4) New York draw.
2. \$, through Amsterdam. £, through Paris.
3. Profit £26·267: discount 3 p.c.

## PAGE 351 (East Indies).

These depend on the Tables given.

## PAGE 353 (East Indies).

1. (1) Calcutta draw. (2) London draw.
2. Rs., through Paris. £, Calcutta remit.

## PAGE 356 (Treaty Ports).

1. 3s. 2d. nearly. 3. 3s. 1½d. nearly. 3. 3s. 0½d. nearly.

## PAGE 357 (Treaty Ports).

1. (1) Hongkong remit. (2) £189·412 (bought).  
(3) \$6517·11 (bought). (4) London remit.
2. £, through Paris. \$, London remit.

## PAGE 359 (China).

1. 5s. 11d. 2. 3s. 10½d. nearly. 3. 4s. 0½d. nearly.

## PAGE 360 (China).

1. (1) £679·652 (bought). (2) Shanghai remit.
2. £, through Paris. T., London remit.

## PAGE 363 (Japan).

1. (1) Y. 4210·38 (bought). (2) £192·745 (sold).
2. £, Yokohama remit. Y., through Hongkong.

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